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Excellence With Respect To The Parameter β_e of A Graph

KEYWORDS : Equivalence set, Equivalence graph, β_e -excellent, rigid β_e-excellent.

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ABSTRACT

Let $G=(V,E)$ be a simple finite undirected graph. A subset S of V is called an equivalence set if every component of the induced sub graph $\langle S\rangle$ is complete. An equivalence number $\beta_e(G)$ is the maximum cardinality of an equivalence set of G [3]. A vertex u in V(G) is said to be β_e-good if u belongs to a β_e set of G. G is said to be β_e-excellent if every vertex of G is β_e-good. G is said to be rigid β_e-excellent if every vertex of G is contained in a unique β_e-set of G. An equivalence graph is a vertex disjoint union of complete graphs. The concept of equivalence set, sub chromatic number, generalized coloring and equivalence covering number were studied in [1],[2],[4],[5],[6],[8],[10]. In this paper the concept of β_e-excellence and rigid β_e-excellence are studied.

1. Introduction.

Gred.H. Fricke et al [7] called a vertex u of a graph G = (V,E) to be μ-good if u is contained in a $\mu(G)$-set of G(where μ is a parameter). G is said to be μ-excellent if every vertex in V is μ-good. A number of results has been proved by taking μ as the domination parameter. Sridharan and Yamuna [12], [13] introduced several types of excellence, one of them being rigid excellence. A graph G is said to be rigid μ-excellent if every vertex of G belongs to a unique μ-set of G. Rigid μ-excellence was studied in [14]. A similar study was made with respect to the parameter β_0 in [11]. A sub set S of V(G) is said to be an equivalence set if every component of $\langle S\rangle$ is complete. A graph G is said to be an equivalence graph if V(G) is an equivalence set. The maximum cardinality of an equivalence set is denoted by $\beta_e(G)$ [3]. In this paper, excellence with respect to $\beta_e(G)$ is introduced, rigid β_e-excellence is defined and several results are derived.

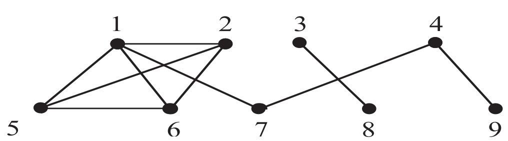
2. β_e -Excellence of a Graph .

Definition 2.1. A vertex u in V(G) is said to be β_e-good if u is contained in a β_e-set of G.

Definition 2.2. A graph G is said to be a β_e-excellent graph if every vertex of G is contained in a β_e-set of G (i.e., every vertex of G is β_e-good).

Example 2.3. K_n, $\overline{K}_n$, C_n are some β_e-excellent graph.

Example 2.4. Consider the graph G in Figure 2.1.



G

A graph which is not β_e-excellent

Volume : 6 | Issue : 10 | October 2016 | ISSN - 2249-555X | IF : 3.919 | IC Value : 74.50

S={1,2,5,6,3,8,4,9} is a maximum equivalence set. Therefore $\beta_e(G)=8$.

{2,5,6,7,8,9,3} is a maximal equivalence set containing 7 and it is not a β_e-set. Therefore 7 is not β_e-good..

Therefore G is not a β_e-excellent graph.

Example 2.5. $K_{1,n}, n \geq 2$ is not β_e-excellent.

β_e values for some standard graphs

1. $$\beta_e(P_n) = 2k \text{ if } n = 3k, \quad = 2k+1 \text{ if } n = 3k+1, \quad = 2k+2 \text{ if } n = 3k+2$$

2. $$\beta_e(C_n) = 2k \text{ if } n = 3k, k \geq 2, \quad = 2k \text{ if } n = 3k+1, k \geq 1, \quad = 2k+1 \text{ if } n = 3k+2, k \geq 1, \quad = 3 \text{ if } n = 3$$

3. $\beta_e(K_{1,n}) = n$

4. $\beta_e(K_{m,n}) = \max(m,n)$

5. $\beta_e(W_n) = \beta_e(C_{n-1}),\ n \geq 6$
$= 3 \text{ if } n = 5$
$= 4 \text{ if } n = 4$

6. $D_{r,s}$ is β_e-excellent iff r = s.

7. $\beta_e(D_{r,s}) = r + s$ and $D_{r,s}$ is not β_e-excellent

8. $\beta_e(P) = 5$ where P is the Petersen graph.

   Petersen graph P is β_e-excellent.

Remark 2.6.

1. $K_{1,n}$ is not an equivalence graph. But there exists a unique β_e-set.
2. $K_{m,n}$ is not an equivalence graph. But there exists a unique β_e-set.

β_e **- Excellence for standard graphs**

1. K_n is β_e - excellent.

2. $\overline{K}_n$ is β_e - excellent.

3. C_n is β_e - excellent.

4. $K_{m,n}$ is β_e - excellent iff m = n

5. P_n is β_e - excellent iff $n \equiv 0,1 \pmod 3$

6. W_n is not β_e - excellent, since W_n has a full degree vertex and W_n is not complete.

3. **Rigid β_e-Excellence of a Graph .**

Definition 3.1. A graph G is rigid β_e-excellent if every vertex of G belongs to a unique β_e - set of G.

Clearly a rigid β_e-excellent graph is β_e-excellent.

Example 3.2.

1. K_n is rigid β_e-excellent.

2. C_6 is β_e-excellent but not rigid β_e-excellent.

For: Let $V(C_6)=\{u_1,u_2,...,u_6\}$. $S_1=\{u_1,u_2,u_4,u_5\}$ and $S_2=\{u_2,u_3,u_5,u_6\}$ are β_e-sets and u_2 belong to S_1 and S_2.

Rigid β_e-excellence of standard graphs:

1. K_n is rigid β_e-excellent.

2. $K_{1,n}$ is not β_e-excellent and hence not rigid β_e-excellent.

3. P_n is not rigid β_e-excellent for all $n \geq 3$.

Proof.

Case i: $n \equiv 0 (\bmod 3)$

Let $V(P_n)=\{v_1,v_2,...,v_{3k}\}$ where n = 3k.

Let $S_1=\{v_1,v_2,v_4,v_5,...,v_{3k-2},v_{3k-1}\}$, $S_2=\{v_1,v_3,v_4,v_6,v_7,...,v_{3k-3},v_{3k-2},v_{3k}\}$. Both S_1 and S_2 are β_e sets of cardinality 2k and v_1 is the common element for S_1 and S_2. Therefore P_{3k} is not rigid β_e-excellent.

Case ii: $n \equiv 1 (\bmod 3)$

Let n = 3k+1. Let where n = 3k+1. $\beta_e(P_{3k+1}) = 2k+1$.

Let $S_1=\{v_1,v_2,v_4,v_5,...,v_{3k-2},v_{3k-1},v_{3k+1}\}$ and $S_2=\{v_1,v_3,v_4,v_6,v_7,...,v_{3k},v_{3k+1}\}$ are β_e-excellent sets of P_{3k+1} containing u_1 therefore P_{3k+1} is not rigid β_e-excellent.

Case iii: $n \equiv 2 (\bmod 3)$ $V(P_n)=\{v_1,v_2,...,v_{3k+1}\}$

$\beta_e(P_{3k+2}) = 2k+2$.

Let $S=\{v_1,v_2,v_4,v_5,...,v_{3k+1},v_{3k+2}\}$. Then S_1 is a β_e-set of P_{3k+2} of cardinality 2k+2.

Consider v_3. There are two equivalent sets of maximum cardinality 2k+1 containing v_3 and they are $\{v_1, v_3, v_4, v_6, v_7, ..., v_{3k}, v_{3k+1}\}$; $\{v_2, v_3, v_5, v_6, .., v_{3k-1}, v_{3k}, v_{3k+2}\}$. Therefore v_3 is not contained in any β_e -set of G. Therefore P_{3k+2} is not β_e-excellent and hence not rigid β_e-excellent. Therefore P_n is not rigid β_e-excellent for any $n \geq 3$. (P_2 is rigid β_e-excellent).

4. C_n is not rigid β_e-excellent for all $n \geq 4$ and C_3 is rigid β_e-excellent.

Proof. Let $n = 3k, k \geq 2$.

$\beta_e(C_{3k}) = 2k$.

Let $V(C_{3k}) = \{v_1, v_2, ..., v_{3k}\}$. $S_1 = \{v_1, v_2, v_4, v_5, ..., v_{3k-2}, v_{3k-1}\}$ and $S_2 = \{v_{3k} v_1, v_3, v_4, ..., v_{3k-3}, v_{3k-2}\}$ are β_e-sets of C_{3k}($k \geq 2$) containing v_1. Therefore C_{3k} is not β_e-excellent when $k \geq 2$.

Let n = 3k+1. $\beta_e(C_{3k+1}) = 2k (k \geq 1)$.

Let $S_1 = \{v_1, v_3, v_4, v_5, ..., v_{3k-1}, v_{3k}\}, S_2 = \{v_2, v_4, v_5, v_7, v_8, ..., v_{3k-1}, v_{3k}\}$. Then S_1 and S_2 are β_e-excellent sets of C_{3k+1} containing v_4. Therefore C_{3k+1} is not rigid β_e-excellent.

Case iii: Let n =3k+2.

$\beta_e(C_{3k+2}) = 2k+1$.

Let $S_1 = \{v_1, v_2, v_4, v_5, ..., v_{3k-2}, v_{3k-1}, v_{3k+1}\}, S_2 = \{v_{3k+2}, v_1, v_3, v_4, v_6, v_7, ..., v_{3k-3}, v_{3k-2}, v_{3k}\}$. Then S_1 and S_2 are β_e sets of C_{3k+2} containing v_4. Therefore C_{3k+2} is not rigid β_e-excellent for $k \geq 1$.

5. $K_{m,n}$ is rigid β_e-excellent iff m = n.

6. $D_{r,s}$ is not rigid β_e-excellent.

Proof. $D_{r,s}$ is not even β_e-excellent.

7. W_n is not rigid β_e-excellent when $n \geq 5$.

8. The Petersen graph P is not rigid β_e-excellent.

Volume : 6 | Issue : 10 | October 2016 | ISSN - 2249-555X | IF : 3.919 | IC Value : 74.50

Theorem 3.3. A graph G is rigid β_e - excellent iff

i). $\beta_e(G)$ divides n.

ii). G has exactly $\frac{n}{\beta_e(G)}$ distinct $\beta_e(G)$ sets.

iii). The maximum cardinality of a partition V(G) into equivalence sets is $n/\beta_e(G)$.

Proof. Let G be rigid β_e -excellent. Let $S_1,S_2,...,S_k$ be the collection of disjoint β_e -sets of G. Since G is rigid β_e - excellent, these sets are pair wise disjoint and their union is V(G). Therefore (i),(ii) and (iii) hold.

Conversely, let G be a graph satisfying conditions (i), (ii) and (iii).

Let $n = m\beta_{e(G)}$. By condition (iii), there exists β_e -set $V_1,V_2,...,V_m$ such that they are pair wise disjoint and $V_1 \cup V_2 \cup ... \cup V_m = V$. Therefore $n = \sum_{i=1}^{m}|V_i| \le m\beta_e(G)$. Since $m = \beta_e(G)$, each V_i is a maximum equivalence set of G. Therefore V is a pair wise disjoint union of β_e -sets. Therefore G is β_e -excellent. Since G has exactly $n/\beta_e(G)(= m)$ distinct β_e -sets, $V_1,V_2,...,V_m$ are the only β_e -sets of G. Therefore G is rigid β_e -excellent.

Property 3.4 . If $G \ne K_1$ is rigid β_e - excellent then for any u in V(G) $\langle V - N[u]\rangle$ does not contain two β_e -sets.

Proof.

Case i: N[u] is non complete.

Since G is rigid β_e - excellent, given u in V(G), there exists a unique β_e -set of G containing u. Suppose V-N[u] contains two maximum β_e - sets. Since $G \ne K_1, \beta_e(G) \ge 2$.Let S be the unique β_e -set of G containing u. Then S-{u} is an equivalence set of G of cardinality $\beta_e - 1$. Since N[u] is not complete, S-{u} is a subset of V-N[u]. Therefore V-N[u] contains an equivalence set of cardinality $\beta_e(G) - 1$. If

$\beta_e\langle V-N[u]\rangle=\beta_e(G)$ then any β_e set of $\langle V-N[u]\rangle$ together with u is an equivalence set of G of cardinality $\beta_e(G)+1$, a contradiction. Therefore $\beta_e\langle V-N[u]\rangle=\beta_e(G)-1$.

Let T_1, T_2 be two maximum equivalence sets of V-N[u]. Then $T_1\cup\{u\}$ and $T_2\cup\{u\}$ are sets of G. Therefore $T_1\cup\{u\}$ and $T_2\cup\{u\}$ are equivalence sets of cardinality $\beta_e(G)$ and these sets containing u, a contradiction. Therefore $\langle V-N[u]\rangle$ does not two maximum equivalence sets.

Case ii: N[u] is complete.

Suppose V-N[u] contains two maximum equivalence sets say T_1, T_2. Then $T_1\cup N[u]$ and w are equivalence sets of G. Since G is rigid β_e -excellent, there exists a unique β_e set S of G containing u. Since N[u] is complete, N[u] is a component of S. Therefore S-N[u] is an equivalence set of V-N[u]. Suppose S-N[u] is not a maximum equivalence set of V-N[u]. Therefore

$$|S-N[u]|\prec\beta_e(\langle V-N[u]\rangle)$$

$$|T_1|=\beta_e(\langle V-N[u]\rangle)$$

Therefore $|T_1\cup N[u]|=\beta_e(\langle V-N[u]\rangle)+|N[u]\succ|S-N[u]|+|N[u]|=|S|$.

$|T_1\cup N[u]|\succ\beta_e(G)$, a contradiction.

Therefore S-N[u] is a maximum equivalence set of V-N[u]. Therefore $|S-N[u]|=|T_1|=|T_2|$. Therefore $|S|=|T_1\cup N[u]|=|T_2\cup N[u]|=\beta_e$.

Therefore $T_1\cup N[u]$ and $T_2\cup N[u]$ are maximum equivalence sets of G containing u, a contradiction. Therefore V-N[u] doses not contain two maximum equivalence sets.

Proposition 3.5. Let G be rigid β_e excellent. Then there exists a unique partition of V(G) into β_e sets of G.

Proof.

By Property (i) G has exactly $\frac{n}{\beta_e(G)}$ distinct β_e sets. Therefore there exists a partition V(G) into β_e sets of G. Suppose G has two partition of V(G) into β_e sets of G. Let them be $\prod_1 = \{V_1, V_2, ..., V_k\}$, $\prod_2 = \{W_1, W_2, ..., W_k\}$. Then there exists a u in V(G) such that u belongs to V_i and W_j for some i and j, $V_i \neq W_j$. Therefore there exists two β_e sets containing u, a contradiction. Therefore there exists a unique partition of V(G) into β_e sets of G.

Property 3.6. Let G be a rigid β_e-excellent graph. Then $|V(G)| = \beta_e(G) \cdot \chi_{eq}(G)$.

Proof.

Since G is rigid β_e-excellent, $n = \beta_e(G) \cdot d$ where d is the number of β_e-sets.

Also $\frac{n}{\beta_e(G)} \leq \chi_{eq}(G)$. That is $d \leq \chi_{eq}(G)$. But $\chi_{eq}(G) \leq d$. Therefore $d = \chi_{eq}(G)$.Therefore $|V(G)| = \beta_e(G) \cdot \chi_{eq}(G)$.

Remark 3.7. There are graphs which are not rigid β_e-excellent in which $|V(G)| = \beta_e(G) \cdot \chi_{eq}(G)$.

For example,

$\beta_e(C_4) = 2$; $\chi_{eq}(C_4) = 2$ and hence $|V(C_4)| = \beta_e(C_4) \cdot \chi_{eq}(C_4)$.

But C_4 is not rigid β_e-excellent.

Proposition 3.8. Let G be a rigid β_e-excellent. Then $\delta(G) \geq \frac{n}{\beta_e(G)} - 1$

Proof. Let $\prod = \{V_1, V_2, ..., V_m\}$ be the unique β_e-partition of V(G).

Let $u \in V_i, 1 \leq i \leq m$.

Volume : 6 | Issue : 10 | October 2016 | ISSN - 2249-555X | IF : 3.919 | IC Value : 74.50

If u is not adjacent with any vertex of V_j, $j \neq i$ then $V_j \cup \{u\}$ is an equivalence set, a contradiction, since V_i is a maximum equivalence set of G. Therefore u is adjacent with every V_j, $j \neq i$. Therefore $\deg(u) \geq m - 1 = \frac{n}{\beta_e(G)} - 1$.

Property 3.9. $\frac{n}{\beta_e(G)} = 1$ iff G is an equivalence graph.

Proposition 3.10. Let G be rigid β_e-excellent. Suppose G has two or more disjoint β_e sets. Then G has no isolates.

Proof. Suppose G has two or more disjoint β_e sets. Let $S_1, S_2, \ldots, S_t$ be the disjoint β_e sets of G. Then $t \geq 2$. Let u be an isolate of G. Then u belongs to a β_e set say S_1. Then $S_2 \cup \{u\}$ is an equivalence set of cardinality $\beta_e(G)+1$, a contradiction. Therefore G has no isolates.

Proposition 3.11. Let G be a rigid β_e-excellent graph. Let G be not an equivalence graph. Then $\delta(G) \geq 2$.

Proof. Suppose $u \in V(G)$. Since G is rigid β_e-excellent, G has no isolates. Therefore $\deg(u) \geq 1$. Let v be the support of u. Let S be the unique β_e set of G containing v. Suppose $u \in S$. Then uv is an component of S. If w is an neighbour of v in G then $\langle u, v, w \rangle$ is a connected subset of V which is not complete. Therefore v has no other neighbour in G. Therefore G has K_2 as a component. Therefore G is an equivalence graph, a contradiction.

Suppose $u \notin S$. Since $G \neq K_1, |S| \geq 2$. Therefore there exist $w \in S, w \neq v$. Then $(S - \{v\}) \cup \{u\}$ is an equivalence set of cardinality $\beta_e(G)$. Therefore w is contained in S as well as $(S - \{v\}) \cup \{u\}$ which are β_e sets, a contradiction, since G is rigid β_e-excellent.

Therefore $\deg(u) \geq 2$

Therefore $\delta(G) \geq 2$.

Volume : 6 | Issue : 10 | October 2016 | ISSN - 2249-555X | IF : 3.919 | IC Value : 74.50

Remark 3.12. The above result, need not hold in equivalence graphs For example, $G = K_2 \cup K_n$, $n \geq 2$ is an equivalence graph in which $\delta(G) = 1$.

Proposition 3.13. Let G be a rigid β_e-excellent graph. Suppose G contains K_2 as a component . Then any component of G is complete(i.e., G is an equivalence graph).

Proof. Let G be a rigid β_e-excellent graph. Suppose G contains K_2 as a component. Let G_1 be a component of G other than K_2. Since G is rigid β_e-excellent, G is β_e-excellent. Let u and v be the vertices of the component K_2. Since G is rigid β_e-excellent, there exists a unique β_e set S containing u. Clearly $v \in S$. Therefore $S - \{u, v\}$ is a unique β_e set of G_1. But $S - \{u, v\} = V(G_1)$. Therefore G_1 is an equivalence graph. Therefore G is an equivalence graph.

Remark 3.14.

1. The above proposition is not true if G is β_e-excellent but not rigid β_e-excellent.

For example,

Let $G = K_2 \cup C_4$. Then G is β_e-excellent, G contains K_2 as a component but G is not an equivalence graph.

2. Let $G = K_2 \cup K_{1,n}$; $n \geq 2$.

Then G is not β_e-excellent, G contains a unique β_e set and G is not an equivalence graph.

3. G is an equivalence graph iff G has a unique equivalence set of cardinality n.

4. Any tree G is not a rigid β_e-excellent graph for if G is a tree then G is connected and G is not an equivalence graph. Also $\delta(G) = 1$. Therefore G is not rigid β_e-excellent.

Proposition 3.15. Let G be a disconnected graph. Let $G = G_1 \cup G_2 \cup \ldots \cup G_k$ where each G_i is connected. G is rigid β_e-excellent iff each G_i $(1 \leq i \leq k)$ is rigid β_e-excellent and each G_i has a unique β_e set.

Proof. Suppose $G = G_1 \cup G_2 \cup \ldots \cup G_k$, where each G_i is connected. Let G be rigid β_e-excellent. Let $u \in G_i (1 \leq i \leq k)$. Then $u \in V(G)$ and hence there exists a unique β_e set S of G containing u. Let

Volume : 6 | Issue : 10 | October 2016 | ISSN - 2249-555X | IF : 3.919 | IC Value : 74.50

$S_i = S \cap V(G_i), (1 \le i \le k)$. Let T be a component of S_i. Then T is a component of S_i. Then T is a component of S. Since S is a β_e set, T is complete. Therefore S_i is an equivalence set of $G_i (1 \le i \le k)$. Suppose $|S_i| \prec \beta_e(G_i)$. Let T_i be a β_e set of G_i. Let $S_1 = (S - S_i) \cup T_i$. Clearly S_1 is a equivalence set of G and $|S_1| \succ |S|$, a contradiction, since S is a β_e set of G. Therefore S_i is a β_e set of G_i . Therefore $u \in G_i$ is an element of S_i which is a β_e set of G_i. Suppose u belongs to β_e sets T_1, T_2 of G_i. Then $(S - S_i) \cup T_1$, $(S - S_i) \cup T_2$ are β_e sets of G containing u, a contradiction. Therefore S_i is a unique β_e set of G_i containing u. Therefore G_i is a rigid β_e-excellent.

Suppose G_i has two β_e sets T_3, T_4. Then $T_3 \cap T_{4=} = \phi$. Let S be a β_e set of G. Let $S_i = S \cap V(G_i)$. Then $(S - S_i) \cup T_1$, $(S - S_i) \cup T_2$ are β_e sets of G containing $(S - S_i)$, a contradiction, since G is rigid β_e-excellent. Therefore each G_i has a unique β_e set.

Conversely, Suppose each G_i is rigid β_e-excellent and each G_i has a unique β_e set. Let T_i be the unique β_e set of G_i. Then $\cup T_i$ is a β_e set of G containing every element of V(G). Therefore G is rigid β_e-excellent.

Proposition 3.16. Let G be a connected graph which is rigid β_e-excellent and which has a unique β_e set. Then G is complete.

Proof. By hypothesis, V(G) is a β_e set. Since V(G) is connected, $\langle V(G) \rangle$ is complete. Therefore G is complete.

Remark 3.17. Let $G = G_1 \cup G_2 \cup ... \cup G_k$ where each G_i is connected. Then G is rigid β_e-excellent iff each G_i is complete. That is iff G is an equivalence graph.

Problem 3.18. Find a connected rigid β_e-excellent graph which is neither K_n nor a complete multi partite graph.

Proposition 3.19. Let G be a rigid β_e-excellent graph. Let $u \in V(G)$. Let S be the unique β_e set of G containing u. Then $\langle pn[u,S] \rangle$ is complete and $|pn[u,S]| \le 2$.

Volume : 6 | Issue : 10 | October 2016 | ISSN - 2249-555X | IF : 3.919 | IC Value : 74.50

Proof. Let $x, y \in pn(u,S)$. Then $x, y \in V - S$ and x,y are adjacent with only u in S. Then $(S-\{u\}) \cup \{x,y\}$ is an equivalence set of cardinality $\beta_e(G)+1$, a contradiction. Therefore $|pn(u,S)| \leq 1$. Therefore $pn[u,S]$ is complete and $|pn[u,S]| \leq 2$.

Problem 3.20. Find an example of a rigid β_e excellent graph G such that for some vertex u in V(G), $pn(u,S)=1$ where S is the unique β_e set of G containing u.

Proposition 3.21. Let G be a rigid β_e-excellent and let G be not an equivalence graph except K_n. Then G is connected.

Proof. Let G be a rigid β_e-excellent. Suppose G is disconnected. Then by the Proposition 3.16, every component of G is complete. That is G is an equivalence graph. By hypothesis, G is not an equivalence graph. Therefore G is connected.

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