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Tri Connectedness in Tritopological Space

KEYWORDS: tri connectedness, tri disconnectedness and tri separation.

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ABSTRACT *The aim of this paper is to introduce new type of connectedness in tri topological spaces and also defined separation properties in tri topological spaces.*

1.Introduction

J.C Kelly [5] introduced the concept of bitopological space. W. J. Pervin [8] was define connectedness in a bitopological space. I.L. Reilly [9] , J. Swart [10] and T. Birsan [2] studied connectedness in bitopological spaces . B. Dvalishi [3] studied connectedness in bitopological space. A. Kandil and others [1] studied connectedness in bitopological ordered spaces and in ideal bitopological spaces. Tri topological space is a generalization of bitopological space. The tri topological space was first initiated by Martin Kovar [6] . S. Palanimmal [7] investigated tri toplogical spaces in 2011.N.F. Hameed and Moh. Yahya Abid [4] gives the definition of 123 open set in tri topological spaces In this paper, we introduce tri connectedness and tri separated sets in tri topological space. Here we are using tri open set in place of 123 open set.

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2. Preliminaries

Definition 2.1[7]: Let X be a nonempty set and T_1, T_2 and T_3 are three topologies on X. The set X together with three topologies is called a tri topological space and is denoted by (X, T_1, T_2, T_3)

Definition 2.2[10]: A subset A of a topological space X is called 123 open set if $A \in T_1 \cup T_2 \cup T_3$ and complement of 123 open set is 123 closed set.

Definition 2.4[8]: A bitopological space is (X, T_1, T_2) said to be connected if and only if X cannot be expressed as the union of two non empty disjoint sets A and B such that A is T_1 open and B is T_2 open. When X can be so expressed ,we write $X = A/B$ and call this a separation of X.

3. tri separated sets in tritopological space

Definition 3.1: Let (X, T_1, T_2, T_3) be a tritopological space, two non empty subsets A and B of X are said to be tri separated if and only if $A \cap tri\,cl(B) = \phi$ and $tri\,cl(A) \cap B = \phi$. These two conditions are equivalent to the single condition

$[A \cap tri\,cl(B)] \cup [tri\,cl(A) \cap B] = \phi$

Example 3.2 : Let, $X = \{a, b, c, d\}$, $T_1 = \{X, \phi, \{a, c\}, \{d\}, \{a, c, d\}\}$, $T_2 = \{X, \phi, \{b, c\}, \{c\}, \{a, c, d\}\}$, $T_3 = \{X, \phi, \{b\}\}$

Open sets in tri topological spaces are union of all three topologies.

tri open sets of X

$=\{X,\phi,\{a,c\},\{b\},\{c\}\{d\},\{b,d\},\{a,b,c\},\{a,d,c\}\}$.

If we take $\{b\}$ and $\{d\}$ Then

$[A\cap tri\,cl(B)]\cup[tri\,cl(A)\cap B]=\{\{b\}\cap\{d\}\}\cup\{\{b\}\cap\{d\}\}=\phi$.

Hence $\{b\}$ and $\{d\}$ are tri separated sets .

Also $\{a,c\}$ & $\{b,d\}$ etc. are tri separated sets.

Theorem 3.3 : If A and B are tri separated subsets of a tritopological space (X,T_1,T_2,T_3) and $C\subset A$ and $D\subset B$, then C and D are also tri separated .

Proof: Since A and B are tri separated sets then $A\cap tri\,cl(B)=\phi$ and $tri\,cl(A)\cap B=\phi$ ....(1)

Also $C\subset A\Rightarrow tri\,cl(C)\subset tri\,cl(A)$ and

$D\subset B\Rightarrow tri\,cl(D)\subset tri\,cl(B)$ ....(2)

By (1) and (2) that $C\cap tri\,cl(D)=\phi$ and $tri\,cl(C)\cap D=\phi$

Hence C and D are tri separated.

Theorem 3.4 : Two tri closed (tri open) subsets A , B of a tritopological space (X,T_1,T_2,T_3) are tri separated if and only if they are disjoint .

Proof : Since any two tri separated sets are disjoint, we have to prove that two disjoint tri closed (tri open) sets are tri separated .

If A and B are both disjoint tri closed , then :

$A\cap B=C,\ tri\,cl(A)=A$ and $tri\,cl(B)=B$ .....[1]

So that $tri\,cl(A)\cap B=\phi$ and $A\cap tri\,cl(B)=\phi$

Therefore A and B are tri separated.

If A and B are both disjoint and tri open , then A^c and B^c are both tri closed so that :

$tri\,cl(A^c)=A^c$ and $tri\,cl(B^c)=B^c$

Also $A\cap B=\phi\Rightarrow A\subset B^c$ and $B^c\subset A$

$\Rightarrow tri\,cl(A)\subset tri\;cl(B^c)=B^c$ and $tri\,cl(B)\subset tri\;cl(A^c)=A^c$

$\Rightarrow tri\,cl(A)\cap B=\phi$ and $A\cap tri\,cl(B)=\phi$

Hence $\Rightarrow A$ and B are tri separated.

4. tri connected and tri disconnected sets in tritopological space

Definition 4.1 : Let (X,T_1,T_2,T_3) be a tritopological space , a subset A of X is said to be tri disconnected if and only if it is the union of two non empty tri separated sets . That is , if and only if there exist two non empty separated sets C and D such that $C\cap tri\,cl(D)=\phi$, $tri\,cl(C)\cap D=\phi$ and $A=C\cup D$, A is said to be tri connected if and only if it is not tri disconnected.

Example 4.2 : Let, $X=\{a,b,c,d\}$, $T_1=\{X,\phi,\{a\},\{b\},\{a,b\}\}$, $T_2=\{X,\phi,\{a\},\{c,d\},\{a,c,d\}\}$, $T_3=\{X,\phi,\{a\},\{b,c,d\}\}$.

Open sets in tri topological spaces are union of all three topologies.

tri open sets of $X=\{X,\phi,\{a\},\{b\},\{c,d\},\{a,b\},\{a,c,d\},\{b,c,d\}\}$.

If we take $A = \{a, c, d\}$, $C = \{a\}$ and $D = \{c, d\}$ Then

$A = C \cup D$ and $\{a\} \cap \{c, d\} = \phi$ and

$C \cap tri\, cl(D) = \{a\} \cap \{c, d\} = \phi$, $tri\, cl(C) \cap D = \{a\} \cap \{c, d\} = \phi$.

Then $\{a\}$ and $\{c, d\}$ are tri separated sets.

Hence the set $\{a, c, d\}$ is tri disconnected.

But $\{a\}, \{b\}, \{c, d\}$ are tri connected sets.

Remarks 4.3 :

(i)The empty set in $tri\, O(X)$ is trivially tri connected.

(ii) Every singleton set in $tri\, O(X)$ is tri connected since it cannot be expressed as a union of two non empty tri separated sets.

Definition 4.4: Two points $\{a\}$ and $\{c, d\}$ of a tritopological space X are said to be tri connected if and only if they are contained in a tri connected subset of X.

Example 4.5 : In the last example the points $\{a\}$ and $\{c, d\}$ are tri connected because they are contained in $\{a, c, d\}$ which is tri connected subset of X.

Theorem 4.6 : A tri topological space (X, T_1, T_2, T_3) is tri disconnected if and only if there exists a non empty proper subset of X which is both tri open and tri closed in X.

Proof : Let A be a non empty proper subset of X which is both tri open and tri closed in X . We have to show that X is tri disconnected :

Let $B = A^c$. Then B is non empty since A is a proper subset of X .Moreover , $A \cup B = X$ and $A \cap B = \phi$, since A is both tri open and tri closed , B is also both tri open and tri closed .

Hence $tri\, cl(A) = A$ and $tri\, cl(B) = B$ it follows that $tri\, cl(A) \cap B = \phi$ and $A \cap tri\, cl(B) = \phi$. Thus X has been expressed as a union of two tri separated sets and so X is tri disconnected .

Conversely ; Let X is tri disconnected . Then there exist non empty subsets A and B of X such that $tri\, cl(A) \cap B = \phi$ and $A \cap tri\, cl(B) = \phi$ and $A \cup B = X$. Since $tri\, cl(A) = A$ and $tri\, cl(B) = B \Rightarrow A \cap B = \phi.$

Hence $A = B^C$ and B is non empty , A is a proper subset of X .

Now $A \cup tri\, cl(B) = X$.

[since $A \cup B = X$ and $B \subset tri\, cl(B) \Rightarrow X \subset A \cup tri\, cl(B)$ but $A \cup tri\, cl(B) \subset X$ always]

Also $A \cap tri\, cl(B) = \phi \Rightarrow A = (tri\, cl(B))^C$ and similarly $B = (tri\, cl(A))^C$.

Since $tri\, cl(A)$ and $tri\, cl(B)$ are tri closed sets, it follows that A and B are tri open sets , and since $A = B^C$, A is also tri closed . Thus A is non empty proper subset of X which is both tri open and tri closed .

In the same way we can show that B is also non empty proper subset of X which is both tri open and tri closed.

Theorem 4.7: Let (X,T_1,T_2,T_3) be a tri topological space and A and B be non empty sets in space X.

(i) If A and B are tri separated and $A_1 \subset A$ and $B_1 \subset B$,then A_1 and B_1 are so.

(ii) If $A \cap B = \phi$ such that each of A and B are both tri open (tri closed) ,then A and B are

separated.

(iii) If each of A and B both tri open (tri closed) and if $H = A \cap (X - B)$ and $G = B \cap (X - A)$, then H and G .

Proof : (i) Since $A_1 \subset A$ then $tri\,clA_1 \subset tri\,clA$ .Then $B \cap tri\,cl(A) = \phi$ implies $B_1 \cap tri\,cl(A) = \phi$ and $B_1 \cap tri\,cl(A_1) = \phi$ .Similarly $A_1 \cap tri\,cl(B_1) = \phi$ .Hence A_1 and B_1 are tri separated.

(ii) Since $A = tri\,cl(A)$ and $B = tri\,cl(B)$ and $A \cap B = \phi$,then $tri\,cl(A) \cap B = \phi$ and $tri\,cl(B) \cap A = \phi$ .Hence A and B are separated. If A and B are tri open , then their complements are tri closed.

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(iii) If A and B are tri open , then $X-A$ and $X-B$ are tri closed .Since $H \subset X-B$, $tri\,cl(H) \subset tri\,cl(X-B) = X-B$ and so $tri\,cl(H) \cap B = \phi$.Thus $G \cap tri\,cl(H) = \phi$.Similarly , $H \cap tri\,cl(B) = \phi$.Hence H and G are tri separated .

Theorem 4.8: Let (X, T_1, T_2, T_3) be a tri topological space and $E \subset X$.If E is tri connected, then so is $tri\,cl\,(E)$.

Proof : Let E be the tri connected subset of a tri topological space (X, T_1, T_2, T_3). To prove that $tri\,cl\,(E)$ is connected .Suppose contrary, Then $tri\,cl\,(E)$ is disconnected .Then their exist non empty sets $A, B \subset X$ such that $\overline{A} \cap B = \phi,\ A \cap \overline{B} = \phi$.

$tri\,cl\,E = A \cup B$

$A \cup B = tri\,cl\,E \supset E,$

$\Rightarrow E \subset A \cup B,\ E$ is connected .

$\Rightarrow E \subset A$ or $E \subset B$ {by theorem 4.8}

$E \subset A \Rightarrow tri\,cl\,E \subset tri\,cl\,A$

$\Rightarrow E \subset A \cup B,$

$\Rightarrow tri\,cl\,E \cap B \subset tri\,cl\,A \cap B = \phi$ .....(i)

$tri\,cl\,E = A \cup B$

$\Rightarrow B \subset tri\,cl\,E$

$\Rightarrow tri\,cl\,E \cap B = B$

$\Rightarrow B = \phi$

For $tri\,cl\,E \cap B = \phi$ (from (i))

H in X such that *tricl* $(E) = G \cup H$ .Since $E = (G \cap E) \cup (H \cap E)$ and $cl(G \cap E) \subset cl(G)$ and $cl(H \cap E) \subset cl(H)$ and $G \cap H = \phi$ then $(cl(G \cap E)) \cap H = \phi$. Hence $(cl(G \cap E)) \cap (H \cap E) = \phi$ .Similarly $(cl(H \cap E)) \cap (G \cap E) = \phi$ .Therefore E is connected a contradiction for $B \neq \phi$ similarly $E \subset B \Rightarrow A = \phi$ .Again we get a contradiction .Hence , If E is tri connected, then so is $tri\, cl\,(E)$.

Corollary 4.9: A tritopological space (X, T_1, T_2, T_3) is tri connected if and only if the only non empty subset of X which is both tri open and tri closed in X is X itself .

Theorem 4.10: A tri topological space (X, T_1, T_2, T_3) is tri disconnected if and only if any one of the following statements holds :

(i) X is the union of two non empty disjoint tri open sets .

(ii) X is the union of two non empty disjoint tri closed sets .

Proof : Let X be a tri disconnected . Then there exists a nonempty proper subset A of X which is both tri open and tri closed . Then A^C is also both tri open and tri closed also $A \cup A^C = X$. Hence the sets A and A^C satisfy the requirements of (i) and (ii) .

Conversely ; let $X = A \cup B$ and $A \cap B = \phi$, where A, B are non empty tri open sets . It follows that $A = B^C$ so that A is tri closed . Since B is non empty , A is a proper subset of X . Thus A is a non empty proper subset of X which is both tri open and tri closed . Hence by the theorem (4.6) , X is tri disconnected .

Again , let $X = C \cup D$ and $C \cap D = \phi$, where C , D are non empty tri closed sets . then

$C = D^C$ so that C is tri open . Since D is non empty , C is a proper subset of X which is both tri open and tri closed . Hence X is tri disconnected by the theorem .

Thus it is shown that if any one of the conditions (i) and (ii) holds , then X is tri disconnected .

Corollary 4.11: A subset Y of a tri topological space X is tri disconnected if and only if Y is the union of two non empty disjoint sets both tri open (tri closed) in Y .

Theorem 4.12: Let (X, T_1, T_2, T_3) be a tri topological space .If A is a tri connected set of X and H, G are tri separated sets of X with $A \subset H \cup G$, then either $A \subset H$ or $A \subset G$.

Proof : Let $A \subset H \cup G$ .Since $A = (A \cap H) \cup (A \cap G)$, then $(A \cap G) \cap tri\, cl(A \cap H) \subset G \cap cl\,(H) = \phi$. By similar reasoning .we have $(A \cap H) \cap tri\, cl(A \cap G) \subset H \cap cl\,(G) = \phi$ .Suppose that

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$A \cap H$ and $A \cap G$ are nonempty. Then A is not tri connected. This is a contradiction. Thus either $A \cap H = \phi$ or $A \cap G = \phi$ .This implies that $A \subset H$ or $A \subset G$.

Theorem 4.13: Let (X, T_1, T_2, T_3) and (Y, T_1', T_2', T_3') are two tri topological space .Let $f : X \to Y$ be a continuous function. if M is tri connected in X ,then $f(M)$ is tri connected in Y .

Proof : Suppose that $f(M)$ is tri disconnected in Y .There exist two tri separated sets P and Q of Y such that $f(M) = P \cup Q$. Set $A = M \cap f^{-1}(P)$ and $B = M \cap f^{-1}(Q)$. Since $f(M) \cap P \neq \phi$ then $M \cap f^{-1}(P) \neq \phi$ and so $A \neq \phi$. Similarly $B \neq \phi$. Since $P \cap Q = \phi$, $A \cap B = M \cap f^{-1}(P \cap Q) = \phi$ and so $A \cap B = \phi$ .Since f is continuous then by Lemma **4.7** , $tri\,cl(f^{-1}(Q)) \subset f^{-1}(tri\,cl(Q))$ and $B \subset f^{-1}(Q)$ then $tri\,cl(B) \subset f^{-1}(tri\,cl(Q))$ .Since $P \cap tri\,cl(Q) = \phi$, then $A \cap f^{-1}(tri\ cl(Q)) \subset f^{-1}(P) \cap f^{-1}(tri\ cl(Q)) = \phi$ and then $A \cap tri\ cl(B) = \phi$. Thus A and B are tri separated.

Theorem 4.14 : If A is a tri connected set of an tri topological space (X, T_1, T_2, T_3) and $A \subset B \subset tri\,cl(A)$ then B tri connected.

Proof: Suppose B is not tri connected. There exist tri separated sets U and V of X such that $B = U \cup V$ .This implies that U and V are nonempty and $tri\,cl(U) \cap V = U \cap cl(V) = \phi$. By Theorem 4.12, we have either $A \subset U$ or $A \subset V$. Suppose that $A \subset U$. Then $cl(A) \subset cl(U)$ and $V \cap cl(A) = \phi$ .This implies that $V \subset B \subset cl(A)$ and $V = cl(A) \cap V = \phi$. Thus, V is an empty set for if V is nonempty, this is a contradiction. Suppose that $A \subset V$. By similar way, it follows that U is empty. This is a contradiction. Hence, B is tri connected.

Theorem 4.15 : If $\{M_i : i \in N\}$ is a nonempty family of tri connected sets of an tri topological space (X, T_1, T_2, T_3) with $\bigcap_{i \in I} Mi \neq \phi$ Then $\bigcup_{i \in I} Mi$ is tri connected.

Proof: Suppose that $\bigcup_{i \in I} Mi$ is not tri connected. Then we have $\bigcup_{i \in I} Mi = H \cup G$, where H and G are tri separated sets in X. Since $\bigcap_{i \in I} Mi \neq \phi$ we have a point x in $\bigcap_{i \in I} Mi$ Since $x \in \bigcup_{i \in I} Mi$, either $x \in H$ or $x \in G$. Suppose that $x \in H$. Since $x \in Mi$ for each $i \in N$, then Mi and H intersect for each $i \in N$. By theorem 4.12; $Mi \subset H$ or $Mi \subset G$.Since H and G are disjoint, $Mi \subset H$ for all $i \in Z$ and hence $\bigcup_{i \in I} Mi \subset H$. This implies that G is empty. This is a contradiction. Suppose that $x \in G$. By similar way, we have that H is empty. This is a contradiction. Thus, $\bigcup_{i \in I} Mi$ is tri connected.

Theorem 4.16: If A and B are tri connected sets which are tri separated ,then $A \cup B$ is tri connected.

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Proof: Suppose $A \cup B$ is tri disconnected and suppose $G \cup H$ is a tri disconnection of $A \cup B$ .Since A is a tri connected subsets of $A \cup B$,either $A \subset G$ or $A \subset H$ .Similarly either $B \subset G$ or $B \subset H$.

If $A \subset G$ and $B \subset H$ then $(A \cup B) \cap G = G$ and $(A \cup B) \cap H = H$ are tri separated but this contradicts the hypothesis. Hence either $A \cup B \subset G$ or $A \cup B \subset B$ and so $G \cup H$ is not a tri disconnection of $A \cup B$ .In other words $A \cup B$ is tri connected.

REFERENCES

[1] A. Kandil , O. Tantwauy ,S.A. El. Sheikh & M. Hosny . (2014), Gen. Math. Notes ,Vol 24, No.2, pp 37-52. ICSRS Publication.

[2] Birsan T.,(1968) , Surles espaces bitopological connexes, An sti. Univ. Al. cuza Iasi sect. I. t. XIV fase 2 ,293-96.

[3] Dvalishi B. (2004) .connectedness of a fine topology and localization in bitopological spaces, gergian. Math. J.,11 (4) , pp 713 -732.

[4] Hameed N.F. & Mohammed Yahya Abid (2011).Certain types of separation axioms in tri topological spaces. Iraqi journal of science. Vol 52,(2),212-217.

[5] Kelly J.C. ,(1963), Bitopological spaces, Proc.LondonMath.Soc.,3 PP. 17-89 .

[6] Kovar, M.,(2000).On 3-Topological version of Thet- Reularity. Internat. J. Matj, Sci. ,23 (6), 393-398.

[7] Palaniammal S. ,(2011).Study of Quad topological spaces, Ph.D Thesis.

[8] Pervin. W.J.(1967), Connectedness in bitopological spaces, Indag Math. ,29, 369-372.

[9] Reilly I.L.. (1971), On pairwise connected bitopological spaces, Kyungpook Math. J., 11(1) 25-28.

[10] Swart J. (1971), Total disconectedness in bitopological spaces and product bitopological spaces, Indag. Math.33,135-145.