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Equivalence Dominator Coloring of A Graph

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N. Saradha
Assistant Professor, Department of Mathematics, SCSVMV University, Enathur, Kanchipuram, Tamilnadu, India

V.Swaminathanb
Coordinator, Ramanujan Research Center in Mathematics, Saraswathi Narayanan College, Madurai, Tamilnadu, India.

ABSTRACT *Let be a simple finite undirected graph. A subset S of V is called an equivalence set if every component of the induced sub graph is complete. The concept of equivalence set, sub chromatic number, generalized coloring and equivalence covering number were studied in [1],[2],[4],[6],[7],[10],[15] A partition is called an equivalence color class domination partition if each Vi is an equivalence set of G and for each Vi there exist ui in V(G) such that ui dominates Vi, . An equivalence dominator coloring is a partition of V(G) into equivalence classes of G such that each vertex of G dominates some equivalence classes in that partition. Since equivalence classes is a generalization of independence, equivalence dominator coloring generalizes dominator coloring [9] in the sense that every dominator color partition is an equivalence dominator partition. In this paper, the properties of equivalence dominator coloring are derived*

Introduction.

Gera etal [9] introduced dominator coloring in graphs . A dominator coloring of a graph G is a proper coloring in which each vertex of the graph dominates every vertex of some color class. The dominator chromatic number is the minimum number of color classes in a dominator coloring of a graph G. A partition $\Pi = \{V_1, V_2, \ldots V_k\}$ is called an equivalence color class domination partition if each V_i is an equivalence set of G and for each V_i there exist u_i in V(G) such that u_i dominates V_i, $1 \le i \le k$. A partition $\Pi = \{V_1, V_2, \ldots V_k\}$ is called an equivalence dominator coloring of G if each V_i is an equivalence set of G and for any $u \in V(G)$, there exist some V_i , $1 \le i \le k$ such that u dominates V_i. Equivalence dominator coloring is a generalization of dominator coloring. That is every dominator color partition is an equivalence dominator color partition. The minimum cardinality of an equivalence color class domination (equivalence dominator coloring partition) is denoted by $\chi_{ecd}(\chi_{ed})$. In this paper, the properties of equivalence dominator coloring is introduced and several results are derived.

2. Equivalence Dominator Coloring of a graph

Definition 2.1. Let G be a simple finite undirected graph. A partition $\Pi = \{V_1, V_2, \ldots V_k\}$ is called an equivalence color class domination partition if each V_i is an equivalence set of G and for each V_i there exist u_i in V(G) such that u_i dominates V_i, $1 \le i \le k$.

Definition 2.2. A partition $\Pi = \{V_1, V_2, \ldots V_k\}$ is called an equivalence dominator coloring if each V_i is an equivalence set of G and for any $u \in V(G)$, there exist some V_i , $1 \le i \le k$ such that u dominates V_i.

Remark 2.3. Equivalence dominator coloring is a generalization of dominator coloring. That is every dominator color partition is an equivalence dominator color partition.

An equivalence color class domination partition is a generalization of color class domination partition.

Remark 2.4. Let $V = \{u_1, u_2, \ldots u_n\}$. Let $\Pi = \{\{u_1\}, \{u_2\}, \ldots \{u_n\}\}$. Then Π is an equivalence color class domination partition as well as equivalence dominator coloring partition ($\{u_i\}$ is assumed to be dominated by u_i).

Definition 2.5. The minimum cardinality if an equivalence color class domination (equivalence dominator coloring partition) is denoted by $\chi_{ecd}(\chi_{ed})$.

Remark 2.6. $\chi_{ecd} \leq \chi_d$ and $\chi_{ecd} \leq \chi_{cd}$.

χ_{ecd} for standard graphs.

1. $\chi_{ecd}(K_n) = 2$.

2 $\chi_{ecd}(K_{1,n}) = 2$.

3. $\chi_{ecd}(\overline{K_n}) = 1$.

4. $\chi_{ecd}(P_n) = \begin{cases} \frac{n}{2} \text{ if n} \equiv 0 (\text{mod } 4) \\ \frac{\text{n}+1}{2} \text{ if n} \equiv 1,3 (\text{mod } 4) \\ \frac{\text{n}+2}{2} \text{ if n} \equiv 2 (\text{mod } 4) \end{cases}$

5. $\chi_{ecd}(C_n) = \begin{cases} \frac{n}{2} \text{ if n} \equiv 0 (\text{mod } 4) \\ \frac{\text{n}+1}{2} \text{ if n} \equiv 1,3 (\text{mod } 4) \\ \frac{\text{n}+2}{2} \text{ if n} \equiv 2 (\text{mod } 4) \end{cases}$

6. $\chi_{ecd}(W_n) = \begin{cases} 3 \text{ if n} \geq 5 \\ 2 \text{ if n} = 4 \end{cases}$

7. $\chi_{ecd}(K_{m,n}) = 2$

Proposition 2.7. Let G be a graph. Then $\max\{\chi_d(G), \gamma(G)\} \leq \chi_{ed}(G) \leq \chi(G) + \gamma(G)$.The bounds are sharp.

Proof. Since dominator coloring is an equivalence dominator coloring , $\chi_d(G) \leq \chi_{ed}(G)$.

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Let $\prod = \{V_1, V_2, ..., V_K\}$ be a χ_d partition of G. Let $D = \{x_1, x_2, ..., x_K\}$ where $x_i \in V_{i,}\ 1 \le i \le k$.

Let $v \in V / D$. Then v dominates V_i for some i, $1 \le i \le k$.

Since $x_i \in V_i$, v and x_i are adjacent. Therefore x_i dominates v. Therefore D is a dominating set of G.

$\gamma(G) \le |D| = k = \chi_{ed}(G)$.

Therefore $\max\{\chi_d(G), \gamma(G)\} \le \chi_{ed}(G)$.

Let c be a proper coloring of G with $\chi(G)$ colors.

Let D be a minimum dominating set of G. Assign colors $\chi(G)+1$, $\chi(G)+2$,..., $\chi(G)+\gamma(G)$ to the vertices of D. Let c_1 be the new coloring. Let $v \in V(G)$. Then v is adjacent to some vertex of D, say x. Then v dominates {x}. Therefore c_1 is an equivalence dominator coloring. Therefore $\chi_{ed}(G) \le |c_1| = \chi(G) + \gamma(G)$.

In C_4, $\chi_d(C_4) = \chi_{ed}(C_4) = \gamma(C_4) = 2$.

In C_8, $\chi_{ed}(C_8) = 5;\ \chi(C_8) = 2; \gamma(C_8) = 3$.

Thus the bounds are attained.

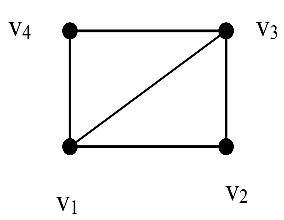
Remark 2.8. In general $\chi(G)$ need not be less than $\chi_{ed}(G)$.

For example,

i). $\chi(K_n) = n$ and $\chi_{ed}(K_n) = 2$.

ii). Let

G=



$\prod = \{\{v_2, v_4\}, \{v_1\}, \{v_3\}\}$.

Then $\prod$ is a chromatic partition. that is $\chi(G) = 3$.

$\prod_1 = \{\{v_1, v_3\}, \{v_2, v_4\}\}$. Then $\prod_1$ is a χ_{ed}-partition of G. Therefore $\chi_{ed}(G) = 2 \prec \chi(G)$.

χ_{ed} for standard graphs:

1. $\chi_{ed}(K_n) = 2$

2. $\chi_{ed}(K_{1,n}) = 2$

3. $\chi_{ed}(\overline{K_n}) = n$

4. $\chi_{ed}(K_{m,n}) = 2$

5. $\chi_{ed}(P_n) = \left\lceil \frac{n}{3} \right\rceil + 1$ for all $n \geq 2$.

Proof. Let $V(P_n) = \{v_1, v_2, ..., v_n\}$.

Let $\prod = \{\{v_2\}, \{v_5\}, \{v_8\}, ... \{v_n\}, \{v_1, v_3, v_4, v_6, v_7, ..., v_{n-1}\}\}$ if $n \equiv 1{,}2 \pmod 3$ and

$\prod = \{\{v_2\}, \{v_5\}, \{v_8\}, ... \{v_{n-1}\}, \{v_1, v_3, v_4, v_6, v_7, ..., v_{n-2}, v_n\}\}$ if $n \equiv 0 \pmod 3$.

Then $\prod$ is a equivalence dominator coloring with $|\prod| = \left\lceil \frac{n}{3} \right\rceil + 1$.

Therefore $\chi_{ed}(P_n) \leq \left\lceil \frac{n}{3} \right\rceil + 1$.

Let $\prod$ be a equivalence dominator coloring of minimum cardinality of P_n. Suppose in $\prod$ there are k singleton classes, l-doubleton classes and r classes with cardinality ≥ 3.

case i. $n \equiv 0 \pmod 3$.

Then $\left\lceil \frac{n}{3} \right\rceil = \frac{n}{3}$.

$k + l + r = \frac{n}{3}$

3(k+l+r)=n

The number of vertices which can dominate at least one color class in $\prod$ is at most $3k + l$. Therefore $n - 2l - 3r$ vertices do not have classes in $\prod$ to dominate. $n - 2l - 3r \geq 1$(since $n - 2l - 3r = 0$ implies $n = 2l + 3r$. That is $3k + 3l + 3r = 2l + 3r$, which implies $3k + l = 0$ $\Rightarrow k = 0, 1 = 0$. That is $\prod$ consists only of color classes with cardinality ≥ 3. That is no vertex can dominate any color class of $\prod$, a contradiction.

Therefore $n - 2l - 3r \geq 1$. That is these vertices do not have classes in $\prod$ to dominate, a contradiction.

Therefore $\frac{n}{3}$ classes are not sufficient for equivalence dominator coloring.

That is $|\prod| \geq \frac{n}{3} + 1$.

case ii. $n \equiv 1 \text{ or } 2 \pmod 3$.

Therefore $\left\lceil \frac{n}{3} \right\rceil = \frac{n+1}{3} \text{ or } \frac{n+2}{3}$.

A similar argument as in case (i) will show that $\left\lceil \frac{n}{3} \right\rceil$ classes are not sufficient for equivalence dominator coloring. Therefore $\chi_{ed}(G) \geq \left\lceil \frac{n}{3} \right\rceil + 1$.

Therefore $\chi_{ed}(G) = \left\lceil \frac{n}{3} \right\rceil + 1$.

6. $\chi_{ed}(C_n) = 2 \text{ if } n = 3,4$
$= \left\lceil \frac{n}{3} \right\rceil + 1 \text{ if } n \geq 5$

Proof. Let $V(C_n) = \{v_1, v_2, ..., v_n\}$. Let $\prod = \{\{v_2\}, \{v_5\}, \{v_8\}, ..., \{v_n\}, \{v_1, v_3, .v_4 .., v_{n-1}\}\}$ if $n \equiv 1,2 \pmod 3$ and $\prod = \{\{v_2\}, \{v_5\}, \{v_8\}, ..., \{v_{n-1}\}, \{v_1, v_3, .v_4 .., v_{n-2,}, v_n\}\}$ if $n \equiv 0 \pmod 3$.

Then $\prod$ is a equivalence dominator coloring .

$|\prod| = \left\lceil \frac{n}{3} \right\rceil + 1.$

Therefore $\chi_{ed}(G) \leq \left\lceil \frac{n}{3} \right\rceil + 1.$

Suppose in $\prod$ there are k singleton color classes, l-doubleton classes and r classes with cardinality ≥ 3.

case i. $n \equiv 0 \pmod 3$.

Then $\left\lceil \frac{n}{3} \right\rceil = \frac{n}{3}$.

$$k + l + r = \frac{n}{3}$$

3(k+l+r)=n

The number of vertices which can dominate at least one color class in $\prod$ is $3k + l$. Therefore $n - 2l - 3r$ vertices do not have classes in $\prod$ to dominate. $n - 2l - 3r \geq 1$(since $n - 2l - 3r = 0$ implies $n = 2l + 3r$. That is $3k + 3l + 3r = 2l + 3r$, which implies $3k + l = 0$ $\Rightarrow k = 0, 1 = 0$. That is $\prod$ consists only of color classes with cardinality ≥ 3. That is no vertex can dominate any color class of $\prod$, a contradiction.

Therefore $n-2l-3r \geq 1$. That is these vertices do not have classes in $\prod$ to dominate, a contradiction.

Therefore $\frac{n}{3}$ classes are not sufficient for equivalence dominator coloring.

That is $|\prod| \geq \frac{n}{3}+1$.

case ii. $n \equiv 1 \text{ or } 2 \pmod 3$.

Therefore $\left\lceil \frac{n}{3} \right\rceil = \frac{n+1}{3}$ or $\frac{n+2}{3}$.

A similar argument as in case (i) will show that $\left\lceil \frac{n}{3} \right\rceil$ classes are not sufficient for equivalence dominator coloring. Therefore $\chi_{ed}(G) \geq \left\lceil \frac{n}{3} \right\rceil + 1$.

Therefore $\chi_{ed}(G) = \left\lceil \frac{n}{3} \right\rceil + 1$.

7 $\chi_{ed}(W_n) = 3$ if $n \geq 5$
$= 2$ if $n = 4$.

Proof. When n = 4, $W_4 = K_4$ and hence $\chi_{ed}(W_4) = 2$. Let $n \geq 5$. Let $V(W_n) = \{u, v_1, v_2, ..., v_{n-1}\}$ where u is the centre of W_n. Let $\prod = \{\{u\}, \{v_1, v_3, ..., v_{n-2}\}, \{v_2, v_4, ..., v_{n-1}\}\}$ if n is odd, let $\prod = \{\{u\}, \{v_1, v_3, ..., v_{n-1}\}, \{v_2, v_4, ..., v_{n-2}\}\}$ if n is even, Then $\prod$ is an equivalence dominator coloring of W_n. $|\prod| = 3$. Therefore $\chi_{ed}(W_n) \leq 3$. Let $\prod$ be an equivalence dominator color partition of W_n. Suppose $|\prod| \leq 2$. Clearly $|\prod \succ 1|$. Also when n =4 W_4 is K_4 and hence $\chi_{ed}(W_4) = 2$.

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Let $n \geq 5$. If u does not appear as singleton, then the class in $\prod$ containing u is not an equivalence set.(since $n \geq 5$). Therefore {u} appears as a class in $\prod$. If $|\prod| = 2$ then the remaining vertices (a

Therefore $\chi_{ed}(W_n) = 3$

Definition 2.9[15] The multi star graph $K_m(a_1, a_2,..., a_m)$ is formed by joining $a_i \geq 1 (1 \leq i \leq m)$ end vertices to each vertex x_i of a complete graph K_m, with $V(K_m) = \{x_1, x_2,..., x_m\}$.

8. $\chi_{ed}(K_n(a_1, a_2,..., a_n)) = n + 1$.

Let $V(K_n(a_1, a_2,..., a_n)) = \{x_1, x_2,...., x_n, x_{11},..., x_{1a_1}, x_{21}, x_{22},..., x_{2a_2},..., x_{n1}, x_{n2},..., x_{na_n}\}$ where $x_1, x_2,..., x_n$ are the vertices of K_n and $x_{i1}, x_{i2},..., x_{ia_i}$ are the pendant vertices attached with x_i, $1 \leq i \leq n$. Let $\prod = \{\{x_1\}, \{x_2\},..., \{x_n\}, \{x_{11},...., x_{1a_1}, x_{21}, x_{22},..., x_{2a_2},..., x_{n1}, x_{n2},..., x_{na_n}\}\}$. Clearly $\prod$ is an equivalence dominator color partition. Therefore $\chi_{ed}(K_n(a_1, a_2,..., a_m)) \leq |\prod| = n + 1$. Suppose $\prod$ is an equivalence dominator color partition of G. Suppose a color class containing x_i and x_j $(i \neq j)(1 \leq i, j \leq n)$. Then the pendance of x_i as well as that of x_j can not dominate any color class. Therefore all the x_i's have to appear in separate classes . If x_i and x_{ja_k} appear in a color class then the pendance at x_i do not have a color class to dominate. If x_i and x_{ia_k} appear in a color class then the pendance at x_i do not have a color class to dominate. Therefore every x_i appears as a singleton in $\prod$. Therefore $|\prod| \geq n + 1$.

Therefore $\chi_{ed}(K_n(a_1, a_2,..., a_n)) \geq n + 1$.

Therefore $\chi_{ed}(K_n(a_1, a_2,..., a_n)) = n + 1$.

9. $\chi_{ed}(K_{a_1, a_2,..., a_k}) = k$ if $a_i \geq k - 1 \forall i$

$= \max(a_1, a_2,..., a_k) + 1$ if $a_i \leq k - 2 \forall i$

Proof. Let $V_1,V_2,...,V_k$ be the k partite sets. Let $\prod = \{V_1, V_2,..., V_k\}$. Then $\prod$ is an equivalence dominator coloring partition of $K_{a_1,a_2,...a_k}$. Therefore $\chi_{ed}(K_{a_1,a_2,...a_k}) \le k$. Suppose $\max(a_i) \ge k-1$. Then $\chi_{ed}(K_{a_1,a_2,...a_k}) \ge k$. Therefore $\chi_{ed}(K_{a_1,a_2,...a_k}) = k$ if $\max(a_i) \ge k-1$. formed by the ith elements of $V_2,...,V_k$. . $\prod_1$ is an equivalence dominator coloring partition of $K_{a_1,a_2,...a_k}$. Therefore $\chi_{ed}(K_{a_1,a_2,...a_k}) = l+1 \prec k$.

Illustration 2.10.

Let $G = K_{a_1,a_2 a_3,a_4,a_5,a_6}$ where $a_1 = 4,\ a_2 = 3,\ a_3 = 2, a_4 = 3, a_5 = 3, a_6 = 4$.

Then $\chi_{ed}(G) = 5$.

10. Let $G = aK_r$ where a and r are positive integers $a \ge 2, r \ge 1$. Then $\chi_{ed}(G) = a+1$.

Proof. Let $D = \{v_1, v_2,..., v_a\}$ where v_i is a vertex in the ith copy of K_r, $1 \le i \le a$ .Let $\prod = \{\{v_1\},\{v_2\},...,\{v_a\},U\}$ where U is obtained from G by deleting $v_1,v_2,...,v_a$.. Then $\prod$ is equivalence color dominating partition of G (note that U is an equivalence set of G).

$\chi_{ed}(G) \le |\prod| = a+1$.

Let $\prod_1$ be a χ_{ed} partition of G. Suppose there are t singleton's one each from a copy of K_r and let $t \prec a$. Then the a-t classes in $\prod_1$ contain more than one element in each class. If a class in the a-t classes contain two elements from the same copy then these two elements can not dominate. Likewise if a class in the a-t classes contain two elements from different classes contain two elements from different copies then also these two elements can not dominate a color class. Therefore $t = a$. Therefore $\chi_{ed}(G) = a+1$.

Proposition 2.11.

1. $2 \le \chi_{ed}(G) \le n$ if $n \ge 2$, $\chi_{ed}(G) = 1$ iff $G = K_1$.

$\chi_{ed}(G) = n$ iff $G = \overline{K_n}$ or $\overline{K_{n-2}} \cup K_2$ or $mK_2 \cup (n-2m)K_1$

2. $\chi_{ed}(G)=2$ iff $G=H_1+H_2$ where H_1 and H_2 are equivalence sub graphs of G.

3. Let $\chi_{ed}(G)=n-1$. Then any χ_{ed} partition consists of n-1 color classes. Of these only one of them is a doubleton and the remaining n-2 classes are singleton. Let $\prod=\{V_1,V_2,...,V_{n-1}\}$ be a χ_{ed} partition of G such that $|V_1|=2, |V_i|=1, 2\le i\le n-1$. Let $V_1=\{v_1,v_2\}$, $V_2=\{v_3\},...,V_{n-1}=\{v_{n-1}\}$.Case i: v_1 and v_2 are independent.

Sub case i: v_1 is adjacent with one vertex say v_i and v_2 is adjacent with either v_i or v_j. In this case $\chi_{ed}(G)=n-1$.

Sub case ii: v_1 is adjacent with two vertices v_i ,v_j and v_2 is adjacent with v_i and v_j. Then $\chi_{ed}(G)=n-2$.

Sub case iii: v_1 is adjacent with two vertices v_i and v_j and v_2 is adjacent with exactly one vertex. Then $\chi_{ed}(G)=n-1$.

Sub case iv: v_1 is adjacent with three vertices and v_2 is adjacent with exactly one vertex. If the three vertices adjacent to v_1 form a P_3 then $\chi_{ed}(G)=n-1$. Otherwise $\chi_{ed}(G)\prec n-1$.

Sub case v: If v_1 is adjacent with four or more vertices then $\chi_{ed}(G)\prec n-1$.

Case ii: v_1 and v_2 are adjacent

The sub cases discussed in case (i) can be repeated and the results are the same.

Therefore $\chi_{ed}(G)=n-1$ iff the doubleton class in a χ_{ed} partition is such that one element is adjacent with at most three elements and the other element is adjacent with exactly one element and the three element is from a P_3.

Proposition 2.12. Given a positive integer a there exists a graph G such that $\chi(G)=\chi_{ed}(G)=a$.

Proof.

Case i: a =1.

When $G = K_1$, $\chi(\mathrm{G}) = \chi_{ed}(G) = 1$

Case ii: a = 2

Let $G = K_2$. Then $\chi(\mathrm{G}) = \chi_{ed}(G) = 2$.

Case iii: $a \geq 3$.

Let $G = (a-1)K_a$. Then $\chi(\mathrm{G}) = \chi_{ed}(G) = a$.

Proposition 2.13. Given a positive integer a, there exists a connected graph G such that $\gamma(G) = 1$ and $\chi(G) = \chi_{ed}(G) = a$.

Proof.

Case i: a = 1.

Let $G = K_1$. Then $\gamma(G) = 1$, $\chi(G) = \chi_{ed}(G) = 1$.

Case ii: a = 2.

Let $G = K_2$. Then $\gamma(G) = 1$ and $\chi(G) = \chi_{ed}(G) = 2$.

Case iii: Let $a \geq 3$.

Let $G = (a-1)K_{a-1}$ .Join one vertex of K_{a-1} to a vertex of the next copy of K_{a-1} and a vertex of last copy of K_{a-1} to a vertex of first copy of K_{a-1} . Add a new vertex u such that u is adjacent with every vertex of every copy of K_{a-1}. Let H be the graph obtained from G by adding a new vertex and making it adjacent with every vertex of G. Also then $\gamma(H) = 1, \chi(H) = a, \chi_{ed}(H) = (a-1) + 1 = a$.

Proposition 2.14. Given a positive integer a, there exists a connected graph G such that $\gamma(G) = 1, \chi(\mathrm{G}) = \mathrm{a}$ and $\chi_{\mathrm{ed}}(G) = a + 1$.

Proof.

Case i: a =1. Let $G = K_1$. Then $\gamma(G) = 1, \chi(\mathrm{G}) = \chi_{\mathrm{ed}}(G) = 1 = a$.

Case ii: a =2. Let $G = K_2$. Then $\gamma(G) = 1, \chi(\mathrm{G}) = \chi_{\mathrm{ed}}(G) = 2 = a$.

Case iii: $a \geq 3$. Let $G = aK_{a-1}$. Join one vertex of K_{a-1} to a vertex of the next copy of K_{a-1} and a vertex of the last copy of K_{a-1} to a vertex of first copy of K_{a-1}. Add a new vertex u such that u is adjacent with every vertex of every copy of K_{a-1}. Let H be the resulting graph. Then $\gamma(H) = 1, \chi(\mathrm{H}) = \mathrm{a}, \chi_{\mathrm{ed}}(H) = a+1$.

Remark 2.15. It has been stated in Theorem 3.5 of [10] that there is no connected graph with $\gamma(G) = 1, \chi(\mathrm{G}) = b$ and $\chi_{\mathrm{ed}}(G) = b+1$. But in the case of equivalence dominator coloring, there exists a connected graph with $\gamma(G) = 1, \chi(\mathrm{G}) = b$ and $\chi_{\mathrm{ed}}(G) = b+1$.

Proposition 2.16. Given positive integers a,b,c, $c \leq a+b, \mathrm{c} \succ \mathrm{a, b, a, b} \geq 2$, there exists a graph G such that $\gamma(G) = a, \chi(\mathrm{G}) = \mathrm{b}$ and $\chi_{\mathrm{ed}}(G) = c$.

Proof. Let $b \succ k$.

Let $G = K_{a_1, a_2, \ldots a_k} \cup rK_b$.

$$\gamma(G) = 2 + r = a$$
$$\chi(G) = \max\{k, b\} = b$$
$$\chi_{ed}(G) = k + r = c$$
$$r = a - 2$$

Therefore $r \geq 1$.

Suppose a = 2

Let $G = K_{a_1, a_2, \ldots a_k}$. Let $b \succ k$. Choose a vertex u in the partite set with a_1 vertices. Construct K_b with one of the vertices as u. Let H be the resulting graph. $\gamma(H) = 2, \chi(H) = b, \chi_{ed}(H) = k+1$

Choose $k = c - 1 \geq 2$. Then H is a connected graph in which $\gamma(H) = 2 = a, \chi(H) = b$ and $\chi_{\mathrm{ed}}(H) = c$.

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