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Inventory Modelling for Ordering Items with Shortages

KEYWORDS: Shortages, Ramp Type Demand, inventory

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ABSTRACT *The present article presents mathematical modeling for ordering items with shortages. In this model inventory is divided into four layers(suppliers, manufacturers, distributors and retailers).Every stage hold inventory in some of the form.. The demand is defined by Ramp type function in which in the first phase the demand increase with time and after that it becomes steady and towards the end in the final phase it decreases and becomes asymptotic.*

Introduction

The demand pattern for fashionable products which initially increases exponentially with time for a period of time after that it becomes steady rather than increasing exponentially. But for fashionable products as well as for the seasonal products the steady demand after its exponential increment with time never be continued indefinitely. Rather it would be followed by exponential decrement with respect to time after a period of time and becomes asymptotic in nature. Thus the demand would be illustrated by three successive time period classified time dependent ramp type function, in which in the first phase the demand increase with time and after that it becomes steady and towards the end in the final phase it decreases and becomes asymptotic.

Goyal and Nebebe (2000) considered a problem of determining economic production from a vendor to a buyer. **Wee (2003)** developed an integrated inventory model with constant rate of deterioration and multiple deliveries. **Lee and Wu (2006)** developed a study on inventory replenishment policies in a two-echelon supply chain system. **Ahmed et. al (2007)** have recently coordinated a two level supply chain in which they considered production interruptions for restoring of the quality of the production process. **Singh (2008)** assumed optimal ordering policy for decaying items under inflation.

Wu (2001) considered an EOQ model with ramp – type demand, weibull distribution deterioration and partial backlogging. The characteristic of ramp – type demand can be found in **Mandal and Pal (1998)** has been taken order level inventory system with ramp–type demand rate for deterioration items**. Wu et al (1999)** developed an EOQ model with ramp type demand rate for items with Weibull deterioration**. Wu and Ouyang (2000)** considered a replenishment policy for deteriorating items with ramp type demand rate**, Manna and Chaudhari (2006)** assumed an EOQ model with ramp type demand rate, time dependent deterioration rate, unit production cost and shortages and **Deng et al. (2007)** considered a note on the inventory models for deteriorating items with ramp type demand rate**.**

2. Assumptions and Notations

The following assumptions and notations are used in developing the model

(i) Shortages in the inventory are allowed and partially backlogged.

(ii) The supply is instantaneous and the lead time is zero.

(iii) A deteriorated unit is not repaired or replaced during a given cycle.

(iv) Single vendor and single buyer model is considered.

3. Model formulation:

In this model we will study only the order cycle in the interval $[\mu_i, \gamma_i]$ and ends in the time interval $[\gamma_i, T_2]$. The deterministic demand rate R(t) is ramp-type time dependent i.e., $D(t) = Ae^{b[t-(t-\mu_i)H(t-\mu_i)]-[(t-\gamma_i)H(t-\gamma_i)]}, i = 1,2$, where A>0 is the initial demand rate and b >0 is the rate with which the demand rate increase. $H(t-\mu_i)$ and $H(t-\gamma_i)$ are well known Heviside functions respectively defined as

$$H(t-\mu_i) = \begin{cases} 1 & \text{if } t \geq \mu_i \\ 0 & \text{if } t < \mu_i \end{cases}, \qquad H(t-\gamma_i) = \begin{cases} 1 & \text{if } t \geq \gamma_i \\ 0 & \text{if } t < \gamma_i \end{cases}$$

The order cycle starts in the interval $[\mu_i, \gamma_i]$ and ends in the time interval $[\gamma_i, T_2]$ and follows the differential equations

$$I_{v1}'(t_1) + \theta I_{v1}(t_1) = (K-1)Ae^{b\mu_1}, \quad 0 \leq t_1 \leq \gamma_1$$
….(1)

$$I_{v1}'(t_1) + \theta I_{v1}(t_1) = (K-1)Ae^{-b(t_1-\overline{\mu_1+\gamma_1})}, 0 \leq t_1 \leq T_1$$
….(2)

$$I_{v2}'(t_2) + \theta I_{v2}(t_2) = -Ae^{b\mu_2}, \quad 0 \leq t_2 \leq \gamma_2$$
….(3)

$$I_{v2}'(t_2) + \theta I_{v2}(t_2) = -Ae^{-b(t_2-\overline{\mu_2+\gamma_2})}, \gamma_2 \leq t_2 \leq T_2$$
….(4)

$$I_b'(t) + \theta I_b(t) = -Ae^{b\mu_2}, \quad 0 \leq t \leq \gamma_2$$
….(5)

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$$I_b^{'}(t)+\theta I_b(t)=-Ae^{-b(t-\overline{\mu_2+\gamma_2})},\gamma_2\le t\le T$$

….(6)

$$I_b^{'}(t)=-\frac{A}{1+\delta(T-t)}e^{b\mu_2}\ ,T\le t\le\frac{T_2}{n}$$

….(7)

with the initial conditions $I_{v1}(0)=0$, $I_{v2}(T_2)=0$ and $I_b(T)=0$, the solution of the above differential equations are.

$$I_{v1}(t_1)=\frac{A(K-1)}{\theta}e^{b\mu}\left[1-e^{-\theta t_1}\right],0\le t_1\le\gamma_1 \qquad ….(8)$$

$$=\frac{A(K-1)}{\theta-b}e^{-b(t_1-(\mu_1+\gamma_1))}+\left[I_{v1}(\gamma)-\frac{A(K-1)}{\theta-b}e^{b\mu_1}\right]e^{\theta(\gamma-t_1)}\ ,\gamma_1\le t_1\le T_1$$

….(9)

$$I_{v2}(t_2)=-\frac{A}{\theta}e^{b\mu_2}+\left[I_{v2}(\gamma_2)+\frac{A}{\theta}e^{b\mu_2}\right]e^{\theta(\gamma_2-t_2)}\ ,0\le t_2\le\gamma_2$$

$$=\frac{A}{\theta-b}e^{b(\mu_2+\gamma_2)}\left[e^{(\theta-b)T_2-\theta t_2}-e^{-bt_2}\right],\gamma_2\le t_2\le T_2$$

….(10)

and

$$I_b(t)=-\frac{A}{\theta}e^{b\mu_2}+\left[I_b(\gamma_2)+\frac{A}{\theta}e^{b\mu_2}\right]e^{\theta(\gamma_2-t)}\ ,0\le t\le\gamma_2$$

….(11)

$$I_b(t)=\frac{A}{\theta-b}e^{b(\mu_2+\gamma_2)}\left[e^{(\theta-b)T-\theta t}-e^{-bt}\right],\gamma_2\le t\le T$$

….(12)

$$I_b(t)=\frac{-A}{\delta}e^{-b\mu_2}\left\{\ln\left[1+\delta(\frac{T_2}{n}-T)\right]-\ln\left[1+\delta(\frac{T_2}{n}-t)\right]\right\}\quad ,T\le t\le\frac{T_2}{n}$$

….(13)

From (10), we have

$$I_{mv}=-\frac{A}{\theta}e^{b\mu_2}+\left[I_{v2}(\gamma_2)+\frac{A}{\theta}e^{b\mu_2}\right]e^{\theta\gamma_2}$$

….(14)

when $I_{mv}=I_{v2}(0)$

We know from previous model that

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$$I_{mb} = -\frac{A}{\theta} e^{b\mu_2} + \left[I_b(\gamma_2) + \frac{A}{\theta} e^{b\mu_2} \right] e^{\theta\gamma_2}$$

….(16)

when $I_{mb} = I_b(0)$

By the boundary condition, $I_{v1}(T_1) = I_{v2}(0)$, one can got the relation between T_1 and T_2 .

The yearly holding cost for buyer and vendor is

$$HC_b = p_b F_b \int_0^{t_1} I_b(t)\,dt$$

$$= p_b F_b \left[\int_0^{\gamma_2} I_b(t)\,dt + \int_{\gamma_2}^{\frac{T_2}{n}} I_b(t)\,dt \right]$$

….(17)

and

$$HC_v = p_v F_v \left[\int_0^{T_1} I_{v1}(t)\,dt + \int_0^{T_2} I_{v2}(t_2)\,dt_2 - \int_0^{T} I_b(t)\,dt \right]$$

$$= p_v F_v \left[\int_0^{\gamma_1} I_{v1}(t_1)\,dt_1 + \int_{\gamma_1}^{T_1} I_{v1}(t_1)\,dt_1 + \int_0^{\gamma_2} I_{v2}(t_2)\,dt_2 + \int_{\gamma_2}^{T_2} I_{v2}(t_2)\,dt_2 - \int_0^{\gamma_2} I_b(t)\,dt - \int_{\gamma_2}^{\frac{T_2}{n}} I_b(t)\,dt \right]$$

….(18)

The annual deteriorated costs for buyer and vendor is

$$DC_b = p_b \left(I_{mb} - \int_0^{\frac{T_2}{n}} D(t)\,dt \right)$$

$$= p_b \left[I_{mb} - \left(\int_0^{\gamma_2} D(t)\,dt + \int_{\gamma_2}^{\frac{T_2}{n}} D(t)\,dt \right) \right]$$

….(19)

and

$$DC_v = p_v (P T_1 - I_{mb})$$

….(20)

respectively

The setup cost per year for buyer and vendor is

$$SC_b = C_{sb}$$

….(21)

and

$$S C_v = C_{s v}$$

….(22)

respectively

The shortage cost for buyer

$$O C_b = S \int_T^{\frac{T_2}{n}} -I_b(t)\, d t$$

….(23)

The opportunity cost for buyer

$$L C_b = \pi \int_T^{\frac{T_2}{n}} A \left[1 - B\left(\frac{T_2}{n} - t\right) \right] d t$$

….(24)

Therefore, the buyer's cost is the sum of (17), (19), (21), (23) and (24) as

$$B C = H C_b + D C_b + S C_b + O C_b + L C_b$$

….(25)

The vendor's cost is the sum of (18), (20) & (22) as

$$V C = H C_v + D C_v + S C_v$$

….(26)

The integrated total cost of the vendor and buyer, is the sum of (95) and (96)

$$T C = B C + V C$$

….(27)

5. Conclusion:

In this paper we have attempted to develop a decaying inventory model with a very realistic and practical demand rate. The procedure presented here may be applied to very practical situations.. To make a better combination of increasing-steady-decreasing demand patterns for perishable seasonal products and finite length of the season this model can be used. The customer neither has the patience nor the requirement to wait. This often results in lost sales. As we compare both models we have seen that total cost without shortages is very high in comparison of with shortages.

An optimal solution of the system is obtained under the assumed conditions. Moreover, we characterize the effects of various parameters of the system on the optimal solution.

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6. Reference:

1. S.K. Goyal (1976), An integrated inventory model for a single supplier-single customer problem, International Journal of Production Research, 107-111
2. M.A. Cohen, H.L. Lee (1988), Strategic analysis of integrated production-distribution system: Model and methods, Operations Research, 36, 216-228
3. D.F. Pake, M.A. Cohen (1993), Performance characteristics of stochastic integrated production-distribution system, European Journal of Operational Research, 68, 23-48
4. R.M. Hill (1995), Inventory model for increasing demand followed by level demand, Journal of the Operational Research Society 46, 1250 – 1259
5. B. Mandal, A.K. Pal (1998), Order level inventory system with ramp–type demand rate for deterioration items, Journal of Interdicplinary Mathematics 1, 49–66
6. J.W. Wu, C. Lin, B. Tan, W.C. Lee (1999), An EOQ model with ramp type demand rate for items with Weibull deterioration, International Journal of Information and Management Sciences 10, 41–51
7. K.S. Wu, L.Y. Ouyang (2000), A replenishment policy for deteriorating items with ramp type demand rate (Short Communication), Proceedings of National Science Council ROC (A) 24, 279-286
8. H.K. Goyal, F. Nebebe (2000), Determination of economic production shipment policy for a single vendor, single buyer system, European Journal of Operational Research, 121, 175-178
9. K.S. Wu (2001), An EOQ model for items with Weibull distribution deterioration, ramp–type demand and partial backlogging Production Planning and Control 12, 787-793
10 Skouri K., Papachristos S., A continuous review inventory model, with deteriorating items, time-varying demand, linear replenishment cost, partially time-varying backlogging, Applied Mathematical Modelling 26 (5) (2002) 603-617.
11. Skouri K., Papachristos S., Note on "deterministic inventory lot-size models under inflation with shortages and deterioration for fluctuating demand" by Yang et ai, Naval Research logistics 49 (5) (2002) 527-529.
12. Skouri K., Papachristos S., Four inventory models for deteriorating items with time varying demand and partial backlogging: a cost comparison, Optimal Control Applications and Methods 24 (6) (2003) 315-330.
13. Skouri K., Papachristos S., Optimal stopping and restarting production times for an EOQ model with deteriorating items and time-dependent , partial backlogging, International journal of Production Economics (2003) 525-531.
14. B.C. Giri, A.K. Jalan, K.S. Chaudhari (2003), Economic order model with Weibull distribution deterioration, shortage and ramp – type demand International Journal of System Science 34, 237-243
15. P.C. Yang, H.M. Wee (2003), An integrated multi lot size production inventory model for deteriorating item, Computers and Operations Research, 30, 671-682
16. H.T. Lee and J.C. Wu (2006), A study on inventory replenishment policies in a two-echelon supply chain system, Computers & Industrial Engineering, 51, 2, 257-263
17. S.K. Manna and K.S. Chaudhuri (2006), An EOQ model with ramp type demand rate, time dependent deterioration rate, unit production cost and shortages, European Journal of Operational Research, 171, 2, 557-566
18. M.A. Ahmed, Y.J. Mohamad (2007), Coordinating a two-level supply chain with production interruptions to restore process quality, Computers and Industrial Engineering
19. P.S. Deng, R.H. Lin and P. Chu (2007), A note on the inventory models for deteriorating items with ramp type demand rate, European Journal of Operational Research, 178, 1, 112-120
20. S.R. Singh, C. Singh (2008), Optimal ordering policy for decaying items with stock-dependent demand under inflation in a supply chain, International Review of Pure and Advanced Mathematics, 1, 31-39
21. Wu B., Sarker B. R. , Optimal manufacturing and delivery schedules in a supply chain system of deteriorating items. International Journal of Production Research, 51(3)(2013), 798-812.
22. Wu J., Chan Y. L. ,Lot-sizing policies for deteriorating items with expiration dates and partial trade credit to credit-risk customers. International Journal of Production Economics, 155(2014), 292-301.
23. Yu J. C., Lin Y. S., Wang K. J. , Coordination-based inventory management for deteriorating items in a two-echelon supply chain with profit sharing. International Journal of Systems Science, 44(9)(2013), 1587-1601.