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On the Ternary Quadratic Equation $2y^2 + xy = 4z^2$

KEYWORDS: Integral solutions, Ternary quadratic.

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ABSTRACT *The ternary Quadratic Diophantine equation is analyzed for its non – zero distinct integer solutions. Four different patterns of non-zero distinct integer solutions to the equation under consideration are obtained. A few interesting relation between the solutions and special numbers are exhibited.*

I. INTRODUCTION

The Ternary Quadratic Diophantine Equation offers an unlimited field for research because of their variety [1-2]. For an extensive review of various problems, one may refer [3-10]. This communication concerns with yet another interesting Ternary Quadratic equation $2y^2 + xy = 4z^2$ representing a homogenous cone for determining its infinitely may non-zero integral solutions. Also a few interesting relations among the solutions have been presented..

II. NOTATIONS

obl_n- Oblong number of rank 'n'

$t_{m,n}$- Polygonal number of rank 'n' with sides'm'

II. METHOD OF ANALYSIS

The ternary quadratic equation to be solved in integers is

$2y^2 + xy = 4z^2$ .... (1)

Now, introducing the linear transformations

$x = u + v;\ y = u - v$ .... (2)

In (1), it leads to

$3u \quad + v \quad - 4uv = 4z$ .... (3)

We solve equation (3) in two different methods and obtain two sets of solutions

METHOD : I

(3) can be written as

$(v - 2u)^2 = u^2 + (2z)^2$

(i) Let $z = pq$, $u = p^2 - q^2$

and $v = 3p^2 - q^2$

Hence $x(p,q) = 4p^2 - 2q^2$

$y(p,q) = -2p^2$

$z(p,q) = pq$

PROPERTIES

(i) $x(n,n+1) + 10y(n,n+1) + 100z(n,n+1) \equiv 0 \pmod 2$

(ii) $y(n,n+1) + 2z(n,n+1) \equiv 0 \pmod 2$

(iii) $x(n,n+1) + y(n,n+1) + 2 = 4obl_n$

(iv) $x(n,1) - y(1,n) + n^2 \equiv 0 \pmod 2$

(v) $x(n,n+1) - z(n,n+1) + 2 \equiv 0 \pmod 3$

(ii) Let $2z = p^2 - q^2$

$u = 2pq$

$v = p^2 + q^2 + 4pq$

By taking $p = 2p : q = 2q : z = p - q$,

$u = 8pq : v = 2p^2 + 2q^2 + 16pq$

Hence $x(p,q) = 2p^2 + 2q^2 + 24pq$

$y(p,q) = 2p^2 + 2q^2 - 8pq$

$z(p,q) = p - q$

PROPERTIES

(i) $x(n,-1) - 3y(n,1) \equiv 0 \pmod 8$

(ii) $x(n,n+1) + 2z(n,n+1) \equiv 0 \pmod 2$

(iii) $10x(n,1) + 30y(1,n) \equiv 0 \pmod 8$

(iv) $x(n,n) + y(-n,-n) \equiv 0 \pmod 2$

(v) $y(n,1) + z(1,n) - 2n^2 \equiv 0 \pmod 3$

METHOD : 2

Writing equation (3) as

$$\frac{u}{(2z+v)} = \frac{(2z-v)}{(3u-4v)} = \frac{a}{b} \quad \text{............... (4)}$$

it is equivalent to the system of equations

$bu - 2az - av = 0$

$-3au + 2bz + (4a - b) = 0$

from which we get

$u = -8a^2 + 4ab; v = 2b^2 - 6a^2$ .... (5)

Using (5) in (2), we obtain the integer solutions to (1) as given below:

$$x = x(a,b) = -14a^2 + 2b^2 + 4ab$$
$$y = y(a,b) = -2a^2 - 2b^2 + 4ab$$
$$z = z(a,b) = 3a^2 + b^2 - 4ab$$

PROPERTIES

(i) $x(n,1) - 3y(n,-1) + 5 + st_n \equiv 0 \pmod 6$

(ii) $y(m,n) + 2z(n,m) \equiv 0 \pmod 4$

(iii) $y(n,1) + z(n,1) + 1 = n^2$

(iv) $x(n,n) - y(n,1) \equiv 0 \pmod 2$

(v) $y(n,1) - x(1,n) \equiv 0 \pmod 8$

It is observed that (4) may also be written in the following three ways

WAY : 1

$$x = x(a,b) = -14a^2 + 2b^2 - 4ab$$
$$y = y(a,b) = -2a^2 - 2b^2 - 4ab$$
$$z = z(a,b) = -3a^2 - b^2 - 4ab$$

PROPERTIES

(i) $x(1,n) + y(n,-1) \equiv 0 \pmod 8$

(ii) $x(1,n) + y(-1,n) \equiv 0 \pmod 3$

(iii) $3y(-1,n) + z(n,-1) - t_{3,4} \equiv 0 \pmod 2$

(iv) $x(m,m) - z(1,m) \equiv 0 \pmod 3$

(v) $10y(1,n) + 100z(n,1) \equiv 0 \pmod 2$

WAY : 2

$$x(a,b) = 2a^2 - 14b^2 - 4ab$$
$$y(a,b) = -2a^2 - 2b^2 - 4ab$$
$$z(a,b) = -a^2 - 3b^2 - 4ab$$

PROPERTIES

(i) $x(n,-1) - y(1,-n) \equiv 0 \pmod 4$

(ii) $4x(n,-1) - 12y(1,-n) \equiv 0 \pmod 8$

(iii) $x(n,-1) + 2zy(n,1) \equiv 0 \pmod 4$

(iv) $x(1,n) + 7y(n,1) + t_{6,7} \equiv 0 \pmod 8$

(v) $12y(n,1) - 3z(1,n) - t_{3,6} \equiv 0 \pmod 3$

WAY: 3

$$x = x(a,b) = 2a^2 - 14b^2 + 4ab$$
$$y = y(a,b) = -2a^2 - 2b^2 + 4ab$$
$$z = z(a,b) = a^2 + 3b^2 - 4ab$$

PROPERTIES

(i) $400x(n,-1) + 200y(1,-n) \equiv 0 \pmod 8$

(ii) $x(n,-1) - z(1,n) + t_{3,5} + n^2 = 0$

(iii) $y(1,-n) - z(1,n) + t_{3,2} \equiv 0 \pmod 5$

(iv) $x(m,m) + y(1,m) + 10z(m,1) \equiv 0 \pmod 4$

(v) $10z(m,1) + 5y(1,m) + 20m = 0$

III.CONCLUSION

To conclude, one may search for other patterns of solutions to the equation under consideration.

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