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A Review: Sixteen Sutras of Vedic Mathematics

KEYWORDS

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ABSTRACT *We in India have good reasons to be proud of a rich heritage in science, philosophy and culture in general, coming to us down the ages. In mathematics, which is my own area of specialization, the ancient Indians not only took great strides long before the Greek advent, which is a standard reference point in the Western historical perspective, but also enriched it for a long period making in particular some very fundamental contributions such as the place-value system for writing numbers as we have today, introduction of zero and so on. Guru ji used to say that he had reconstructed the sixteen mathematical formulae (given in this text) from the Atharveda after assiduous research and 'Tapas' for about eight years in the forests surrounding Sringeri. Obviously these formulae are not to be found in the present recensions of Atharvaveda; they were actually reconstructed, on the basis of intuitive revelation, from materials scattered here and there in the Atharvaveda. Revered Gurudeva used to say that he had written sixteen volumes on these sutras one for each sutra. In this paper I try to present a review on given sixteen sutras.*

INTRODUCTION

In this paper I just recall some notions given by Jagadguru Swami Sri Bharati Krsna Tirthaji Maharaja (Sankaracharya of Govardhana Matha, Puri, Orissa, India), Before
I proceed to discuss the Vedic Mathematics that he professed here I give a brief sketch of his heritage.
In 1957, he rewrote from his memory the present volume of Vedic Mathematics giving an introductory account of the sixteen formulae reconstructed by him. This is the only work on mathematics that has been left behind by him. Now I proceed on to give the 16 sutras (aphorisms or formulae) and their corollaries . list of these main 16 sutras and of their sub-sutras or corollaries is prefixed in the beginning of the text and the style of language also points to their discovery by Sri Swamiji himself. This is an open acknowledgement that they are not from the Vedas. The vast merit of these rules should be a matter of discovery for each intelligent reader.
Now having known that even the 16 sutras are the Jagadguru Sankaracharya's invention here I mention the name of the sutras and the sub sutras or corollaries as given by him.

SIXTEEN SUTRAS

Sr. No.	Sutras	Sub sutras or Corollaries
1	Ekādhikena Pūrvena (also a corollary)	Ānurūpyena
2	Nikhilam Navataścaramam Daśatah	Śisyate Śesamjnah
3	Ūrdhva - tiryagbhyām	Ādyamādyenantyamantyena
4	Parāvartya Yojayet	Kevalaih Saptakam Gunỹat
5	Sūnyam Samyasamuccaye	Vestanam
6	(Ānurūpye) Śūnyamanyat	Yāvadūnam Tāvadūnam
7	Sankalana vyavakalanābhyām	Yāvadūnam Tāvadūnīkrtya Vargaňca Yojayet

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8	Puranāpuranābhyām	Antyayordasake' pi
9	Calanākalanābhyām	Antyayoreva
10	Yāvadūnam	Samuccayagunitah
11	Vyastisamastih	Lopanasthāpanabhyām
12	Śesānyankena Caramena	Vilokanam
13	Sopantyadvayamantyam	Gunitasamuccayah Samuccayagunitah
14	Ekanyūnena Pūrvena	
15	Gunitasamuccayah	
16	Gunakasamuccayah	

Following is the spectacular illustrations and a brief descriptions of the sutras.

The First Sutra: Ekādhikena Pūrvena

The relevant Sutra reads Ekādhikena Pūrvena which rendered into English simply says "By one more than the previous one". Its application and "modus operandi" are as follows.
(1) The last digit of the denominator in this case being 1 and the previous one being 1 "one more than the previous one" evidently means 2. Further the proposition 'by' (in the sutra) indicates that the arithmetical operation prescribed is either
multiplication or division. We illustrate this example from pp. 1to 3.
Let us first deal with the case of a fraction say 1/19. 1/19 where denominator ends in 9.
By the Vedic one - line mental method.

A. First method

$$\frac{\mathbf{1}}{\mathbf{19}} = .\mathbf{0\ 526315\ 7894736842}\boldsymbol{i}$$

1 1 111 1 1 11

B. Second Method

$$\frac{\mathbf{1}}{\mathbf{19}} = .\mathbf{0\ 526315\ 78\ /\ 94736842}\boldsymbol{i}$$

1 1 111 1 1 11

This is the whole working. And the modus operandi is explained below.

A. First Method

Modus operandi chart is as follows:
(i) We put down 1 as the right-hand most digit 1
(ii) We multiply that last digit 1 by 2 and put the 2 down as the immediately preceding digit. 2 1
(iii) We multiply that 2 by 2 and put 4 down as the next previous digit. 4 2 1
(iv) We multiply that 4 by 2 and put it down thus 8 4 2 1
(v) We multiply that 8 by 2 and get 16 as the product. But this has two digits. We therefore put the product. But this has two digits we therefore put the 6 down immediately to the left of the 8 and keep the 1 on hand to be carried over to the left at the next step (as we always do in all multiplication e.g. of 69 × 2 =138 and so on). 6 8 4 2 1
1
(vi) We now multiply 6 by 2 get 12 as product, add thereto the 1 (kept to be carried over from the right at the last step), get 13 as the consolidated product, put the 3 down and keep the 1 on hand for carrying over to the left at the next step. 3 6 8 4 2 1
1 1
(vii) We then multiply 3 by 2 add the one carried over from the right one, get 7 as the consolidated product. But as this is a single digit number with nothing to carry over to the left, we put it down as our next multiplicand. 7 3 6 8 4 2 1
1 1
((viii) and xviii) we follow this procedure continually until we reach the 18th digit counting leftwards from the right, when we find that the whole decimal has begun to repeat itself. We therefore put up the usual recurring marks (dots) on the first and the last digit of the answer (from betokening that the whole of it is a Recurring Decimal) and stop the multiplication there.
Our chart now reads as follows:

$$\frac{\mathbf{1}}{\mathbf{19}} = .\mathbf{0\ 526315\ 78\ /\ 94736842}\boldsymbol{i}$$

1 1 1 1 1 1 / 1 11

B. Second Method

The second method is the method of division (instead of multiplication) by the self-same "*Ekādhikena Pūrvena*" namely
2. And as division is the exact opposite of multiplication it stands to reason that the operation of division should proceed not from right to left (as in the case of multiplication as expounded here in before) but in the exactly opposite direction; i.e. from left to right. And such is actually found to be the case.

Its application and modus operandi are as follows:

(i) Dividing 1 (The first digit of the dividend) by 2, we see the quotient is zero and the remainder is 1. We therefore set 0 down as the first digit of the quotient and prefix the remainder 1 to that very digit of the quotient (as a sort of reverse-procedure to the carrying to the left process used in multiplication) and thus obtain 10 as our next dividend. 0
1
(ii) Dividing this 10 by 2, we get 5 as the second digit of the quotient, and as there is no remainder to be prefixed thereto we take up that digit 5 itself as our next dividend. .0 5
1
(iii) So, the next quotient – digit is 2, and the remainder is 1. We therefore put 2 down as the

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third digit of the quotient and prefix the remainder 1 to that quotient digit 2 and thus have 12 as our next dividend.

.0 5 2
1 1

(iv) This gives us 6 as quotient digit and zero as remainder. So we set 6 down as the fourth digit of the quotient, and as there is no remainder to be prefixed thereto we take 6 itself as our next digit for division which gives the next quotient digit as 3.

. 0 5 2 6 3 1
1 1 1

(v) That gives us 1 and 1 as quotient and remainder respectively. We therefore put 1 down as the 6th quotient digit prefix the 1 thereto and have 11 as our next dividend.

. 0 5 2 6 3 1 5
1 1 1 1

(vi to xvii) Carrying this process of straight continuous division by 2 we get 2 as the 17th quotient digit and 0 as remainder.
(xviii) Dividing this 2 by 2 are get 1 as 18th quotient digit and 0 as remainder. But this is exactly what we began with. This means that the decimal begins to repeat itself from here. So we stop the mental division process and put down the usual recurring symbols (dots) on the 1st and 18th digit to show that the whole of it is a circulating decimal.

.052631578
1 1 1111
947368421
1 11

Now if we are interested to find 1/29 the student should note down that the last digit of the denominator is 9, but the penultimate one is 2 and one more than that means 3. Likewise for 1/49 the last digit of the denominator is 9 but penultimate is 4 and one more than that is 5 so for each number the observation must be memorized by the student and remembered.
The following are to be noted.

1. Student should find out the procedure to be followed. The technique must be memorized. They feel it is difficult and cumbersome and wastes their time and repels them from mathematics.

2. "This problem can be solved by a calculator in a time less than a second. Who in this modernized world take so much strain to work and waste time over such simple calculation?" asked several of the students.

3. According to many students the long division method was itself more interesting.

The Second Sutra: Nikhilam Navataścaramam Daśatah

Now we proceed on to the next sutra "*Nikhilam sutra*" The sutra reads "*Nikhilam Navataścaramam Daśatah"*, which literally translated means: all from 9 and the last from 10". We shall presently give the detailed explanation presently of the meaning and applications of this cryptical-sounding formula and then give details about the three corollaries.
He has given a very simple multiplication.

Suppose we have to multiply 9 by 7.

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1. We should take, as base for our calculations that power of 10 which is nearest to the numbers to be multiplied. In this case 10 itself is that power.

(10)
9 – 1
7 – 3
----
6 / 3
----

2. Put the numbers 9 and 7 above and below on the left hand side (as shown in the working alongside here on the right hand side margin);
3. Subtract each of them from the base (10) and write down the remainders (1 and 3) on the right hand side with a connecting minus sign (–) between them, to show that the numbers to be multiplied are both of them less than 10.
4. The product will have two parts, one on the left side and one on the right. A vertical dividing line may be drawn for the purpose of demarcation of the two parts.
5. Now, the left hand side digit can be arrived at in one of the 4 ways

a) Subtract the base 10 from the sum of the given numbers (9 and 7 i.e. 16). And put (16 – 10) i.e. 6 as the left hand part of the answer

9 + 7 – 10 = 6

or b) Subtract the sum of two deficiencies (1 +3 = 4) from the base (10) you get the same answer (6) again

10 – 1 – 3 = 6

or c) Cross subtract deficiency 3 on the second row from the original number 9 in the first row. And you find that you have got (9 –3) i.e. 6 again

9 – 3 = 6

or d) Cross subtract in the converse way (i.e. 1 from 7), and you get 6 again as the left hand side portion of the required answer

7 – 1 = 6.

Note: This availability of the same result in several easy ways is a very common feature of the Vedic system and is great advantage and help to the student as it enables him to test and verify the correctness of his answer step by step.

6. Now vertically multiply the two deficit figures (1 and 3). The product is 3. And this is the right hand side portion of the answer

(10) 9 – 1
7 – 3
----
6 / 3

7. Thus 9 × 7 = 63.

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This method holds good in all cases and is therefore capable of infinite application. Now we proceed on to give the interpretation and working of the 'Nikhilam' sutra and its three corollaries.

The Third Sutra: Ūrdhva Tiryagbhyām

Ūrdhva Tiryagbhyām sutra which is the General Formula applicable to all cases of multiplication and will also be found very useful later on in the division of a large number by another large number.
The formula itself is very short and terse, consisting of only one compound word and means "vertically and cross-wise." The applications of this brief and terse sutra are manifold.
A simple example will suffice to clarify the modus operandi thereof. Suppose we have to multiply 12 by 13.

(i) We multiply the left hand most digit 1 of the multiplicand vertically by the left hand most digit 1 of the multiplier get their product 1 and set down as the left hand most part of the answer;

12
13

1:3+2:6 = 156

(ii) We then multiply 1 and 3 and 1 and 2 crosswise add the two get 5 as the sum and set it down as the middle part of the answer; and

(iii) We multiply 2 and 3 vertically get 6 as their product and put it down as the last the right hand most part of the answer.

Thus 12 × 13 = 156.

The Fourth Sutra: Parāvartya Yojayet

The term *Parāvartya Yojayet* which means "Transpose and Apply." Here he claims that the Vedic system gave a number is applications one of which is discussed here. The very cceptance of the existence of polynomials and the consequent remainder theorem during the Vedic times is a big question so we don't wish to give this application to those polynomials.
However the four steps given by them in the polynomial division are given below:
Divide x3 + 7x2 + 6x + 5 by x – 2.

i. x3 divided by x gives us x2 which is therefore the first term of the quotient

$$\frac{x^3 + 7x^2 + 6x + 5}{x - 2}$$

$\therefore Q = x^2 + \ldots.$

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ii. $x^2 \times -2 = -2x^2$ but we have $7x^2$ in the divident. This means that we have to get $9x^2$ more. This must result from the multiplication of x by 9x. Hence the 2nd term of the divisor must be 9x

$$\frac{x^3 + 7x^2 + 6x + 5}{x - 2}$$

$\therefore$ $Q = x^2 + 9x + \ldots.$

iii. As for the third term we already have $-2 \times 9x = -18x$. But we have 6x in the dividend. We must therefore get an additional 24x. Thus can only come in by the multiplication of x by 24. This is the third term of the quotient.
$\therefore$ $Q = x^2 + 9x + 24$
iv. Now the last term of the quotient multiplied by – 2 gives us – 48. But the absolute term in the dividend is 5. We have therefore to get an additional 53 from some where. But
there is no further term left in the dividend. This means that the 53 will remain as the remainder
$\therefore$ $Q = x^2 + 9x + 24$ and $R = 53$.
This method for a general degree is not given. However this does not involve anything new. Further is it even possible that the concept of polynomials existed during the period of Vedas Itself ?
Now we give the 5th sutra.

The Fifth Sutra: Sūnyam Samyasamuccaye

We begin this section with an exposition of several special types of equations which can be practically solved at sight with the aid of a beautiful special sutra which reads *Sūnyam Samyasamuccaye* and which in cryptic language which renders its applicable to a large number of different cases. It merely says "when the Samuccaya is the same that Samuccaya is zero i.e. it should be equated to zero."
Samuccaya is a technical term which has several meanings in different contexts which we shall explain one at a time.
Samuccaya firstly means a term which occurs as a common factor in all the terms concerned.
Samuccaya secondly means the product of independent terms.
Samuccaya thirdly means the sum of the denominators of two fractions having same numerical numerator.
Fourthly Samuccaya means combination or total.
Fifth meaning: With the same meaning i.e. total of the word
(Samuccaya) there is a fifth kind of application possible with quadratic equations.
Sixth meaning – With the same sense (total of the word –Samuccaya) but in a different application it comes in handy to solve harder equations equated to zero.
Thus one has to imagine how the six shades of meanings have been perceived by the Jagadguru Sankaracharya that too from the Vedas when such types of equations had not even been invented in the world at that point of time. However the immediate application of the subsutra Vestnam is not given but extensions of this sutra are discussed.
So we next go to the sixth sutra given by His Holiness Sankaracharya.

The Sixth Sutra: Ānurūpye Śūnyamanyat

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As said by Dani [32] we see the 6th sutra happens to be the subsutra of the first sutra. Its mention is made in {pp. 51, 74,249 and 286 of }. The two small subsutras (i) *Anurpyena* and (ii) *Adayamadyenantyamantyena* of the sutras 1 and 3 which mean "proportionately" and "the first by the first and the last by the last". Here the later subsutra acquires a new and beautiful double application and significance. It works out as follows:

i. Split the middle coefficient into two such parts so that the ratio of the first coefficient to the first part is the same as the ratio of that second part to the last coefficient. Thus in the quadratic 2x2 + 5x + 2 the middle term 5 is split into two such parts 4 and 1 so that the ratio of the first coefficient to the first part of the middle coefficient i.e. 2 : 4 and the ratio of the second part to the last coefficient i.e. 1 : 2 are the same. Now this ratio i.e. x + 2 is one factor.

ii. And the second factor is obtained by dividing the first coefficient of the quadratic by the first coefficient of the factor already found and the last coefficient of the quadratic by the last coefficient of that factor. In other words the second binomial factor is obtained thus

$$\frac{2x^2}{x} + \frac{2}{2} = 2x + 1$$

Thus $2x^2 + 5x + 2 = (x + 2)(2x + 1)$. This sutra has *Yavadunam Tavadunam* to be its subsutra which the book claims to have been used.

The Seventh Sutra: Sankalana Vyavakalanābhyām

Sankalana Vyavakalan process and the Adyamadya rule together from the seventh sutra. The procedure adopted is one of alternate destruction of the highest and the lowest powers by a suitable multiplication of the coefficients and the addition or subtraction of the multiples.
A concrete example will elucidate the process.

Suppose we have to find the HCF (Highest Common factor) of $(x^2 + 7x + 6)$ and $x^2 – 5x – 6$.
$X^2 + 7x + 6 = (x + 1)(x + 6)$ and
$X^2 – 5x – 6 = (x + 1)(x – 6)$

$\therefore$ the HCF is x + 1

but where the sutra is deployed is not clear.
This has a subsutra *Yavadunam Tavadunikrtya*. However it is not mentioned in chapter 10 of Vedic Mathematics .

The Eight Sutra: Puranāpuranābhyām

Puranāpuranābhyām means "by the completion or not completion" of the square or the cube or forth power etc. But when the very existence of polynomials, quadratic equations etc. was not defined it is a miracle the Jagadguru could contemplate of the completion of squares (quadratic) cubic and forth degree equation. This has a subsutra *Antyayor dasake'pi*
use of which is not mentioned in that section.

The Ninth Sutra: Calanā kalanābhyām

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The term (*Calanā kalanābhyām*) means differential calculus according to Jagadguru Sankaracharya. It is mentioned in page 178 that this topic will be dealt with later on. We have not dealt with it as differential calculus not pertaining to our analysis as it means only differential calculus and has no mathematical formula or sutra value.

The Tenth Sutra: Yāvadūnam

Yāvadūnam Sutra (for cubing) is the tenth sutra. However no modus operandi for elementary squaring and cubing is given in this book . It has a subsutra called *Samuccayagunitah*.

The Eleventh Sutra: Vyastisamastih Sutra

Vyastisamastih sutra teaches one how to use the average or exact middle binomial for breaking the biquadratic down into a simple quadratic by the easy device of mutual cancellations of the odd powers. However the modus operandi is missing.

The Twelfth Sutra: Śesānyankena Caramena

The sutra *Śesānyankena Caramena* means "The remainders by the last digit". For instance if one wants to find decimal value of 1/7. The remainders are 3, 2, 6, 4, 5 and 1. Multiplied by 7 these remainders give successively 21, 14, 42, 28, 35 and 7. Ignoring the left hand side digits we simply put down the last digit of each product and we get 1/7 = .14 28 57!
Now this 12th sutra has a subsutra *Vilokanam*. *Vilokanam* means "mere observation" He has given a few trivial examples for the same.
Next we proceed on to study the 13th sutra Sopantyadvayamantyam.

The Thirteen Sutra: *Sopantyadvayamantyam*

The sutra *Sopantyadvayamantyam* means "the ultimate and twice the penultimate" which gives the answer immediately. No mention is made about the immediate subsutra.
The illustration given by them.

$$\frac{1}{(x+2)(x+3)}+\frac{1}{(x+2)(x+4)}=\frac{1}{(x+2)(x+5)}+\frac{1}{(x+3)(x+4)}$$

Here according to this sutra L + 2P (the last + twice the penultimate)

= (x + 5) + 2 (x + 4) = 3x + 13 = 0
∴ $x = -4\frac{1}{3}$.

The proof of this is as follows.

$$\frac{1}{(x+2)(x+3)}+\frac{1}{(x+2)(x+4)}=\frac{1}{(x+2)(x+5)}+\frac{1}{(x+3)(x+4)}$$

∴

$$\frac{1}{(x+2)(x+3)}-\frac{1}{(x+2)(x+5)}=\frac{1}{(x+3)(x+4)}-\frac{1}{(x+2)(x+4)}$$

∴

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$$\frac{1}{(x+2)}\left[\frac{2}{(x+3)(x+5)}\right]=\frac{1}{(x+4)}\left[\frac{-1}{(x+2)(x+3)}\right]$$

Removing the factors (x + 2) and (x + 3);

$$\frac{2}{(x+5)}=\frac{-1}{(x+4)}$$

i.e.

$$\frac{2}{L}=\frac{-1}{P}$$

∴ L + 2P = 0.

The General Algebraic Proof is as follows.

$$\frac{1}{AB}+\frac{1}{AC}=\frac{1}{AD}+\frac{1}{BC}$$

(where A, B, C and D are in A.P).

Let d be the common difference

$$\frac{1}{A(A+d)}+\frac{1}{A(A+2d)}=\frac{1}{A(A+3d)}+\frac{1}{(A+d)(A+2d)}$$

$$\frac{1}{A(A+d)}-\frac{1}{A(A+3d)}=\frac{1}{(A+d)(A+2d)}+\frac{1}{A(A+2d)}$$

$$\frac{1}{A}\left\{\frac{2d}{(A+d)(A+3d)}\right\}=\frac{1}{(A+2d)}\left\{\frac{-d}{A(A+d)}\right\}$$

Canceling the factors A (A + d) of the denominators and d of the numerators:

∴

$$\frac{2}{(A+3d)}=\frac{-1}{(A+2d)}$$

In other words,

$$\frac{2}{L}=\frac{-1}{P}$$

∴ L + 2P = 0

It is a pity that all samples given by the book form a special pattern.

The Fourteenth Sutra: Ekanyūnena Pūrvena

The *Ekanyūnena Pūrvena* Sutra sounds as if it were the converse of the Ekadhika Sutra. It actually relates and provides for multiplications where the multiplier the digits consists entirely of nines. The procedure applicable in this case is therefore evidently as follows.

For instance 43 × 9.

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i. Divide the multiplicand off by a vertical line into a right hand portion consisting of as many digits as the multiplier; and subtract from the multiplicand one more than the whole excess portion on the left. This gives us the left hand side portion of the product or take the Ekanyuna and subtract it from the previous i.e. the excess portion on the left and

ii. Subtract the right hand side part of the multiplicand by the Nikhilam rule. This will give you the right hand side of the product

43 × 9
4 : 3
:–5 : 3
----
3 : 8 : 7

This Ekanyuna Sutra can be utilized for the purpose of postulating mental one-line answers to the question.

The Fifthteen Sutra: Gunitasamuccayah

Gunitasamuccayah rule i.e. the principle already explained with regard to the Sc of the product being the same as the product of the Sc of the factors. Let us take a concrete example and see how this method (p. 81) can be made use of. Suppose we have to factorize $x^3 + 6x^2 + 11x + 6$ and by some method, we know $(x + 1)$ to be a factor. We first use the corollary of the 3rd sutra viz. Adayamadyena formula and thus mechanically put down x2 and 6 as the first and the last coefficients in the quotient; i.e. the product of the remaining two binomial factors. But we know already that the Sc of the given expression is 24 and as the S_c of $(x + 1) = 2$ we therefore know that the Sc of the quotient must be 12. And as the first and the last digits thereof are already known to be 1 and 6, their total is 7. And therefore the middle term must be $12 – 7 = 5$. So, the quotient $x^2 + 5x + 6$.

This is a very simple and easy but absolutely certain and effective process.

As per pp. XVII to XVIII of the book there is no corollary to the 15th sutra i.e. to the sutra Gunitasamuccayah but in p. 82 of the same book they have given under the title corollaries 8 methods of factorization which makes use of mainly the Adyamadyena sutra. The interested reader can refer pp. 82-85 of .

The Sixteen Sutra :Gunakasamuccayah.

"It means the product of the sum of the coefficients in the factors is equal to the sum of the coefficients in the product".

In symbols we may put this principle as follows:
S_c of the product = Product of the S_c (in factors).

For example

$$(x + 7) (x + 9) = x^2 + 16 x + 63$$

and we observe

$$(1 + 7) (1 + 9) = 1 + 16 + 63 = 80.$$

Similarly in the case of cubics, biquadratics etc. the same rule holds good.
For example

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$$(x + 1)(x + 2)(x + 3) = x^3 + 6x^2 + 11x + 6$$
$$2 \times 3 \times 4 = 1 + 6 + 11 + 6$$
$$= 24.$$

Thus if and when some factors are known this rule helps us to fill in the gaps.
It will be found useful in the factorization of cubics, biquadratics and will also be discussed in some other such contexts later on.
In several places in the use of sutras the corollaries are subsutras are dealt separately. One such instance is the subsutra of the 11th sutra i.e., *Vyastisamastih* and its corollary viz. *Lapanasthapanabhyam* finds its mention in page 77 which is cited verbatim here. The *Lapana Sthapana* subsutra however removes the whole difficulty and makes the factorization of a quadratic of this type as easy and simple as that of the ordinary quadratic already explained. The procedure is as follows: Suppose we have to factorise the following long quadratic.

$$2x^2 + 6y^2 + 6z^2 + 7xy + 11yz + 7zx$$

i. We first eliminate by putting $z = 0$ and retain only x and y and factorise the resulting ordinary quadratic in x and y with Adyam sutra which is only a corollary to the 3rd sutra viz. Urdhva tryyagbhyam.

ii. We then similarly eliminate y and retain only x and z and factorise the simple quadratic in x and z.

iii. With these two sets of factors before us we fill in the gaps caused by our own deliberate elimination of z and y respectively. And that gives us the real factors of the given long expression. The procedure is an argumentative one and is as follows:

If $z = 0$ then the given expression is $2x^2 + 7xy + 6y^2 = (x + 2y)(2x + 3y)$. Similarly if $y = 0$ then $2x^2 + 7xz + 3z^2 = (x + 3z)(2x + z)$.

Filling in the gaps which we ourselves have created by leaving out z and y, we get $E = (x + 2y + 3z)(2x + 3y + z)$

Note:
This Lopanasthapana method of alternate elimination and retention will be found highly useful later on in finding HCF, in solid geometry and in co-ordinate geometry of the straight line, the hyperbola, the conjugate hyperbola, the asymptotes etc. In the current system of mathematics we have two methods which are used for finding the HCF of two or more given expressions.
The first is by means of factorization which is not always easy and the second is by a process of continuous division like the method used in the G.C.M chapter of arithmetic. The latter is a mechanical process and can therefore be applied in all cases. But it is rather too mechanical and consequently long and cumbrous.

The Vedic methods provides a third method which is applicable to all cases and is at the same time free from this disadvantage.
It is mainly an application of the subsutras or corollaries of the 11th sutra viz. *Vyastisamastih*, the corollary *Lapanasthapana* sutra the 7th sutra viz. *Sankalana Vyavakalanabhyam* process and the subsutra of the 3rd sutra viz. *Adyamādyenantyamantyena*. The procedure adopted is one of alternate destruction of the highest and the lowest powers by a suitable multiplication of the coefficients and the addition or subtraction of the multiples.

A concrete example will elucidate the process.

Suppose we have to find the H.C.F of $x^2 + 7x + 6$ and x2 –5x – 6

i. $x^2 + 7x + 6 = (x + 1)(x + 6)$ and $x^2 - 5x - 6 = (x + 1)(x - 6)$. HCF is $(x + 1)$. This is the first method.

ii. The second method the GCM one is well-known and need not be put down here.

iii. The third process of 'Lopanasthapana' i.e. of the elimination and retention or alternate destruction of the highest and the lowest powers is explained below.
Let E1 and E2 be the two expressions. Then for destroying the
highest power we should substract E_2 from E_1 and for destroying the lowest one we should add the two. The chart is as follows:

subtraction

$x^2 + 7x + 6$
$x^2 + 5x + 6$

$12x + 12$
$12)12x + 12$

$X + 1$

addition

$x^2 - 5x - 6$
$x^2 + 7x + 6$

$2x^2 + 2x$
$2x)\ 2x^2 + 2$

$X + 1$

We then remove the common factor if any from each and we find x + 1 staring us in the face i.e. x + 1 is the HCF. Two things are to be noted importantly.

(1) We see that often the subsutras are not used under the main sutra for which it is the subsutra or the corollary. This is the main deviation from the usual mathematical principles of theorem (sutra) and corollaries (subsutra).

(2) It cannot be easily compromised that a single sutra (a Sanskrit word) can be mathematically interpreted in this manner even by a stalwart in Sanskrit except the Jagadguru Puri Sankaracharya. We wind up the material from the book of Vedic Mathematics and proceed on to give the opinion/views of great personalities on Vedic Mathematics given by Jagadguru. Since the notion of integral and differential calculus was not in vogue in Vedic times, here we do not discuss about the authenticated inventor, further we have not given the adaptation of certain sutras in these fields. Further as most of the educated experts felt that since the

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Jagadguru had obtained his degree with mathematics as one of the subjects, most of the results given in book on Vedic Mathematics were manipulated by His Holiness.

CONCLUSION

"The "Vedic" methods of mental calculations in the decimal system are all based on the book Vedic Mathematics by Jagadguru (world guru) Swami (monk) Sri (reverend) Bharati Krsna Tirthaji Maharaja, which appeared in 1965 and which has been reprinted many times. The book contains sixteen brief sutras that can be used for mental calculations in the decimal place-value system. An example is the sutra Ekadhikena Purvena, meaning: by one more than the previous one. The Guru explains that this sutra can for example be used in the mental computation of the period of a recurring decimal fraction such as 1/19 = 0.052631578947368421. as follows:

The word "Vedic" suggests that these calculations are authentic Vedic Mathematics. The question now arises how the Vedic mathematicians were able to write the recurrent decimal fraction of 1/19, while decimal fractions were unknown in India before the seventeenth century.

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