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Two Stage Data Envelopment Analysis-Indian Public Sector Banks Performance

KEYWORDS: Commercial Banks, Pure Technical Efficiency, Scale Efficiency, BCC

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ABSTRACT *In India the financial sector is dominated by the Commercial Banks' activities. The public sector banks account for 73 per cent of Commercial Banks' business. PSBs net work is found wide spread in rural India. This study aimed to explain inter bank efficiency differences of PSBs based on the secondary data published by the Reserve Bank of India (2016). For PSBs it is found that pure technical efficiency is positively related to the bank size. Put together the sector of Public Banks is scale efficient. The most robust bank is State Bank of India. Due to infeasibility in BCC-Super efficiency problems to rank the efficient banks it is suggested that the predicted probabilities,* $P(y=1/z)$ *may be used.*

1. PUBLIC SECTOR BANKS

The financial sector is dominated by the Commercial Banks' activities. The traditional bank activities are collection and distribution of funds. In recent years there has been shift of focus to pursue other income generating activities. Between reforms period (1991-2001) and post reforms era, commercial banks' performance was found significantly improved (Bhatia and Mahendra, 2015). Indian Commercial banking sector is comprised of Public, Private and Foreign sector banks. Indian Public Sector Banks account for 73 per cent of Commercial Banks' business. Public Sector Banks are more labour intensive than the private and foreign sector banks. The statistics of 2014 reveal that 88.02 per cent of bank branches and 69 per cent of ATMs belonged to PSBs, whose net work is found wide spread in rural India. PSBs are more efficient than private and foreign sector banks. Banking reforms have shown a significant impact on both efficiency and total factor productivity (Rajan et. al 2011). In post reforms period efficiency gap between efficient and inefficient PSBs kept closing (Kumar and Gulati, 2009). Major determinants of efficiency of PSBs are off balance sheet business, productivity of employees, market share and size (Kumar and Gulati, 2008).

2. BANK MODELS

Efficiency studies require banks modeled suitably. Two early and popular approaches applied very widely in Banks' research are the production and intermediation approaches. One newest Bank model is based on profit approach. The other approaches were rarely applied. The production and intermediation approaches are based on the theory of firm, differ in the specification of bank activities. Former approach assumes bank's inputs, capital and labour produce deposits, advances and services, suitable for bank branch efficiency measurement.

Some studies based on Production Approach refer to Camanho and Dyson (1999), Athanassopoulos and Gioka (2000), Drake (2002), Paster *et al.,* (2003), Paradi and Claire (2004), Camanho and Dyson (2005), Portela and Thanassoulis (2005), Yang (2009), Sherman and Zhu (2006), Eken and Kale (2011). Most widely used input variables are arranged below in the decreasing order of their frequency: (i) Number of employees, (ii) Non-employee expenditure, (iii) Location (measured in terms of area or rent), and (iv) Equipment. Similarly, the output variables, too, are arranged in the decreasing order of their frequency: (i) Deposits, (ii) Loans, (iii) Non-interest income, (iv) Other transactions, (v) Other products, (vi) Number of loan accounts or transactions, (vii) Number of deposit accounts, and (viii) Interest income.

Most widely used approach for bank efficiency measurement is intermediation approach, first suggested by Sealey and Lindley (1977), views banks as financial intermediaries, use labour, capital and deposits to produce investments and risky products. The two bank models differ in the treatment of deposits as input or output. Some studies based on intermediation approach are due to Tahir and Haron (2008), Casu and Molynex (2003), Daley and Matthews (2009), Eken and Kale (2011), Kamau (2011), Karray and Chichite (2013), Akinsoyinu (2015), and Boda and Zimkova (2015).

The input and output variable employed by different researchers are as follows, arranged in the decreasing order of their frequency:

Inputs: (i) Deposits, (ii) Labour / Employees/ Labour expenses/ Staff Costs, (iii) Capital, (iv) Operational Costs, and (v) Total Cost. Labour expenses /Staff costs are proxy for employees.

Outputs: (i) Total loans/ Advances/ Net Loans/ Gross Loans, (ii) Investments, (iii) Net interest income,(iv) Interest income, (v) Non-interest income, (vi) Total earning assets, and (vii) NPAs.

PROFIT APPROACH

Although, there is no general consensus in the choice of inputs and outputs, there is consensus if one models a bank as Profit Maximizing Agent. The 'Profit approach' (Kamecka, 2010; Ahn and Le 2014) assumes bank to maximize its profits. Profit of a bank is embedded in the variables,

Inputs: (i) Interest Expenditure, (ii) Non-Interest Expenditure (other expenditure).

Outputs: (i) Interest income, (ii) Non- interest income.

This study views banks as profit maximizing agents, in a competitive environment.

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3.ANALYTICAL APPROACHES TO MEASURE BANK EFFICIENCY

To measure Banks' technology and to explain their performance structural or non-structural approaches may be followed. The later implements financial ratios that capture financial institution's performance and the evaluations are absolute. The structural approach relies on a microeconomic theoretical model and a principle of optimization (Hughes and Mester, 2008).

The structural approach requires a frontier which is a cost frontier if the banker is cost minimizer, profit frontier if the bank is a profit maximizing agent, production function if the bank attempts to estimate technical efficiency. In structural approach evaluations are relative. Cost minimization requires input prices and a given output vector. Profit maximization can be implemented if input and output prices are known. Given the production frontier, often hypothesized, there are different approaches to evaluate efficiency scores required for performance evaluation of Commercial Banks.

(i) Thick Frontier Approach (TFA), (ii) Stochastic Frontier Approach (SFA), (iii) Data Envelopment Analysis Approach (DEA)

Thick Frontier Approach: (Berger and Humphrey, 1992; Berger, Cummins and Weiss, 1997) deals with estimation of two thick frontiers one for the lowest and one for the highest average costs quartile of firm.

The Stochastic Frontier Approach (SFA): requires explicit specification of Production frontier/ Cost frontier/ Profit frontier. With the frontier specification two disturbance terms are augmented, one representing inefficiency variations and the other accommodates random fluctuations. The probability distributions of the two disturbance terms are explicitly specified, before estimation is implemented by appropriate statistical methods of estimation (Berger et. al 2004), Green and Segal, 2004; Klumps, 2004; Cummins *et al.*, 2006; Hardwick, P, 1977; Fenn *et al.,* 2008; Rai, A., 1996; Ward, D, 2002, Tahir and Haron, 2008).

Data Envelopment Analysis (DEA): based efficiency studies of Financial Institutions dominate the literature. DEA is a linear programming tool implemented to measure efficiency scores, and to establish efficient targets to inefficient decision making units, such as commercial banks in the present study. The approach is deterministic, but not stochastic. In DEA, efficiency scores are obtained projecting inefficient production plan on to the envelopment frontier. Projection requires choice of distance function, radial or non-radial.

Choice of a distance function is not trivial in Data Envelopment Analysis. Short run target settings require radial distance functions as projection tools. Along radial path input mix/output mix remains to be the same implying that neither input substitution nor output transformation is possible which means the technique of production remains to be the same. In DEA literature envelopment frontiers extensively used are convex frontier (Charnes *et al.,* 1978; Banker *et al.,* 1984). The non-convex frontier of Free Disposable Hull (FDH) can be implemented to set the shortest efficient targets to the inefficient Banks. (Deprins, Simar and Tulkens, 1984; Tulkens 1993). The production possibility sets of CCR, BCC and FDH are related as follows:

$$P_{FDH} \subseteq P_{BCC} \subset P_{CCR}$$

CCR targets are the largest targets. BCC targets are shorter than CCR targets. For very short run the appropriate distance function is radial and the envelopment frontier is non-convex. FDH envelopment frontier can be viewed as expost production frontier. To measure short run technical efficiency and set short run targets, the BCC radial distance function and the convex envelopment frontier are the most appropriate. The performance studies which implement BCC frontier to measure technical efficiency scores of Banks are equivalent to short run performance studies.

Cost/profit frontier based performance studies can be viewed as long run studies, since in long run technique can be changed. DEA based efficiency measurement and target settings are deterministic. Some of the several DEA studies are due to, Brocket *et al.,* (1998, 2004) Athanassopoulos and Giokas (2000); Drake (2002); Camanho and Dyson (1999,2005) Paster *et al.,* (2003); Paradi and Claire (2004), Barros *et al.,* (2005,2007); Sherman and Zhu (2006); Portela and Thanassoulis (2007);Cummins *et al.,* (2006); Subrahmanyam and CS Reddy (2008), Kumar and Gulati (2008,2009); Yang, Z (2009); Kameka, M (2010); Kamau (2011); Eken and Kale (2011); Casu and Molynex (2003); Daley and

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Matthews (2009); Rajan *et al.,*(2011); Karray and Chichiti (2013); Ahn and Le (2014); Makina (2014); Boda and Zimkovia (2015); Akinsoyunu (2015).

The objectives of this study are (i) to evaluate CCR/BCC efficiency scores under input orientation (ii) to examine Scale Efficiency of Indian Public Sector Banks (iii) to set efficient targets to the inefficient PSBs. (iv) to perform regression analysis to explain inter bank efficiency differences. (v) to Rank Public Sector Banks.

DATA

The data are collected from the Reserve Bank of India Bulletins (2016), refer to the year 2015. The input variables are (i) Interest Expenditure (x_1), & (ii) Other expenditure (x_2) and output variables are (i) Interest income (y_1) & (ii) Other income (y_2).

Environmental Variables:

Net Non-Performing Assets (Z_N), Size of the Bank (Z_S), Income from Off Balance Sheet Business (Z_O)

(4) CCR envelopment problems is solved to obtain input overall technical efficiency scores.

$$\bar{\theta}_{CCR} = \text{Min}\,\theta - \in\left(\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+\right)$$

$$\text{s.t} \quad \sum_{j=1}^{n} \lambda_j x_{ij} = x_{ij_0} - s_i^- \ \ldots (1)$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} = y_{rj_0} + s_r^+ ,$$

$$\lambda_j \geq 0 .$$

Bank_{j_0} is said to be overall input technical efficient if and only if, $\bar{\theta}_{CCR} = 1$.CCR technical efficiency approach cannot distinguish scale differences among the Commercial Banks. In CCR measure scale effects are confounded with effects of pure technical efficiency. To purge over all technical efficiency scores from scale effects the convexity constraint, $\sum_{j=1}^{n} \lambda_j = 1$ .... (2)

is augmented to (1) to obtain, input pure technical efficiency scores. The optimal value of the objective functions of (1), (2) is $\bar{\theta}_{BCC}$. Bank_{j_0} is said to be efficient if and only if, $\bar{\theta}_{BCC} = 1$. Since every feasible solution of BCC problem is feasible to the CCR problem and the converse is not true, we have, at the optimum,

$$\bar{\theta}_{CCR} = \theta_{CCR} - \varepsilon\left(\sum_{i=1}^{m} s_i^{-*} + \sum_{r=1}^{s} s_r^{+*}\right)$$

$$\bar{\theta}_{BCC} = \theta_{BCC} - \varepsilon\left(\sum_{i=1}^{m} s_i^{-*} + \sum_{r=1}^{s} s_r^{+*}\right)$$

$$\text{and} \quad \theta_{BCC} \leq \theta_{CCR}$$

θ_{CCR} can be multiplicatively decomposed into input pure technical efficiency (θ_{BCC}) and input scale efficiency.

$$\theta_{CCR} = \theta_{BCC}\ \theta_{SE}$$

$$0 \leq \theta_{SE} = \frac{\theta_{BCC}}{\theta_{CCR}} \leq 1$$

Table (1): Input overall/pure/ Scale efficiency scores

Bank No.	Bank Name	$\theta_{j_0}^{CCR}$	$\theta_{j_0}^{BCC}$	$\theta_{j_0}^{SE}$
1	SB of Bikaner and Jaipur	0.9839	0.9912	0.9926
2	SB of Hyderabad	0.9666	0.9746	0.9918
3	State Bank of India	1	1	1
4	State Bank of Mysore	0.932	0.9403	0.9912
5	State Bank of Patiala	0.9054	0.91	0.9949
6	State Bank of Travancore	0.887	0.8918	0.9946
7	Allahabad Bank	0.9764	0.9829	0.9934
8	Andhra Bank	0.9573	0.9603	0.9969
9	Bank of Baroda	0.9791	1	0.9791

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10	Bank of India	0.9223	0.9539	0.9669
11	Bank of Maharashtra	0.957	0.9641	0.9926
12	Bharatiya Mahila Bank Ltd	1	1	1
13	Canara Bank	0.9004	1	0.9004
14	Central Bank of India	0.9128	0.9202	0.9919
15	Corporation Bank	1	1	1
16	Dena Bank	0.9026	0.9044	0.998
17	IDBI Bank Limited	1	1	1
18	Indian Bank	0.952	0.9562	0.9956
19	Indian Overseas Bank	0.8961	0.9015	0.994
20	Oriental Bank of Commerce	0.9379	0.9392	0.9986
21	Punjab and Sind Bank	0.8856	0.899	0.9851
22	Punjab National Bank	1	1	1
23	Syndicate Bank	0.9337	0.9363	0.9972
24	UCO bank	1	1	1
25	Union Bank of India	0.9212	0.932	0.9884
26	United Bank of India	1	1	1
27	Vijaya Bank	0.8791	0.8856	0.9927

Table (2): Scores Summary

	$\hat{\theta}_{j_0}^{CCR}$	$\hat{\theta}_{j_0}^{BCC}$	$\hat{\theta}_{j_0}^{SE}$
Mean	0.9477	0.9572	0.9902
SD	0.0425	0.0405	0.0194
Min	0.8791	0.8856	0.9004
Max	1.00	1.00	1.00

H_{01} : The Sampled plans arise from input overall technical efficient population

$$t_1 = \frac{\hat{\theta}_{CCR} - 1}{SE\left(\hat{\theta}_{CCR}\right)}$$

H_{01} is rejected at one percent of level of significance

H_{02} : The sampled plans come from input pure technical efficient population.

$$t_2 = \frac{\hat{\theta}_{BCC} - 1}{SE\left(\hat{\theta}_{BCC}\right)}$$

H_{02} is rejected at $P < 0.01$

(5) H_{03} : There is no significant difference between $\hat{\theta}_{CCR}$ and $\hat{\theta}_{BCC}$

$$t_3 = \frac{\hat{\theta}_{BCC} - \hat{\theta}_{CCR}}{SE\left(\hat{\theta}_{BCC} - \hat{\theta}_{CCR}\right)}$$

The null hypothesis is accepted at one percent level of significance. (t_1, t_2 and $t_3$are assumed to follow Student's t- distribution) PSB is Scale Efficient.

Conclusions: (1) PSB is high Overall Technical Efficient (2) PSBs is high Pure Technical Efficient (3) PSB is Scale Efficient.

(6) EFFICIENT TARGETS

For efficient target setting, Cooper *et al.,* (1999) proposed additive DEA model under L_1 norm. Additive DEA problem seeks input specific reduction and output specific augmentation to reach the frontier. It fails to provide a direct measure of efficiency. However, utilizing the slacks at the optimum non-radial measures such as Range Adjusted Measure (Cooper *et al.,*(2007) and BRWZ measure (Brockett, *et al.,*) can be obtained. The additive problem is as follows:

$$S_{j_0} = Max\left(\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+\right)$$

such that $\sum_{j=1}^{n} \lambda_j x_{ij} = x_{ij_0} - s_i^-, \quad i \in M$

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$$\sum_{j=1}^{n}\lambda_j y_{rj} = y_{rj_0} + s_r^+,\ r \in S$$

$$\sum_{j=1}^{n}\lambda_j = 1\ ,\ \lambda_j \geq 0,\ j \in N$$

Efficient Input Targets for $Bank_{j_0}$:

$$x^*_{1j_0} = x_{1j_0} - s^*_{1j_0} \qquad x^*_{2j_0} = x_{2j_0} - s^*_{2j_0}$$

Efficient output targets for DMU_{j_0} **:**

$$y^*_{1j_0} = y_{1j_0} + s^{+*}_{1j_0} \qquad y^*_{2j_0} = y_{2j_0} + s^{+*}_{2j_0}$$

Table (3): Efficient targets of DMU_{j_0}

Bank No.	Bank Name	Targets			
		$x_{1j_0} - s^{-*}_{1j_0}$	$x_{2j_0} - s^{-*}_{2j_0}$	$y_{1j_0} + s^{+*}_{1j_0}$	$y_{2j_0} + s^{+*}_{2j_0}$
1	SB of Bikaner and Jaipur	78881.57	22689.82	91504.17	10713.57
2	SB of Hyderabad	122681.1	36065.07	141894.2	16907.18
3	State Bank of India	1819608	600970.3	176242.1	659082.6
4	State Bank of Mysore	63129.6	19595.53	70648.5	8924.69
5	State Bank of Patiala	100870.5	25777.85	106346.7	12900.28
6	State Bank of Travancore	94770.35	24769.15	97793.43	12254.41
7	Allahabad Bank	176107.3	47771.55	200488.4	23287.27
8	Andhra Bank	153893.7	35234.59	167369.7	18681.99
9	Bank of Baroda	462464.4	191693.6	635909.7	45602.95
10	Bank of India	306800.9	156540.6	438421.6	46449.47
11	Bank of Maharashtra	114346.8	32488.42	132060.9	15466.26
12	Bharatiya Mahila Bank Ltd	517.07	1548.79	1331.09	285.89
13	Canara Bank	384.697	545366.1	101822.7	136588.8
14	Central Bank of India	249258.2	71797.77	279204.3	34058.82
15	Corporation Bank	157760	348008.1	484832.5	15002.06
16	Dena Bank	108170.8	23652.18	113245.2	12823.69
17	IDBI Bank Limited	269948.3	169260.9	3089.17	130153.2
18	Indian Bank	148184.2	36154.02	163448.6	18552.74
19	Indian Overseas Bank	197003	75631.14	245586.9	27589.49
20	Oriental Bank of Commerce	193524.5	41916.5	201304.9	22905.03
21	Punjab and Sind Bank	67450.69	38498.06	91463.28	9865.34
22	Punjab National	743289.3	182733.1	862072.5	65109.93

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23	Syndicate Bank	170889.2	65230.26	220553.4	25497.76
24	UCO Bank	188207	89853.22	242743.8	35307.9
25	Union Bank of India	251001.2	110621.7	323675	38065.4
26	United Bank of India	121483.8	49727.99	111898.3	59481.33
27	Vijaya Bank	98342.37	55246.99	128266.4	14320.75

(7) SECOND STAGE DATA ENVELOPMENT ANALYSIS

The focus of Second Stage Data Envelopment Analysis is to explain inter bank efficiency differences reflected in BCC input oriented pure technical efficiency scores, which requires regression specification, useful as an analytical tool. Choice of regression for second stage DEA is not a trivial econometric problem, due to the nature of technical efficiency scores distributed over the fractional interval]0,1]. The standard linear regression, whose parameters can be estimated by the method of Ordinary Least Squares, if applied to BCC scores, can not prevent the predictions falling out of the fractional range.

Mitchel and Anvural (1996), Miller and Noulas (1996), Berger and Mester (1997), Gold Berg and Rai (1996), Sathey (2001), Kumar and Gulati (2008), Banker and Natarajan (2008), Simar and Wilson (2007), Hoff (2007) and Mcdonold (2009), tried to explain inter bank differences reflected in CCR/BCC/Cost/ efficiency scores by means of (explanatory) Environmental Variables. The environmental variables involved in studies were, Net NPAs: Ratio of net NPAs to net advances; Off Balance Sheet Business Earnings: Size of the bank (measured by total assets); Number of bank branches; Ownership (Viewed as a limited independent variable); non-interest income were some important environmental variable used to explain inter bank differences.

This study selected net NPAs, Size of the Bank and environmental variables. There are sev alternative regression specifications for D Second Stage study, some important of them Linear Probability Regression, Logit Regress Probit Regression, Latent Variable Regression, I Log model Regression, Complementary Log- model Regression, Fractional Regress Generalized fractional regression. The li probability regression suffers from the problem non-normality of disturbances heteroscedasticity. Logistic and Probit regres models stemmed from dose –response problem biology (D. Mc Fadden, 1970; D.R. Cox, 1970).

The latent variable model (Bolen, 1989) may postulated as,

$$y_i^* = x_i\beta + \varepsilon_i,\ i = 1,2.....n$$

y^* is a continuous variable, but unobserva which is related to a binary variable,

$$y = \begin{cases} 1 & \text{if} \quad y^* > 0 \\ 0 & \text{if} \quad y^* \le 0 \end{cases}$$

From this representation, the Probit and L representations can be directly obtained as follo

$$y = 1 \Leftrightarrow y^* > 0 \Leftrightarrow \varepsilon = y^* - x\beta$$

$$\Leftrightarrow \varepsilon > -x\beta$$

$$P(y = 1/x) = p(y^* > 0/x) = p(\varepsilon > -x\beta)$$

$$P(y = 1/x) = F(x\beta)$$

If F is cumulative distribution function (c. d .f normal distribution we get probit model. If F is f of logistic distribution we get the logit m (Aldrich and Nelson,(1984)). In this study assume F is the c. d. f of logistic distribution. T we have,

$$P(y = 1/x) = \frac{\exp(x\beta)}{1 + \exp(x\beta)}$$

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Log-Log Model:

$$F(x\beta) = 1 - \exp(-\exp(x\beta))$$

Complementary Log-Log model:

$$F(x\beta) = \exp(\exp(-x\beta))$$

Popke and Wooldridge (1996) introduced the fractional regression model. The model with out disturbance term can be formulated as follows:

$$E(y/x) = G(x\beta) \quad \text{.... (3)}$$

where $0 \le G(.) \le 1$, $G(.)$ is some nonlinear function. The parameters of (7.6) can be estimated by Quasi Maximum Likelihood Method base on Bernoulli Log Likelihood function. Ramalho, Ramalho and Murtarea (2010) generalized Popke and Wooldridge fraction regression model, leading to the following generalized fraction regression models:

(i) $E(y/x) = (G(x\beta))^{\alpha}$, (ii)

$$E(y/x) = 1 - [1 - G(x\beta)]^{\alpha}$$

where $\alpha > 0$, such that $0 < E(y/x) < 1$

Binary Regression

$$y = \beta + \beta_N Z_N + \beta_S Z_S + \beta_O Z_O + \varepsilon \quad \text{.... (4)}$$

where y is binary dependent variable

$$y_j = \begin{cases} 1 & \text{if } DMU_j \text{ is efficient} \\ 0 & \text{Otherwise,} \end{cases}$$
$$j = 1, 2, ...n$$

The fit of (4) obtained regressing all the three explanatory variables emerged to be inappropriate. The explanatory variables based regression fits were also failed. Finally, the regression fit of,

$$y = \beta + \beta_S Z_S + \varepsilon$$

emerged, meaningful for which $\hat{\beta}$ is significant at 2 percent and $\hat{\beta}_S$ significant at 8 percent. The following results are SPSS based:

Table(4):Classification Table

Observed	Predicted		Percentage Correct
	Inefficient	Efficient	
Inefficient	17	1	94.4
Efficient	5	4	44.4
Overall Percentage			77.8

77.8 per cent of model predictions come true. One inefficient Bank is identified to be efficient. But 5 efficient banks are identified inefficient. The Count R^2 (Maddala, 1992) is summary statistic behaves like R^2 in classical linear regression.

$$R^2_{count} = \frac{\text{Number of Correct Predictions}}{\text{Number of observations}} = 77.8$$

Mis identified efficient banks are those whose BCC efficiency scores are unity, but size is small. One such bank is Bharatiya Mahila Bank Ltd owned by the Central Government. Other such banks are United bank of India, UCO Bank, Corporation Bank and IDBI Bank Ltd. The probabilities of these banks to remain efficient are lower than State Bank of India, Bank of Baroda, Punjab National Bank and Canara bank. But, Bank of Baroda and Canara bank are scale inefficient banks, these are inefficient in CCR but efficient under BCC formulation.

Table (5): Variables in the equation

	B	SE	Wald	d.f	Sig.
Z_S	0.000045	0.000026	3.015	1	0.083
Constant	-2.005	0.845	5.635	1	0.018

$$\hat{\beta} = -2.005, \qquad \hat{\beta}_S = 0.000045$$

Wald statistic follows Chi-Square distribution with one degree of freedom.

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Intercept estimate $\left(\hat{\beta}\right)$ is significant at 2 per cent level of significance. The estimate of regression co-efficient of size $\left(\hat{\beta}_S\right)$ is significant at 8 per cent level of signification.

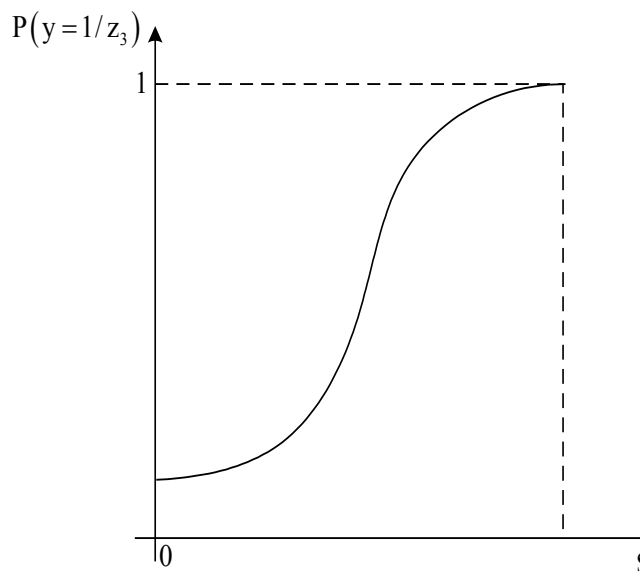
Table (6): Model Summary

-2 Log likelihood	Nagalkerke R^2
28.798	0.259

$$P\left(y=1/z_S\right)=F\left(-2.005+0.000045\ z_s\right)$$

Table: 7

S.No.	Name of the Bank	$p\left(y=1/z_S\right)$
1.	SB of Bikaner and Jaipur	0.1722
2.	SB of Hyderabad	0.2077
3.	State Bank of India	0.9989(*)
4.	State Bank of Mysore	0.1598
5.	State Bank of Patiala	0.1846
6.	State Bank of Travancore	0.1780
7.	Allahabad Bank	0.2693
8.	Andhra Bank	0.2293
9.	Bank of Baroda	0.7481(*)
10.	Bank of India	0.6628
11.	Bank of Maharashtra	0.2027
12.	Bharatiya Mahila Bank Ltd	0.1194(*)
13.	Canara Bank	0.5832(*)
14.	Central Bank of India	0.3427
15.	Corporation Bank	0.2691(*)
16.	Dena Bank	0.1928
17.	IDBI Bank Limited	0.3861(*)
18.	Indian Bank	0.2406
19.	Indian Overseas Bank	0.3222
20.	Oriental Bank of Commerce	0.2707
21.	Punjab and Sind Bank	0.1719
22.	Punjab National Bank	0.6435(*)
23.	Syndicate Bank	0.3194
24.	UCO bank	0.2863(*)
25.	Union Bank of India	0.4131
26.	United Bank of India	0.1904(*)
27.	Vijaya Bank	0.2018



The above figure represents Logistic Probability Distribution function. Total assets are measured along horizontal axis and $P\left(y=1/z_S\right)=F\left(x\hat{\beta}\right)$ is measured along vertical axis.

CONCLUSIONS:

(i) For Indian Public Sector Banks Total Assets and Pure Technical Efficiency are Positively related, (ii) State Bank of India is the most robust bank.

8 (a) RANKING OF PUBLIC SECTOR BANKS – SUPER EFFICIENCY

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Petersen and Andersen (1996) introduced the concept of 'super efficiency' for the extremely efficient decision making units. To find input super efficiency of an extremely efficient decision making unit, its input and output plan is removed from the reference technology and the modified frontier points are represented by

$$\left(\sum_{\substack{j=1 \\ j \neq j_0}}^{n} \lambda_j x_j, \ \sum_{\substack{j=1 \\ j \neq j_0}}^{n} \lambda_j y_j \right)$$

The input and output plan of DMU_{j_0} is projected onto the modified frontier. The super efficiency problem is formulated in CCR frame work. The CCR input technical super efficiency,

$$\theta_{CCR}^{Super} > 1$$

θ_{CCR}^{Super} is a metric that indicates the stability of efficient $Bank_{j_0}$ to remain efficient under input expansion. For two banks j_1 and j_2,

$\theta_{CCR}^{Super}(j_1) > \theta_{CCR}^{Super}(j_2)$ implies $Bank_{j_1}$ attains better rank than $Bank_{j_2}$. Thus, super efficiency enhances the discriminatory power of DEA.

The CCR-Super efficiency problem can be extended to BCC frame work. However, BCC-Super efficiency problems are not always feasible (Seiford and Zhu, 1998) with the following being observed:

(i) If input SE-BCC problem is infeasible, then output SE-BCC problem is feasible.

(ii) If output SE-BCC problem is infeasible, then input SE-BCC problem is feasible.

Due to the infeasibility for the purpose of ranking efficient DMUs, the BCC super efficiency approach is not recommended. An alternative DEA model to rank Banks can be obtained from the class of directional distance functions. Chambers *et al.,* (1996) formulated directional distance function, that can be expressed in BCC frame work as follows:

$$\overrightarrow{D}\left(x_{j_0}, y_{j_0}; g_x, g_y\right) = \text{Max } \beta$$

such that $\sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{ij_0} - \beta g_{x_i}, \ i \in M$ … (5)

$$\sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{rj_0} + \beta g_{y_r}, \ r \in S$$

$$\sum_{j=1}^{n} \lambda_j = 1, \ \lambda_j \geq 0, \ j \in N$$

$$\overleftarrow{D}\left(x_{j_0}, y_{j_0}; g_x, g_y\right) \geq 0$$

$\overleftarrow{D}\left(x_{j_0}, y_{j_0}; g_x, g_y\right) = 0$ implies $Bank_{j_0}$ is efficient, Otherwise inefficient

Chambers *et al.,* (1996) suggested the directional vectors as the input and output vectors of DMU_{j_0}.

$$g_x = x_{j_0}, \ g_y = y_{j_0} \ \ldots. (6)$$

Substituting (6) in (5) we obtain,

$$\overleftarrow{D}\left(x_{j_0}, y_{j_0}; x_{j_0}, y_{j_0}\right) = \text{Max } \beta$$

such that $\sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{ij_0}(1-\beta), \ i \in M$

$$\sum_{j=1}^{n} \lambda_j y_{rj} \leq y_{rj_0}(1+\beta), \ r \in S$$

$$\sum_{j=1}^{n} \lambda_j = 1$$

S. Ray (2004) formulated Super Efficiency problem for efficient $\left(x_{j_0}, y_{j_0}\right)$ as follows:

$$\beta_{j_0}^{Super} = \text{Max}\beta$$

such that $\sum_{\substack{j=1 \\ j \neq j_0}}^{n} \lambda_j x_{ij} \leq x_{ij_0}(1-\beta), \ i \in M$

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$$\sum_{\substack{j=1 \\ j\neq j_0}}^{n} \lambda_j y_{rj} \geq y_{rj_0}\left(1+\beta\right),\ r \in M \quad \ldots\ldots (7)$$

$$\sum_{\substack{j=1 \\ j\neq j_0}}^{n} \lambda_j = 1,\ \lambda_j \geq 0,\ j \in N$$

$$\beta_{j_0}^{Super} < 0 \text{ for efficient } Bank_{j_0}$$

Problem (7) is always feasible (Seiford and Zhu, 1999; Zhu, 2006; Cooper *et al.,* 2007). Smaller values of $\beta_{j_0}^{Super}$ implies greater stability of DMU_{j_0} under input expansion and output contraction.

Table (8): Ranking of Indian Public Sector Banks (Directional Distance Orientation)

S.No	Name of the Bank	Efficiency / super efficiency	RANK
1.	SB of Bikaner and Jaipur	0.0044	10
2.	SB of Hyderabad	0.0129	12
3.	State Bank of India	-0.7391 (*)	2
4.	State Bank of Mysore	0.0307	17
5.	State Bank of Patiala	0.0471	22
6.	State Bank of Travancore	0.0572	26
7.	Allahabad Bank	0.0086	11
8.	Andhra Bank	0.0202	14
9.	Bank of Baroda	-0.0216(*)	8
13.	Canara Bank	-0.0337(*)	6
14.	Central Bank of India	0.0416	21
15.	Corporation Bank	-0.0305(*)	7
16.	Dena Bank	0.0502	23
17.	IDBI Bank Limited	-0.1678(*)	3
18.	Indian Bank	0.0224	16
19.	Indian Overseas Bank	0.0508	24
20.	Oriental Bank of Commerce	0.0314	18
21.	Punjab and Sind Bank	0.0539	25
22.	Punjab National Bank	-0.0085(*)	9
23.	Syndicate Bank	0.0328	19
24.	UCO bank	-0.0360(*)	5
25.	Union Bank of India	-0.0344	20
26.	United Bank of India	-0.0837 (*)	4
27.	Vijaya Bank	0.0613	27

The most supper efficient of all the Banks is Bharatiya Mahila bank owned by the Government of India.

Findings of the Study

(i) Public Sector Banks are highly input technical efficient. (ii) This sector put together is scales efficient. (iii) Pure technical efficiency is positively related with size. (iv) The most robust bank, based on predicted probabilities of efficient banks to stay efficient, is State Bank of India. (v) Bharatiya Mahila Bank is the most DDF super efficient, consequently attained first rank. (vi) The predicted probabilities for

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