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# CHROMATIC PRIME NUMBER ON TENSOR PRODUCT GRAPH

KEYWORDS: Chromatic number and Chromatic prime number. P+ and P- are positive and negative edge prime numbers.

**Kanneeswari. K**
Asst.Prof Dept. of Mathematics, Sree Saraswathi Thyagaraja College, Pollachi, India

**Seshavarthini. K**
M.Phil Scholar, Dept. of Mathematics, Sree Saraswathi Thyagaraja College, Pollachi, India

**ABSTRACT** In this paper, we introduced the chromatic prime number on tensor product graph and we found some results regards positive and negative prime numbers for tensor product graph

A graph consists of a set of vertices V(G) and a set of edges E(G). For every vertices $u_1, u_2 \in V(G)$, the edge connecting $u_1$ and $u_2$ is denoted by $e_1$. Here, we introduced the chromatic prime number for the tensor product graph $\boldsymbol{\chi_p(G \otimes H)} \le \min\{\chi_p(G), \chi_p(H)\}$ D. Duffus B. Sands and R.E Woodrow[2]. An assignment of colors to the vertices of a graph, so that no two adjacent vertices get the same color is called a coloring of the graph. The chromatic number of a graph G is the minimum number of colors need to color the graph G by J.A. Bondy and U.S.R. Murty[1]. An assignment of prime numbers to the vertices of a graph $\boldsymbol{\chi_p(G \otimes H)}$ so that no two adjacent vertices get the same prime numbers is called chromatic prime. We can take the particular prime numbers only in this paper such as 2,3,5,7,11. If we add (or) subtract the adjacent vertices, then we get edge weight as prime numbers only.

## 2. Preliminaries

In this section, we introduced the chromatic prime number on tensor product graph with respect to the positive and negative prime number

Definition 1: The Tensor product, $\boldsymbol{G \otimes H}$, of graph G and H is the graph with vertex set $v(G) \times v(H)$ and $(a,x)(b,y) \in E(\boldsymbol{G \otimes H})$ whenever $ab \in E(G)$ and $xy \in E(H)$.

Definition 2: The chromatic number, $\chi_p(G)$, of G is the smallest number n for $n_p$ which G has an $n_p$-coloring.

Definition 2: The chromatic prime number, $\chi_p(G)$, of G is the smallest number np for which G has an np-coloring.

## 3. Chromatic Prime Number on Tensor Product graphs

**Theorem 1:** Let and H be the 2-prime colorable connected and butterfly graph respectively, then $\boldsymbol{\chi_p(G \otimes H)} \le \min\{\chi_p(G), \chi_p(H)\}$.

**Proof:** Let G and H be a 2-prime colorable complete graph and butterfly prime colorable graph with vertices $u_1, u_2$ and $v_1, v_2, \ldots, v_8$ respectively.

Let the value of the vertices of graph G, $u_1 = 2$ and $u_2 = 3$; $(u_1 + u_2)$ is only positive prime number and their

$$\chi(G) = 2 = \chi_p(G) \qquad (1)$$

Let the value of the vertices of graph H, $v_1, v_8, v_7, v_4, v_6 = 2$; $v_2, v_5 = 5$; and $v_3 = 5$, $(v_1 + v_2)$, $(v_3 + v_4)$, $(v_4 + v_5)$, $(v_3 + v_6)$, $(v_3 + v_7)$, $(v_3 + v_8)$, $(v_1 + v_3)$ and $(v_3 - v_5)$, $(v_3 - v_2)$ are positive and negative prime numbers respectively.

$$\chi(H) = 3 = \chi_p(H) \qquad (2)$$

Let the graph $\boldsymbol{(G \otimes H)}$, we divide the vertex of $\boldsymbol{\chi_p(G \otimes H)}$ into two disjoint sets as below:

$T_1 = \{u_1 v_i\} / 0 < i \le 8\}$
$T_2 = \{(u_2 v_i) / 0 < I \le 8\}$

The graph $\boldsymbol{(G \otimes H)}$, which has prime numbers of vertices on addition and subtraction, we get positive and negative prime numbers respectively.

$$\boldsymbol{\chi(G \otimes H) = 2 = \chi_p(G \otimes H)} \qquad (3)$$

From (1),(2) and (3),we get

$\boldsymbol{\chi(G \otimes H) \le \min\{\chi(G), \chi(H)\}}$
$\boldsymbol{\chi_p(G \otimes H) \le \min\{\chi_p(G), \chi_p(H)\}}$

**Example 1:**

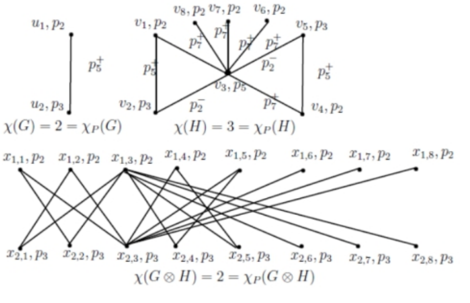


**Fig. 1**

**Theorem 2:** Let G and H be 3-prime colorable complete and 3-prime colorable connected graph respectively, then $\boldsymbol{\chi_p(G \otimes H)} \le \min\{\chi_p(G), \chi_p(H)\}$.

**Proof:** Let G and H be a 3-prime colorable complete graph and star prime colorable graph with vertices $u_1, u_2, u_3$ and $v_1, v_2, \ldots, v_5$ respectively.

Let the value of the vertices of graph G, $u_1 = 2$, $u_2 = 3$ and $u_3 = 5$; $(u_1 + u_2)$, $(u_1 + u_3)$ and $(u_3 - u_2)$ are positive and negative prime numbers,

$$\chi(G) = 3 = \chi_p(G) \qquad (1)$$

Let the value of the vertices of graph H, $v_1, v_2 = 2$; $v_3$, v4 = 3 and v5 = 5. $(v_1 + v_3)$, $(v_1 + v_4)$, $(v_2 + v_4)$, $(v_2 + v_5)$, $(v_3 + v_1)$ and $(v_5 - v_3)$ are positive and negative prime numbers respectively.

$$\chi(H) = 3 = \chi_p(H) \qquad (2)$$

Let the graph $\boldsymbol{\chi_p(G \otimes H)}$, we divide the vertex of $\boldsymbol{\chi_p(G \otimes H)}$ into three disjoint sets as below:

$T_1 = \{(u_1 v_i) / 0 < i \le 5\}$
T2 = {(u2vi) / 0 < i ≤ 5}
T3 = {(u3vi) / 0 < i ≤ 5}

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The graph $\chi_p(G \otimes H)$, which has prime numbers of vertices on addition and subtraction, we get positive and negative prime numbers respectively.

$\chi(G \otimes H) = 3 = \chi_p(G \otimes H)$ (3)

From (1),(2) and (3),we get

$\chi(G \otimes H) \leq \min\{\chi(G), \chi(H)\}$

$\chi_p(G \otimes H) \leq \min\{\chi_p(G), \chi_p(H)\}$

**Example 2:**

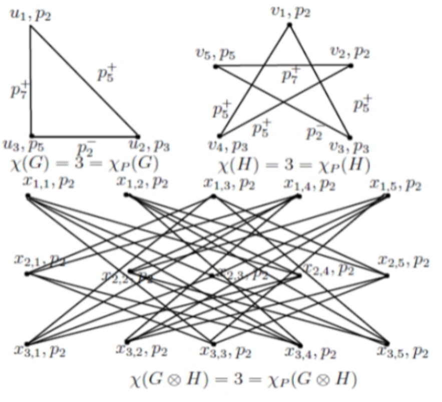


**Fig. 2**

**Theorem 3:** The tensor product of any complete graph and complete bipartite graph Bn,n is bipartite graph, then $\chi_p(G \otimes H) \leq \min\{\chi_p(G), \chi_p(H)\}$ does not exit.

**Example 3:**

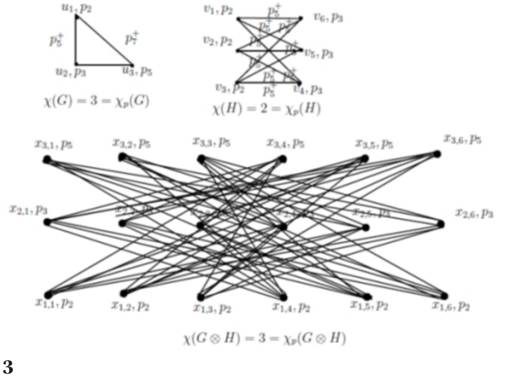


**Fig. 3**

**Theorem 4:** The tensor product of any connected graph and complete bipartite graph, then $\chi_p(G \otimes H) \leq \min\{\chi_p(G), \chi_p(H)\}$.

**Remark:** The complete graph Kn (n>3) whose edges are satisfies the prime numbers except some edges.

Ex: $K_5, K_6, K_7$

**Conclusion:** In this paper, we observed that the chromatic prime number of tensor product graph does not possess prime chromatic number for all order and size.

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