ORIGINAL RESEARCH PAPER | Mathematics | Volume - 7 | Issue - 4 | April-2017 | ISSN - 2249-555X | IF : 4.894 | IC Value : 79.96

PRIME LABELING FOR SOME VANESSA RELATED GRAPHS

KEYWORDS: Prime Labeling, Prime Graph, Duplication, Fusion, Switching.

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ABSTRACT In this paper, we investigate prime labeling for some graphs related to Vanessa graph. We discuss prime labeling in the context of some graph operations namely duplication, fusion, switching in Vanessa graph V_n.

1. Introduction

In this paper, we consider only simple, finite, undirected and non – trivial graph $G = (V(G), E(G))$ with the vertex set $V(G)$ and the edge set $E(G)$. For notations and terminology we refer to Bondy and Murthy[1]. The notion of a prime labeling was introduced by Roger Entringer and was discussed in a paper by Tout. A[9]. Many researchers have studied prime graph for example in Fu. H[4] have proved that the path P_n on n vertices is a prime graph. In Deretsky. T [3] have proved that the Cycle C_n on n vertices is a prime graph. Lee. S [6] have proved that Wheel W_n is a prime graph iff. n is even. In [8] S. Meena and K. Vaithilingam have proved the prime labeling 'for some Fan related graphs. In [7] S. Meena and P. Kavitha have proved the prime labeling for some Butterfly related graphs. For latest survey on graph labeling we refer to [5] (Gallian. J. A., 2009). Vast amount of literature is available on different types of graph labeling and more than 1000 research papers have been published so far in past four decades.

2. Preliminary Definitions

Definition[8]

Let $G = (V(G), E(G))$ be a graph with p vertices. A bijection $f : V(G) \to \{1, 2, \dots, p\}$ is called a *prime labeling* if for each edge $e = uv$, $\gcd\{f(u), f(v)\} = 1$. A graph which admits prime labeling is called a *prime graph*

Definition [8]

Duplication of a vertex v_k of a graph G produces a new graph G_1 by adding a vertex v_k' with $N(v_k') = N(v_k)$. In other words a vertex v_k' is said to be a duplication of v_k if all the vertices which are adjacent to v_k are now adjacent to v_k' also.

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Definition [8]

Let u and v be two distinct vertices of a graph G. A new graph G_1 is constructed by *identifying(fusing)* two vertices u and v by a single vertex x is such that every edge which was incident with either u or v in G is now incident with x in G_1.

Definition [8]

A *vertex switching* G_v of a graph G is obtained by taking a vertex v of G, removing all the entire edges incident with v and adding edges joining v to every vertex which are not adjacent to v in G.

Definition[2]

A $k-$ coloring of a graph $G = (V, E)$ is a function $c : V \to C$, where $|c| = k$. (Most often we use $c = [k]$). Vertices of the same color form a color class. A coloring is *proper* if adjacent vertices have different colors. A graph is $k-$ *colorable* if there is a proper $k-$ coloring. The chromatic number $\chi(G)$ of a graph G is the minimum k such that G is $k-$ colorable.

3. Prime Labeling For Some Vanessa Related Graphs

3.1. Vanessa graph

A *Vanessa graph* $V_n, (n \geq 2)$ can be constructed by two fan graphs $F_n, (n \geq 2)$ of the same order sharing a common vertex with n number of pendant vertices, where n is any positive integer. i.e., $V_n = 2F_n + K_{1,n}$.

Example 3.2.

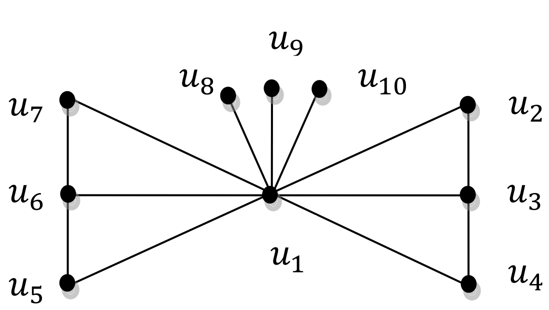

Figure 3.1. Vanessa graph V_n

Theorem 3.3. The Vanessa graph V_n is a prime graph, where n is any positive integer.

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Proof. Let V_n be the Vanessa graph with vertices $\{u_1, u_2, \ldots, u_{3n+1}\}$. Let $E(V_n)$ be the edges of the Vanessa graph where $E(V_n) = \{u_1u_i / 2 \le i \le 3n+1\} \cup \{u_iu_{i+1} / 2 \le i \le n-1\} \cup \{u_iu_{i+1} / n+1 \le i \le 2n-2\}$. Here $|V(V_n)| = 3n+1$, where n is any positive integer.

Define a labeling $f : V(V_n) \to \{1, 2, \ldots, 3n+1\}$ as follows.

$$f(u_1) = 1$$

$$f(u_i) = i \quad \text{for } 2 \le i \le 3n+1$$

Clearly vertex labels are distinct. Then for any edge $e = u_1u_i \in V_n$, $\gcd(f(u_1), f(u_i)) = \gcd(1, f(u_i)) = 1$ for $i = 1, 2, \ldots, n, n+1, \ldots, 3n+1$ and for any edge $e = u_iu_{i+1} \in V_n$, $\gcd(f(u_i), f(u_{i+1})) = 1$ for $1 \le i \le n-1$ and $n+1 \le i \le 2n-2$. Since it is consecutive positive integers. Thus labeling defined above gives a prime labeling for a graph V_n. Thus V_n is a prime graph.

Example 3.4.

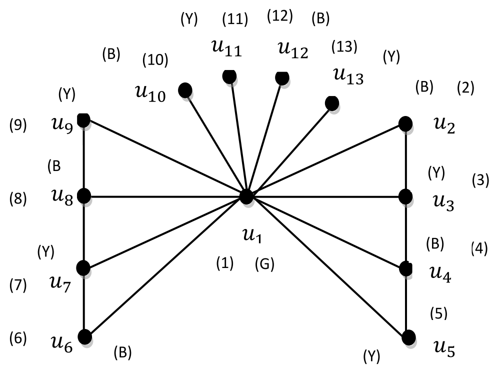

Figure 3.2. Prime labeling for V_4

Theorem 3.5. The graph obtained by duplicating a vertex u_k to u_k' of vanessa graph V_n is a prime graph, where n is any positive integer.

Proof Let G_k be the graph obtained by duplicating the vertex u_k in V_n , where n is any positive integer. Let u_k' be the duplication of u_k in G_k. Then $|V(G_k)| = 3n+2$.

We define a labeling $f : V(G_k) \to \{1, 2, \ldots, 3n+2\}$ as follows.

$$f(u_1) = 1$$

$$f(u_2) = 2$$

$$f(u_k) = k+1$$

$$f(u_k') = k$$

$$f(u_i) = i+1 \quad \text{for } k+1 \le i \le 3n+1$$

Clearly vertex labels are distinct. Then f admits prime labeling. Thus G_k is a prime graph.

Example 3.6.

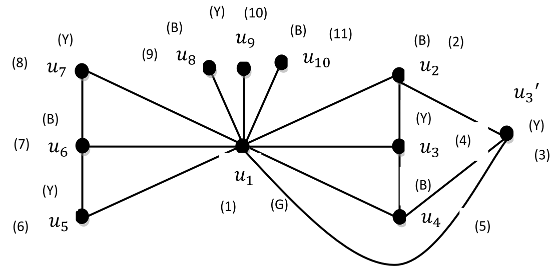

Figure 33. Duplication of u_3 in V_3.

Theorem 3.7. The graph obtained by duplicating a pendant vertex u_k to u_k' of vanessa graph V_n is a prime graph, where n is any positive integer.

Proof Let G_k be the graph obtained by duplicating the pendant vertex u_k in V_n , where n is any positive integer. Let u_k' be the duplication of pendant vertex u_k in G_k. Then $|V(G_k)| = 3n + 2$.

We define a labeling $f : V(G_k) \rightarrow \{1, 2, \ldots, 3n + 2\}$ as follows.

$$f(u_1) = 1$$

$$f(u_k) = k + 1$$

$$f(u_k') = k$$

$$f(u_i) = i \quad \text{for } 2 \leq i \leq 2n + 1$$

$$f(u_i) = i + 1 \quad \text{for } 2n + 2 \leq i \leq 3n + 1$$

Clearly vertex labels are distinct. Then f admits prime labeling. Thus G_k is a prime graph.

Example 3.8.

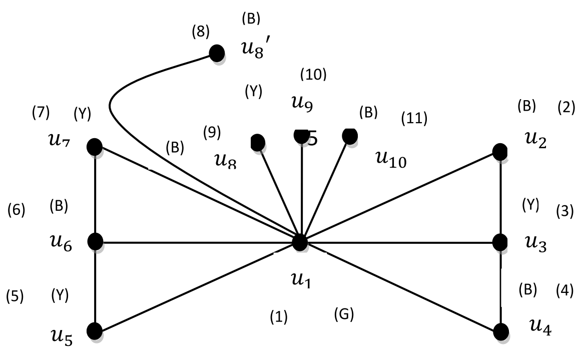

Figure 3.4. Duplication of a pendant vertex u_8 in V_3.

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Theorem 3.9. The graph obtained by duplicating an apex vertex u_1 to u_1' of vanessa graph V_n is a prime graph, where n is any positive integer.

Proof Let G_k be the graph obtained by duplicating an apex vertex u_1 in V_n , where n is any positive integer. Let u_1' be the duplication of an apex vertex u_1 in G_k. Then $|V(G_k)| = 3n + 2$.

We define a labeling $f : V(G_k) \to \{1, 2, \dots, 3n + 2\}$ as follows.

$$f(u_i) = i \quad \text{for } 1 \le i \le 2n$$

$$f(u_i) = i + 1 \quad \text{for } 2n + 1 \le i \le 3n + 1$$

$$f(u_1') = 7$$

Clearly vertex labels are distinct. Then f admits prime labeling. Thus G_k is a prime graph.

Example 3.10.

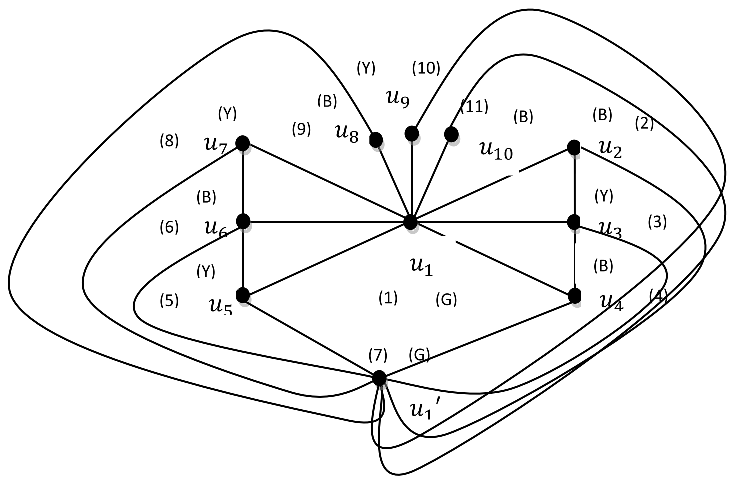

Figure 3.5. Duplication of an apex vertex u_1 in V_3.

Theorem 3.11. The graph obtained by fusing the vertex u_i with u_k (where $d(u_i, u_k) \ge 3$) in vanessa graph V_n is a prime graph, where n is any positive integer.

Proof Let $V(V_n) = \{u_1, u_2, \dots, u_{3n+1}\}$.

$$E(V_n) = \{u_1 u_i / 2 \le i \le 3n + 1\} \cup \{u_i u_{i+1} / 2 \le i \le n - 1\} \cup$$
$$\{u_i u_{i+1} / n + 1 \le i \le 2n - 2\}.$$

Let G_k be the graph obtained by fusing the vertex u_i with u_k in V_n. Here $|V(G_k)| = 3n$. For n and k are both odd or even.

Define a labeling $f: V(G_k) \to \{1,2, \dots ,3n\}$ as follows

$$f(u_5) = 5 = f(u_6)$$

$f(u_i) = i$ for $1 \leq i \leq n-1$

$f(u_i) = i-1$ for $n+1 \leq i \leq 3n+1$

Then f admits prime labeling. According to this pattern the vertices are labeled such that for any edge $e = u_i u_k \in G_k$, $\gcd(f(u_i), f(u_k)) = 1$.Clearly vertex labels are distinct. Thus we proved that the graph under consideration admits prime labeling. That is, the graph obtained by fusing (identifying) any two vertices u_i and u_k (where $d(u_i, u_k) \geq 3$) of Vanessa graph V_n is a prime graph.

Example 3.12.

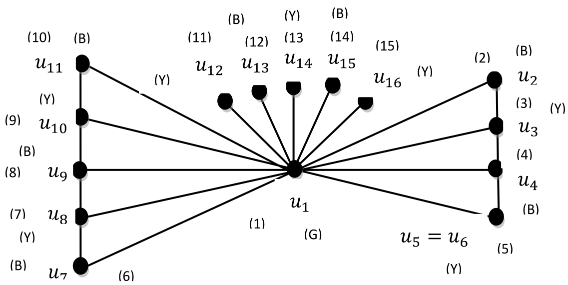

Figure 3.6. Fusion of u_5 and u_6 in V_5.

Theorem 3.13. The graph obtained by fusing the pendant vertex u_i with u_k (where $d(u_i, u_k) \geq 3$) in vanessa graph V_n is a prime graph, where n is any positive integer.

Proof Let $V(V_n) = \{u_1, u_2, \dots, u_{3n+1}\}$.

$$E(V_n) = \{u_1 u_i / 2 \leq i \leq 3n+1\} \cup \{u_i u_{i+1} / 2 \leq i \leq n-1\} \cup$$
$$\{u_i u_{i+1} / \, n+1 \leq i \leq 2n-2\}.$$

Let G_k be the graph obtained by fusing the pendant vertex u_i with u_k in V_n. Here $|V(G_k)| = 3n$. For n and k are both odd or even.

Define a labeling $f: V(G_k) \to \{1,2, \dots, 3n\}$ as follows

$f(u_i) = i$ for $1 \leq i \leq 2n+1$

$f(u_{12}) = 12 = f(u_{13})$

$f(u_i) = i-1$ for $2n+4 \leq i \leq 3n+1$

Then f admits prime labeling. According to this pattern the vertices are labeled such that for any edge $e = u_i u_k \in G_k$, $\gcd(f(u_i), f(u_k)) = 1$. Clearly vertex labels are distinct. Thus we proved that the graph under consideration admits prime labeling. That is, the graph obtained by fusing (identifying) any two pendant vertices u_i and u_k (where $d(u_i, u_k) \geq 3$) of vanessa graph V_n is a prime graph.

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Example 3.14.

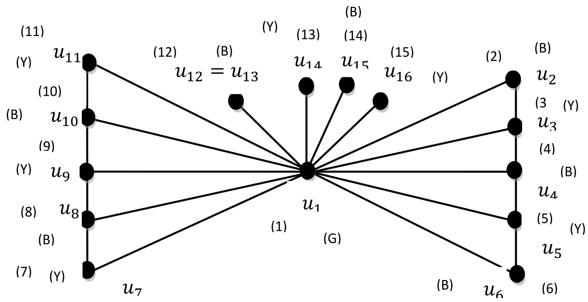

Figure 3.7. Fusion of the pendant vertices u_{12} and u_{13} in V_5.

Theorem 3.15. The graph obtained by fusing an apex vertex u_1 with u_k (where $d(u_1, u_k) \geq 3$) in vanessa graph V_n is a prime graph, where n is any positive integer.

Proof Let $V(V_n) = \{u_1, u_2, \ldots, u_{3n+1}\}$.

$$E(V_n) = \{u_1u_i / 2 \leq i \leq 3n+1\} \cup \{u_iu_{i+1} / 2 \leq i \leq n-1\} \cup$$
$$\{u_iu_{i+1} / \, n+1 \leq i \leq 2n-2\}.$$

Let G_k be the graph obtained by fusing an apex vertex u_1 with u_k in V_n. Here $|V(G_k)| = 3n$. For n and k are both odd or even.

Define a labeling $f: V(G_k) \to \{1,2, \ldots, 3n\}$ as follows

$$f(u_1) = 1 = f(u_2)$$
$$f(u_i) = i - 1 \qquad \text{for } 3 \leq i \leq 3n+1$$

Then f admits prime labeling. According to this pattern the vertices are labeled such that for any edge $e = u_1u_k \in G_k$, $\gcd(f(u_1), f(u_k)) = 1$. Clearly vertex labels are distinct. Thus we proved that the graph under consideration admits prime labeling. That is, the graph obtained by fusing (identifying) any two vertices u_1 with u_k (where $d(u_1, u_k) \geq 3$) of vanessa graph V_n is a prime graph.

Example 3.16.

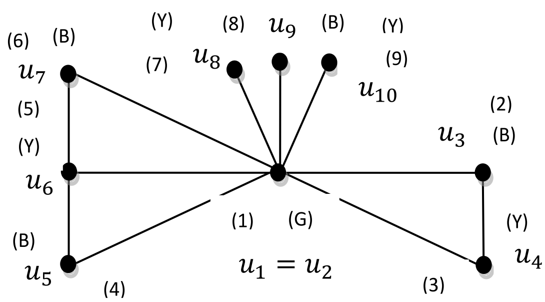

Figure 3.8. Fusion of u_1 and u_2 in V_3.

Theorem 3.17. The switching of any vertex u_k in a vanessa graph V_n produces a Prime graph, where n is any positive integer.

Proof Let $V(V_n) = \{u_1, u_2, \dots, u_{3n+1}\}$.

$$E(V_n) = \{u_1u_i/2 \le i \le 3n+1\} \cup \{u_iu_{i+1}/2 \le i \le n-1\} \cup \{u_iu_{i+1}/\, n+1 \le i \le 2n-2\}.$$

Let G_u be the graph obtained by switching any vertex u_k in V_n. Here $|V(G_u)| = 3n+1$.

Define a labeling $f: V(G_u) \to \{1,2,\dots,3n+1\}$ as follows

$$f(u_1) = 1$$
$$f(u_2) = 7$$
$$f(u_i) = i-1 \text{ for } 3 \le i \le 3n+1$$

Then for any edge $e = u_iu_{i+1} \in G_u$, $\gcd(f(u_i), f(u_{i+1})) = 1$ and for any edge $e = u_1u_i \in G_u$, $\gcd(f(u_1), f(u_i)) = \gcd(1, f(u_i)) = 1$. Clearly vertex labels are distinct. Then f admits prime labeling. Thus G_u is a prime graph.

Example 3.18.

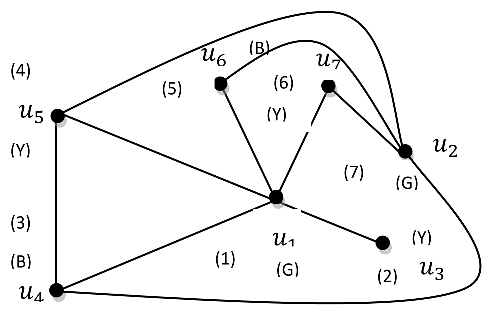

Figure 3. 9.Switching the vertex u_2 in V_2.

Theorem 3.19. The switching of a pendant vertex u_k in a vanessa graph V_n produces a Prime graph, where n is any positive integer.

Proof Let $V(V_n) = \{u_1, u_2, \dots, u_{3n+1}\}$.

$$E(V_n) = \{u_1u_i/2 \le i \le 3n+1\} \cup \{u_iu_{i+1}/2 \le i \le n-1\} \cup \{u_iu_{i+1}/\, n+1 \le i \le 2n-2\}.$$

Let G_u be the graph obtained by switching any pendant vertex u_k in V_n. Here $|V(G_u)| = 3n+1$.

Define a labeling $f: V(G_u) \rightarrow \{1,2,\ldots,3n+1\}$ as follows

$$f(u_i) = i \quad \text{for} \quad 1 \leq i \leq 3n+1$$

Then for any edge $e = u_i u_{i+1} \in G_u$, $\gcd\big(f(u_i), f(u_{i+1})\big) = 1$ and for any edge $e = u_1 u_i \in G_u$, $\gcd\big(f(u_1), f(u_i)\big) = \gcd\big(1, f(u_i)\big) = 1$. Clearly vertex labels are distinct. Then f admits prime labeling. Thus G_u is a prime graph.

Example 3.20.

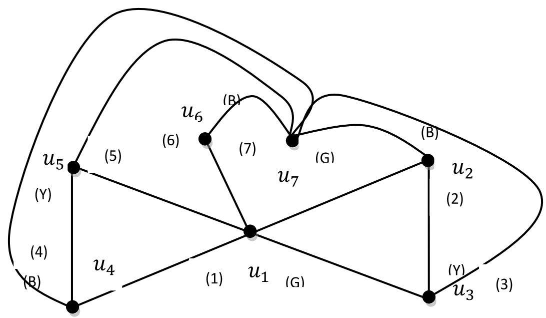

Figure 3.10 Switching the pendant vertex u_7 in V_2.

Theorem 3.21. The switching of an apex vertex u_1 in a vanessa graph V_n produces a Prime graph, where n is any positive integer.

Proof Let $V(V_n) = \{u_1, u_2, \ldots, u_{3n+1}\}$.

$$E(V_n) = \{u_1 u_i / 2 \leq i \leq 3n+1\} \cup \{u_i u_{i+1} / 2 \leq i \leq n-1\} \cup$$

$$\{u_i u_{i+1} / \ n+1 \leq i \leq 2n-2\}.$$

Let G_u be the graph obtained by switching an apex vertex u_1 in V_n. Here $|V(G_u)| = 3n+1$.

Define a labeling $f: V(G_u) \rightarrow \{1,2,\ldots,3n+1\}$ as follows

$$f(u_i) = i \quad \text{for} \quad 1 \leq i \leq 3n+1$$

Clearly vertex labels are distinct. Then f admits prime labeling. Thus G_u is a prime graph and it is a disconnected graph.

Example 3.22.

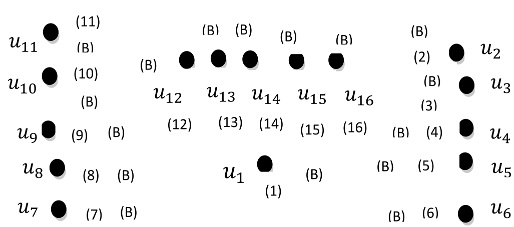

Figure 3.11 Switching an apex vertex u_1 in V_5.

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Conclusion

In this paper we proved that the Vanessa graph V_n, duplication of the Vanessa graph V_n, fusing of the Vanessa graph V_n, switching of the Vanessa graph V_n are prime graphs. There may be many interesting prime graphs can be constructed in future.

References:

[1] Bondy.J.A and Murthy.U.S.R, "Graph Theory and Applications" (North-Holland), Newyork, 1976.
[2] Brooks R. L., "On colouring the nodes of a network". Proc. Cambridge Phil. Soc. 37:194–197, 1941.
[3] Dretskyetal.T "on Vertex Prime labeling of graphs in graph theory", Combinatories and applications vol.1 Alari.J (Wiley. N.Y.) 299-359, 1991.
[4] Fu.H.C and Huany.K.C "on Prime labeling Discrete Math",127, 181-186, 1994.
[5] Gallian J.A, "A dynamic survey of graph labeling". The Electronic Journal of Combinations 16 # DS6, 2009.
[6] Lee.S.M, Wui.L and Yen.J "on the amalgamation of Prime graphs Bull", Malaysian Math.Soc.(Second Series) 11, 59-67, 1988.
[7] Meena .S and Kavitha .P "Prime labeling for some butterfly related graphs", International Journal of Mathematical Archive – 5(10), 15 – 25, 2014.
[8] Meena .S and Vaithilingam .K "Prime labeling for some fan related graphs", International Journal of Engineering Research & Technology(IJERT) Vol. 1, Issue 9, 2012.
[9] Tout.A Dabboucy.A.N and Howalla.K "Prime labeling of graphs". Nat.Acad.Sci letters 11, 365-368, 1982.