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ECONOMIC EFFICIENCY ESTIMATION - INTERVAL DATA

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ABSTRACT Data Envelopment Analysis is linear programming based procedure that can be used to assess Economic efficiency of decision making units. If data uncertainty prevails where inputs and outputs are assumed to lie in intervals then economic efficiencies also belong to intervals. In the presence of interval data we formulated two pairs of economic efficiency problems under weak and strong optimistic and pessimistic view points. The economic efficiency intervals are shown as nested.

KEYWORDS : Data Envelopment Analysis, Interval Data, Economic Efficiency

1. INTRODUCTION

Data Envelopment Analysis (DEA) is linear programming based technique, implemented to measure efficiency scores of decision making units (DMU). Input and output vectors of firms that are in competition determine the production possibility set whose boundary plays a predominant role, not only yielding efficiency scores but targets to the interior firms that are inefficient. The inefficient decision making unit shall strive hard to reach the frontier travelling along the path determined by a distance function. Choice of distance function from its class depends on the objectives of the production manger or the policy maker. Ex post production possibilities do not allow input or output substitution, in this case the production manager chooses radial distance functions to reach the boundary of the production possibility set. In short run, input or output substitution are not possible, consequently the appropriate distance function is radial distance function that allows input contraction or output expansion along a ray. Ex ante production possibilities allow movements along input or output isoquant. In long run, input or output substitution is possible and the policy maker chooses non-radial distance functions.

Among the technology sets the convex production possibility sets (CCR, 1978; BCC, 1984) are very widely used for efficiency measurement. The CCR technology set is based on the axioms of inclusion, free disposability, ray unboundedness and minimum extrapolation. It is a convex cone. The BCC technology set is based on the axioms of inclusion, convexity, free disposability and minimum extrapolation.

$$T^{BCC} \subseteq T^{CCR}$$

where T^{CCR} and T^{BCC} are the CCR and BCC production possibility sets respectively. The boundary of these production possibility sets is piecewise linear and an arbitrary point of the boundary is denoted by,

$$\left(\sum_{j=1}^{n} \lambda_j x_j , \sum_{j=1}^{n} \lambda_j y_j \right)$$

Varying λ_j any point on the boundary can be reached.

where $x_j \in R_+^m,\ y_j \in R_+^s$,

(i) $\lambda_j \geq 0$, in CCR formulation and

(ii) $\lambda_j \geq 0, \sum_{j=1}^{n} \lambda_j = 1$ in BCC formulation

The frontiers of CCR and BCC production possibility sets are determined by the inputs and outputs of extremely efficient decision making units. The CCR and BCC problems, respectively assume constant and variable returns to scale. Following the axioms of CCR/BCC several non-oriented distance functions were introduced. Very widely implemented of these are the Russell non-radial slack based, Hyperbolic Graph (Fare et.al 1978) and directional (Chambers et.al 1996) distance functions. The Russell non-radial efficiency measure seeks component wise reduction of inputs and / or component wise augmentation of outputs before the distance function reaches the frontier. The slack based efficiency measurement optimizes sum of slacks, producing non-radial movements before frontier of the production possibility set is reached. The Hyperbolic Graph efficiency measurement seeks simultaneous reduction of inputs and augmentation of outputs along hyperbolic path to reach the boundary of the production possibility set. The directional distance functions provide a wide class of distance functions which include radial distance functions.

2. FACTOR MINIMAL COST FUNCTION

Data Envelopment Analysis (DEA) can handle the assessment of not only the profitable, but also the non-profitable organizations with comfortable ease. In addition to input and output values, if input prices are available factor minimal cost can be evaluated solving the following linear programming problem (Fare et.al 1978):

$$Q(y,p) = \underset{x}{Min}\ px$$

s.t
$$\sum_{j=1}^{n} \lambda_j x_{ij} \le x_i,\ i \in M$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{r0},\ r \in S$$

$$\lambda_j \ge 0,\ j \in N$$

The ratio of potential cost to actual cost defines input cost efficiency.

$$CE(y,p) = \frac{Q(y,p)}{px}$$

$$0 \le CE(y,p) \le 1$$

$Q(y,p)$ possesses the following properties:

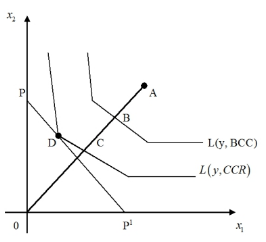
P.1. $Q(0,p) = 0$. The minimal cost incurred to produce null output vector is zero

P.2. $Q(y,0) = 0$. No free lunch.

P.3. $y_1 \ge y_2 \Rightarrow Q(y_1,p) \ge Q(y_2,p)$. Larger output production incurs larger minimal cost.

P.4 . $Q(y,\lambda p) = \lambda Q(y,p)$. If prices change by λ , then minimal cost also changes by λ . P.5. $p_1 \ge p_2 \Rightarrow Q(y_1,p_1) \ge Q(y_2,p_2)$.

Achievement of Cost efficiency $(CE(u,p) = 1)$ requires movement along the isoquant which requires input substitution. Input substitution requires change in technique which is possible in ex ante production. Thus, achievement of cost efficiency is a long run phenomena, that may be achieved by an interior firm in three steps. In step one, in short run, the entrepreneur should reach the variable returns to scale frontier, in step two radial movement from variable returns to constant returns to scale frontier by radial reduction of inputs and in step three a point on the frontier at which factor cost is minimized is reached, which is a non-radial movement.



$A \rightarrow B$ (Radial movement to reach the BCC frontier)

$B \rightarrow C$ (Radial movement to reach the CCR frontier)

$C \rightarrow D$ (Non-radial movement to reach the cost minimized targets of DMUs)

$L(y,BCC)$: Input level set of BCC (cross section of BCC PP set)

$L(y,CCR)$: Input level set of CCR (cross section of PP set)

PP' : Cost line

The targets assigned by cost minimization are larger than the radial input targets, for an inefficient decision making unit. Achievement of these targets is possible only in long run. To implement Data Envelopment Analysis, the decision making units are assumed to combine similar inputs to produce similar outputs. Following a property of linear programming problem, an increase in the number of inputs and / or outputs leads to increase in the efficiency of decision making units, even if the input or output freshly augmented is irrelevant. Therefore, the researcher shall take appropriate care while the inputs and outputs are selected for the study. In distorted data pictures, crisp data of inputs and outputs are not available. Data Envelopment Analysis can handle data with missing values or in the form of intervals with lower and upper bounds specified for input and output variables. The radial efficiency measurement of CCR and BCC formulations can be extended to interval data. Two production frontiers, namely, the optimistic and pessimistic frontiers are visualized onto which the input and output vectors of interior firms are projected. Optimistic frontier is determined by upper bounds of output variables and lower bounds of input variables. Pessimistic frontier is determined by lower bounds of output variables and upper bounds of input variables.

3. OPTIMISTIC AND PESSIMISTIC FRONTIERS

An optimistic view point is to produce outputs at upper bounds, employing inputs at lower bounds. On the other hand, a pessimistic view point is to employ inputs at upper bounds to produce outputs at lower bounds. Thus, bounds of inputs and outputs are projected onto optimistic and pessimistic frontiers to arrive at efficiencies in bounded form. The projections result in upper and lower bounds of true efficiency scores. Following, we visualize two situations. (Wang et.al 2005, Toloo et.al 2008, M.Venkata Subba Reddy, 2015)

(i) **Weak optimistic view point:** Weak optimistic producer assumes best performance by him, but worst performance by his rivals and rivals include him also, since his inputs and outputs are augmented to the reference technology.

(ii) **Weak pessimistic view point:** under this hypothesis the entrepreneur assumes, worst performance by him and best performance by his rivals including himself.

Weak optimistic constraints:

$$\sum_{j=1}^{n} \lambda_j x_j^U \le x_0^L$$

$$\sum_{j=1}^{n} \lambda_j y_j^L \geq y_0^U \quad .... \quad (3.1)$$

$$\lambda_j \geq 0$$

Weak pessimistic constraints:

$$\sum_{j=1}^{n} \lambda_j x_j^L \leq x_0^U$$

$$\sum_{j=1}^{n} \lambda_j y_j^U \geq y_0^L \quad .... \quad (3.2)$$

$$\lambda_j \geq 0$$

(iii) **Strong optimistic view point:** The producer assumes best performance by him and worst performance by his rivals excluding himself.

(iv) **Strong pessimistic view point:** Under this hypothesis the producer under evaluation assumes worst performance by him and best performance by his rivals excluding himself.

Strong optimistic constraints:

$$\sum_{j \neq 0} \lambda_j x_j^U + \lambda_0 x_0^L \leq x_0^L$$

$$\sum_{j \neq 0} \lambda_j y_j^L + \lambda_0 y_0^U \geq y_0^U \quad ....(3.3)$$

$$\lambda_j \geq 0, \forall j$$

Strong optimistic constraints:

$$\sum_{j \neq 0} \lambda_j x_j^L + \lambda_0 x_0^U \leq x_0^U$$

$$\sum_{j \neq 0} \lambda_j y_j^U + \lambda_0 y_0^L \geq y_0^L \quad .....(3.4)$$

$$\lambda_j \geq 0$$

4. ECONOMICS EFFICIENCY – INTERVAL DATA:

Let $p_i^{\min} = \underset{j}{Min}\, p_{ij}$, i = 1, 2, ….., m

To measure economic efficiency, we formulate and solve the following linear programming problem:

$$EE = \underset{x,\lambda}{Min} \frac{\sum_{i=1}^{m} p_i^{\min} x_i}{\sum_{i=1}^{m} p_{i0} x_{i0}}$$

s.t $\sum_{j=1}^{n} \lambda_j x_{ij} = x_i$, i = 1, 2, ……, m

…… (4.1)

$\sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{i0}$, r = 1, 2, ……., s

$\lambda_j \geq 0$, j = 1, 2, ……., n

Problem (4.1) can be alternatively expressed as,

$$EE = \underset{u}{Max} \sum_{r=1}^{s} y_{r0} u_r$$

s.t $$\sum_{r=1}^{s} y_{rj} u_r \leq \frac{\sum_{i=1}^{m} p_i^{\min} x_{ij}}{\sum_{i=1}^{m} p_{i0} x_{i0}}, \ j \neq 0$$

……..(4.2)

$$\sum_{r=1}^{s} y_{r0} u_r \leq \frac{\sum_{i=1}^{m} p_i^{\min} x_{i0}}{\sum_{i=1}^{m} p_{i0} x_{i0}}, \ j \neq 0$$

$u_r \geq 0$, r = 1, 2, ……., s

5. WEAK OPTIMISTIC VIEW POINT

Let $x_{ij} \in \left[x_{ij}^L, x_{ij}^U \right]$

$y_{rj} \in \left[y_{rj}^L, y_{rj}^U \right]$

$$EE_U = \underset{u}{Max} \sum_{r=1}^{s} y_{r0}^U u_r$$

s.t $$\sum_{r=1}^{s} y_{rj}^L u_r \leq \frac{\sum_{i=1}^{m} p_i^{\min} x_{ij}^U}{\sum_{i=1}^{m} p_{i0} x_{i0}^L}, \ j \neq 0$$

… (5.1)

$$\sum_{r=1}^{s} y_{r0}^L u_r \leq \frac{\sum_{i=1}^{m} p_i^{\min} x_{i0}^U}{\sum_{i=1}^{m} p_{i0} x_{i0}^L}$$

WEAK PESSIMISTIC VIEW POINT

$$EE_L = \underset{u}{Max} \sum_{r=1}^{s} y_{r0}^{L} u_r$$

$$\text{s.t} \quad \sum_{r=1}^{s} y_{rj}^{U} u_r \le \frac{\sum_{i=1}^{m} p_i^{\min} x_{ij}^{L}}{\sum_{i=1}^{m} p_{i0} x_{i0}^{U}}, \; j \ne 0$$

…… (5.2)

$$\sum_{r=1}^{s} y_{r0}^{L} u_r \le \frac{\sum_{i=1}^{m} p_i^{\min} x_{i0}^{L}}{\sum_{i=1}^{m} p_{i0} x_{i0}^{U}}, \; j = 0$$

THEOREM (1): $EE_L \le EE_U$

Proof:

$$\sum_{r=1}^{s} y_{rj}^{L} u_r \le \sum_{r=1}^{s} y_{r0}^{U} u_r \le \frac{\sum_{i=1}^{m} p_i^{\min} x_{ij}^{L}}{\sum_{i=1}^{m} p_{i0} x_{i0}^{U}} \le \frac{\sum_{i=1}^{m} p_i^{\min} x_{ij}^{U}}{\sum_{i=1}^{m} p_{i0} x_{i0}^{L}}$$

$$\sum_{r=1}^{s} y_{rj}^{U} u_r \le \frac{\sum_{i=1}^{m} p_i^{\min} x_{ij}^{L}}{\sum_{i=1}^{m} p_{i0} x_{i0}^{U}} \Rightarrow \sum_{r=1}^{s} y_{r0}^{L} u_r \le \frac{\sum_{i=1}^{m} p_i^{\min} x_{ij}^{U}}{\sum_{i=1}^{m} p_{i0} x_{i0}^{L}}$$

$$\sum_{r=1}^{s} y_{r0}^{L} u_r \le \sum_{r=1}^{s} y_{r0}^{U} u_r \le \frac{\sum_{i=1}^{m} p_i^{\min} x_{ij}^{L}}{\sum_{i=1}^{m} p_{i0} x_{i0}^{U}} \le \frac{\sum_{i=1}^{m} p_i^{\min} x_{i0}^{U}}{\sum_{i=1}^{m} p_{i0} x_{i0}^{L}}$$

$$\sum_{r=1}^{s} y_{r0}^{U} u_r \le \frac{\sum_{i=1}^{m} p_i^{\min} x_{ij}^{L}}{\sum_{i=1}^{m} p_{i0} x_{i0}^{U}} \Rightarrow \sum_{r=1}^{s} y_{r0}^{L} u_r \le \frac{\sum_{i=1}^{m} p_i^{\min} x_{i0}^{U}}{\sum_{i=1}^{m} p_{i0} x_{i0}^{L}}$$

Every feasible solution of (5.2) is feasible for (5.1)

Optimal solution of (5.2) is feasible for (5.1)

Let $\hat{u}_r$ be optimal for (5.2) then, we have

$$EE_L \le EE_U$$

STRONG OPTIMISTIC VIEW POINT

$$\overline{EE_U} = \underset{u}{Max} \sum_{r=1}^{s} y_{r0}^{U} u_r$$

$$\text{s.t} \quad \sum_{r=1}^{s} y_{rj}^{L} u_r \le \frac{\sum_{i=1}^{m} p_i^{\min} x_{ij}^{U}}{\sum_{i=1}^{m} p_{i0} x_{i0}^{L}}, \; j \ne 0$$

…… (5.3)

$$\sum_{r=1}^{s} y_{r0}^{U} u_r \le \frac{\sum_{i=1}^{m} p_i^{\min} x_{ij}^{U}}{\sum_{i=1}^{m} p_{i0} x_{ij}^{L}}, \; j = 0$$

$u_r \ge 0$, r = 1, 2, …….., s

THEOREM (2): $\overline{EE_U} \le EE_U$

Proof:

$$\sum_{r=1}^{s} y_{r0}^{L} u_r \le \sum_{r=1}^{s} y_{r0}^{U} u_r \le \frac{\sum_{i=1}^{m} p_i^{\min} x_{i0}^{L}}{\sum_{i=1}^{m} p_{i0} x_{ij}^{L}} \le \frac{\sum_{i=1}^{m} p_i^{\min} x_{i0}^{U}}{\sum_{i=1}^{m} p_{i0} x_{i0}^{U}}$$

$$\sum_{r=1}^{s} y_{r0}^{U} u_r \le \frac{\sum_{i=1}^{m} p_i^{\min} x_{i0}^{L}}{\sum_{i=1}^{m} p_{i0} x_{i0}^{L}} \Rightarrow \sum_{r=1}^{s} y_{r0}^{L} u_r \le \frac{\sum_{i=1}^{m} p_i^{\min} x_{i0}^{U}}{\sum_{i=1}^{m} p_{i0} x_{i0}^{L}}$$

Every feasible solution of (5.3) is feasible solution of (5.2)

$$\Rightarrow \overline{EE_U} \le EE_U$$

THEOREM (3): $EE_L \le \overline{EE_L}$

Proof:

$$\overline{EE_L} = \underset{u}{Max} \sum_{r=1}^{s} y_{r0}^{L} u_r$$

s.t $$\sum_{r=1}^{s} y_{rj}^{U} u_r \le \frac{\sum_{i=1}^{m} p_i^{\min} x_{ij}^{L}}{\sum_{i=1}^{m} p_i x_{i0}^{U}}, \quad j \ne 0$$

… (5.4)

$$\sum_{r=1}^{s} y_{r0}^{L} u_r \le \frac{\sum_{i=1}^{m} p_i^{\min} x_{i0}^{U}}{\sum_{i=1}^{m} p_{i0} x_{i0}^{U}}, \quad j = 0$$

$$\sum_{r=1}^{s} y_{r0}^{L} u_r \le \sum_{r=1}^{s} y_{r0}^{U} u_r \le \frac{\sum_{i=1}^{m} p_i^{\min} x_{i0}^{L}}{\sum_{i=1}^{m} p_{i0} x_{i0}^{U}} \le \frac{\sum_{i=1}^{m} p_i^{\min} x_{i0}^{U}}{\sum_{i=1}^{m} p_{i0} x_{i0}^{U}}$$

$$\sum_{r=1}^{s} y_{r0}^{U} u_r \le \frac{\sum_{i=1}^{m} p_i^{\min} x_{i0}^{L}}{\sum_{i=1}^{m} p_{i0} x_{i0}^{U}} \Rightarrow \sum_{r=1}^{s} y_{r0}^{L} u_r \le \frac{\sum_{i=1}^{m} p_i^{\min} x_{i0}^{U}}{\sum_{i=1}^{m} p_{i0} x_{i0}^{U}}$$

Every feasible solution of (5.3) is a feasible solution of (5.4)

$$\Rightarrow EE_L \le \overline{EE}_L$$

THEOREM (4):

(i) $EE_L \le CE_L$ (ii) $EE_U \le CE_U$

Proof:

(i) $$CE_L = Max \sum_{r=1}^{s} y_{r0}^{L} u_r$$

s.t $$\sum_{r=1}^{s} y_{rj}^{U} u_r \le \frac{\sum_{i=1}^{m} p_{i0} x_{ij}^{L}}{\sum_{i=1}^{m} p_{i0} x_{i0}^{U}}, \quad j \ne 0$$

$$\sum_{r=1}^{s} y_{r0}^{L} u_r \le \frac{\sum_{i=1}^{m} p_{i0} x_{i0}^{L}}{\sum_{i=1}^{m} p_{i0} x_{i0}^{U}}, \quad j = 0$$

$$u_r \ge 0$$

$$\sum_{r=1}^{s} y_{r0}^{L} u_r \le \sum_{r=1}^{s} y_{r0}^{U} u_r \le \frac{\sum_{i=1}^{m} p_i^{\min} x_{i0}^{L}}{\sum_{i=1}^{m} p_{i0} x_{i0}^{U}} \le \frac{\sum_{i=1}^{m} p_i x_{i0}^{L}}{\sum_{i=1}^{m} p_{i0} x_{i0}^{U}}$$

$$\sum_{r=1}^{s} y_{r0}^{U} u_r \le \frac{\sum_{i=1}^{m} p_i^{\min} x_{i0}^{L}}{\sum_{i=1}^{m} p_{i0} x_{i0}^{L}} \Rightarrow \sum_{r=1}^{s} y_{r0}^{L} u_r \le \frac{\sum_{i=1}^{m} p_i x_{i0}^{L}}{\sum_{i=1}^{m} p_{i0} x_{i0}^{U}}$$

Every feasible solution of EE_L is feasible solution of CE_L

$$\Rightarrow EE_L \le CE_L$$

(ii) $$CE_U = \underset{u}{Max} \sum_{r=1}^{s} y_{r0}^{U} u_r$$

s.t $$\sum_{r=1}^{s} y_{rj}^{L} u_r \le \frac{\sum_{i=1}^{m} p_{i0} x_{ij}^{U}}{\sum_{i=1}^{m} p_{i0} x_{i0}^{L}}, \quad j \ne 0$$

$$\sum_{r=1}^{s} y_{r0}^{L} u_r \le \frac{\sum_{i=1}^{m} p_{i0} x_{i0}^{U}}{\sum_{i=1}^{m} p_{i0} x_{i0}^{L}}, \quad j = 0$$

$$u_r \ge 0$$

$$\sum_{r=1}^{s} y_{rj}^{L} u_r \le \frac{\sum_{i=1}^{m} p_i^{\min} x_{ij}^{U}}{\sum_{i=1}^{m} p_{i0} x_{i0}^{L}} \le \frac{\sum_{i=1}^{m} p_i x_{ij}^{U}}{\sum_{i=1}^{m} p_{i0} x_{i0}^{L}}$$

$$\sum_{r=1}^{s} y_{rj}^{L} u_r \le \frac{\sum_{i=1}^{m} p_i^{\min} x_{ij}^{U}}{\sum_{i=1}^{m} p_{i0} x_{i0}^{L}}$$

$$\Rightarrow \sum_{r=1}^{s} y_{rj}^{L} u_r \le \frac{\sum_{i=1}^{m} p_i x_{ij}^{U}}{\sum_{i=1}^{m} p_{i0} x_{i0}^{L}}$$

$$\sum_{r=1}^{s} y_{r0}^{L} u_r \le \frac{\sum_{i=1}^{m} p_i^{\min} x_{i0}^{U}}{\sum_{i=1}^{m} p_{i0} x_{i0}^{L}} \Rightarrow \sum_{r=1}^{s} y_{r0} u_r \le \frac{\sum_{i=1}^{m} p_{i0} x_{i0}^{U}}{\sum_{i=1}^{m} p_{i0} x_{i0}^{L}}$$

Every feasible solution of EE_U is a solution of CE_U

$$EE_U \le CE_U$$

6. CONCLUSIONS

(1) We have established nestedness as follows

$$\overline{EE_L} \le \overline{EE_L} \le \overline{EE_U} \le \overline{EE_U}$$

(2) The economic and cost efficiency are related as follows

$$EE_L \le CE_L$$

$$EE_U \le CE_U$$

$$\therefore CE_L \le CE_U$$

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