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Statistics

DIRECTIONAL DISTANCE FUNCTIONS-INTERVAL DATA

Dr.H.Ravi Sankar	Lecturer in Statistics, Loyola Degree College, Pulivendula,
Dr. M. Venkata Subba Reddy	Lecturer in Statistics, Loyola Degree College, Pulivendula,
Dr.Y.M.Chenna Reddy	Lecturer in Statistics, S.V.Degree College, Kadapa.
Dr. A. Naga Bhusana Reddy	Government College Men, Kadapa.

ABSTRACT In Data Envelopment Analysis applications, if crisp data are not available, and if the data are expressed in intervals, one seeks to estimate efficiency scores for decision making units. The scores obtained under optimistic and pessimistic hypotheses are also bounds but not crisp values. This study provides a methodology to evaluate bounds for directional distance efficiency scores, the production possibility set being the Free Disposable Hull. The methodology is implemented to obtain numerical result for total manufacturing sectors of 28 Indian States.

KEYWORDS : Data Envelopment Analysis, Directional Distance Functions, Free Disposable Hull, Interval Data.

1. INTRODUCTION:

Data Envelopment Analysis is a Linear programme based deterministic approach that estimates efficiencies of decision making units. The production possibility set that promoted DEA is built on axioms such as, inclusion, free disposability, convexity and minimum extrapolation. This technique was launched by Charnes, Cooper and Rhodes (1978) and improved by Bankar, Charnes and Cooper (1984). R.W. Shephard (1970) published a book on production and cost functions in which the concept of distance function and its properties were discussed. Shephard's distance functions are closely related to the CCR and BCC technical efficiency measures. Farrell (1957) was the first one who suggested a practical approach to measure radial efficiencies of production units. Farrell's efficiency measures are inversely related to Shephard's distance functions. DEA compares an inefficient Decision Making Unit's (DMU) coordinates with the coordinates of virtual DMU that operates on the frontier of the production possibility set. The efficient DMUs linear combinations or convex combinations refer to best practices. Tulkens (1993) removed the convexity axiom, with inclusion and free disposability and the minimum extrapolation axioms active, a non-convex production possibility set was introduced, resulting in Free Disposable Hull (FDH). Projections on to the non-convex frontier (FDH) provide not only shorter targets, but a single efficient peer for an inefficient decision making unit.

2. DISTNACE FUNCTIONS:

For measuring efficiency and setting targets for inefficient firms, the chief tool is distance function. Following Farrell and Shephard, numerous distance functions were introduced and combined with DEA constraint inequalities, by researchers.

3. DIRECTIONAL DISTANCE FUNCTION:

An important class of distance functions, called the Directional Distance Functions (DDF) were formulated by Chambers et.al (1996, 1998), for which the Farrell's (Shephard's) distance functions can be obtained as special cases. For any suitably structured production possibility set T, the directional distance function is formulated as,

$$D_T\left(x_0, y_0; g_x, g_y\right) = \text{Max } \beta$$

such that $\left(x_0 - \beta g_x, y_0 + \beta g_y\right) \in T$ …… (3.1)

g_x and g_y are the directional vectors along which inputs are contracted and outputs are expanded simultaneously. There are several applications of DDF. If only inputs are to be contracted, one can solve,

$$D_T\left(x_0, y_0; g_x, 0\right) = \text{Max } \beta$$

s.t $\left(x_0 - \beta g_x, y_0\right) \in T$ …… (3.2)

If situation demands only output expansion but not input contraction, the appropriate directional distance problem is,

$$D_T\left(x_0, y_0; 0, g_y\right) = \text{Max } \beta$$

s.t $\left(x_0, y_0 + \beta g_y\right) \in T$ …… (3.3)

The directional distance function is sensitive to the choice of the direction choosen for input contraction and / or output expansion. The directions are exogeneous for policy and regulatory applications, but endogeneous for internal performance evaluation.

4. IMPRECISE DATA:

In live situations, a firm manager is forced to take decisions based on imprecise data. One source of imprecise data is interval data where lower and upper bound values are available in the place of crisp data, Interval input and output data produce bounds for efficiency scores. This study provides closed form solution to lower and upper bounds of efficiency scores in the frame work of non-convex production possibility sets and directional distance functions. The developed measures are implemented for total manufacturing sectors of Indian States.

5. DIRECTIONAL EFFICIENCY – INTERVAL DATA – FDH ASSESSMENT:

For input and output variables if crisp data are not available, but bounds of the variable are known, the consequent efficiency scores also emerge in bounds. Let x_i^L and x_i^U be lower and upper bounds of input variable x_i; y_r^L and y_r^U be lower and upper bounds of output variable.

$$x_i^L \le x_i \le x_i^U, \; i \in M$$

$$y_r^L \le y_r \le y_r^U, \; r \in S$$

(i). Under pessimistic view point we postulate the following optimization problem (Wang et.al (2005), Toloo et.al (2008), Raza Farzipoor Sean (2010), Silva Portela et.al (2003), Soharb Kordrostami et.al (2008), Tome Entani et.al (2002), M. Venkata Subba Reddy et.al (2015))

$$\beta_{FDH}^{P} = \text{Max}\,\beta$$

$$\text{s.t} \quad x_k^L \le x_0^U - \beta x_k^L \qquad \ldots\ldots (5.1)$$

$$y_k^U \ge y_0^L + \beta y_k^U, \; k \in R$$

where R is the index set of the inputs and outputs of firms which dominate the decision making unit whose efficiency is under evaluation and the problem admits variable returns to scale.

The optimal solution of model (5.1) provides upper bound for the unknown directional distance function.

$$\beta_{FDH}^{P} = \underset{k}{\text{Max}}\,\beta_k = \underset{k}{\text{Max}}\,\underset{i}{\text{Min}}\left\{ \underset{i}{\text{Min}}\frac{x_{i0}^U}{x_{ik}^L} - 1,\; 1 - \text{Max}\frac{y_{r0}^L}{y_{rk}^U} \right\}$$

(ii). Under optimistic view point the following FDH directional distance problem is postulated:

$$\text{Max}\,\beta$$

$$\text{s.t} \quad x_k^U \le x_0^L - \beta x_k^U \qquad \ldots\ldots (5.2)$$

$$y_k^U \ge y_0^L + \beta y_k^U, \; k \in R$$

The problem admits variable returns to scale.

The optimal solution of problem (5.2) is as follows:

$$\beta_{FDH}^{O} = \underset{k}{\text{Max}}\,\beta_k = \underset{k}{\text{Max}}\left\{ \underset{i}{\text{Min}}\left(\frac{x_{i0}^L}{x_{ik}^U}\right) - 1,\; 1 - \underset{r}{\text{Max}}\left(\frac{y_{r0}^U}{y_{rk}^L}\right) \right\}$$

6. DATA:

The numerical example worked out refers to the data collected from Annual Survey of Industries for 2012-13. The variables of study are Fixed Capital and Total Persons Engaged as inputs and Net Value Added as output.

7. EMPERICAL RESULTS:

Model (5.1) is based on pessimistic approach, in which the producer whose efficiency is under evaluation is hypothesised employ inputs at upper bound but produce outputs at lower bound. It is further assumed that his rivals employ inputs at lower bounds but produce outputs at upper bound $\left(x_0^U, y_0^L\right)$ is projected onto the pessimistic frontier production function which serves as boundary of FDH technology in the direction of one of the rival decision making unit that dominates. This leads to the directional distance efficiency score β_{FDH}^{P}. This provides upper bound to the unknown efficiency score.

LOWER AND UPPER BOUNDS OF DIRECTIONAL EFFICIENCY

S.No	Total Manufacturing Sector	β_{FDH}^{O}	Efficient Peer	β_{FDH}^{P}	Efficient Peer
1	Maharastra (MH)	0	-	0	-
2	Gujarat (GUJ)	0	-	0	-
3	Tamilnadu (TN)	0	-	0	-
4	Karnataka (KA)	0	-	0	-
5	Uttar Pradesh (UP)	0	-	0.1338	UK
6	Haryana (HA)	0	-	0.1247	UK
7	Uttarakhund (UK)	0	-	0	-
8	Rajasthan (RA)	0	-	0.3206	UK
9	Telangana (TEL)	0	-	0.3484	UK
10	Andhra Pradesh (AP)	0	-	0.4630	UK
11	West Bengal (WB)	0	-	0.4978	UK
12	Himachal Pradesh (HP)	0	-	0	-
13	Madhya Pradesh (MP)	0	-	0.2107	HP
14	Jharkhand (JHA)	0	-	0.2309	HP
15	Punjab (PUN)	0	-	0	-
16	Odisha (ODI)	0	-	0.3359	HP
17	Chettisgarh	0	-	0.1445	HP
18	Daman and Diu (DD)	0	-	0	-
19	Kerala	0	-	0.2325	(DD)
20	Goa	0	-	0	-
21	Dadra Nagar Haveli	0.0482	Goa	0.2534	Goa
22	Delhi	0.0472	Goa	0.3917	Goa
23	Assam	0.2744	Goa	0.3467	Goa
24	Jammu & Kashmere (JK)	0	-	0.1094	Sikkim
25	Sikkim (SIK)	0	-	0	-
26	Bhihar	0.4289	Sikkim	0.6177	Sikkim
27	Puducheri	0.5633	Sikkim	0.7076	Sikkim
28	Meghalaya	0	-	0	-

Model (5.2) is based on optimistic approach. It is hypothesised the firm under evaluation employs lower bound inputs to produce higher bound outputs: Further, it is assumed its rivals employ inputs at upper bounds to produce outputs at lower bounds. The optimistic production plan $\left(x_0^L, y_0^U\right)$ is projected on to the optimistic frontier in the direction of one of the dominating production plans. The directional efficiency score so obtained serves as lower bound of the unknown efficiency score. Let this score be denoted by β_{FDH}^{O} . This represents greatest efficiency score.

$$\beta_{FDH}^{O} \leq \beta \leq \beta_{FDH}^{P}$$

To implement the directional efficiency closed form solutions in the frame work of interval data under FDH technology, the data confronted with the earlier models, inputs being Fixed Capital and Number of Employees, and output being the Net Value Added were increased by 10 percent to get upper bounds and decreased by 10 percent to get lower bounds.

(A) OPTIMISTIC FRONTIER:

Under optimistic approach it is found that 23 out of 28 total manufacturing sectors are found to be directional distance efficient. Only five total manufacturing sectors experienced inputs and output losses.

The total manufacturing sector of Dadra & Nagar Haveli (DNH) experienced input and output losses of 4.8 percent times of inputs and output of the total manufacturing sector of Goa. The input and output targets of Dadra & Nagar Haveli are as follows:

Input targets: $\hat{x}_{DNH} = x_{DNH} - 0.0482\ x_{Goa}$

Output targets: $\hat{y}_{DNH} = y_{DNH} + 0.0482\ y_{Goa}$

The total manufacturing sector of Delhi suffered from input and output losses marginally at the rate 4.7 percent and input contraction and output expansion are in the direction of inputs and output of the total manufacturing sector of Goa.

$$\hat{x}_{Delhi} = x_{Delhi} - 0.0472\ x_{Goa}$$

$$\hat{y}_{Delhi} = y_{Delhi} + 0.0472\ y_{Goa}$$

The total manufacturing sector of Assam experienced input and output losses by 27 percent due to free disposability. For this state input contraction and output expansion have occurred in the direction of the production plan of Goa.

$$\hat{x}_{Assam} = x_{Assam} - 0.2744\ x_{Goa}$$

$$\hat{y}_{Assam} = y_{Assam} + 0.2744\ y_{Goa}$$

The total manufacturing sectors of Bihar and Puducheri input and output losses are 43 and 56 percent respectively. For both these states the total manufacturing sector of Sikkim is the efficient peer in whose direction inputs are contracted and outputs are expanded to attain directional distance efficiency.

$$\hat{x}_{Bihar} = x_{Bihar} - 0.4289\ x_{Sikkim}$$

$$\hat{y}_{Bihar} = y_{Bihar} + 0.4289\ y_{Sikkim}$$

$$\hat{x}_{Pudhecheri} = x_{Puducheri} - 0.5633\ x_{Sikkim}$$

$$\hat{y}_{Pudhecheri} = y_{Puducheri} + 0.5633\ y_{Sikkim}$$

(B) PESSIMISTIC FRONTIER:

Model (5.2) is based on pessimistic view points and attempts are made to project inefficient production plans to the pessimistic frontier in the frame work of interval data and FDH technology. This model identifies 11 total manufacturing sectors as efficient. These belong to the states of Maharastra, Gujarat, Tamilnadu, Karnataka, Uttarakhund, Himachal Pradesh, Punjab, Daman & Diu, Goa, Sikkim and Meghalaya. There are six total manufacturing sectors for which the state of Uttarakhund serves as efficient peer, in whose direction inputs are contracted and outputs are expanded for the inefficient production plans. These states are Uttar Pradesh, Haryana, Rajasthan, Telangana, Andhra Pradesh and West Bengal. In this group of states Haryana looses 12 per cent of inputs and outputs, while the state that succeeds it, namely Uttar Pradesh experiences 13 per cent of inputs and output losses due to free disposability. The total manufacturing sector of Rajasthan follows Uttar Pradesh. The input and output losses it experiences are

$$0.3206\ x_{UK}$$

$$0.3206\ y_{UK}$$

Telangana competes closely with Rajasthan suffering from input and output losses as follows.

$$0.3484\ x_{UK}$$

$$0.3484\ y_{UK}$$

The total manufacturing sectors of Andhra Pradesh and West Bengal competes with each other closely, experiencing input and output losses of 46 and 50 per cent respectively. The input and output targets for these states are as follows:

$$\hat{x}_{AP} = x_{AP} - 0.463\, x_{UK}$$

$$\hat{y}_{AP} = y_{AP} + 0.463\, y_{UK}$$

$$\hat{x}_{WB} = x_{WB} - 0.4978\, x_{UK}$$

$$\hat{y}_{WB} = y_{WB} + 0.4978\, y_{UK}$$

Under pessimistic view point the total manufacturing sectors of Chattisgarh, Madya Pradesh, Jharkhand and Odisha are directional distance efficient. For all these states, the total manufacturing sector of Himachal Pradesh is efficient peer along whose direction inputs of the inefficient production plans are contracted and outputs are expanded. Had the total manufacturing sector of Chattisgarh been directional distance efficient, it could have saved 14 per cent of inputs and outputs measured in the direction of the production plan of Himachal Pradesh.

$$\hat{x}_{C} = x_{C} - 0.1445\, x_{HP}$$

$$\hat{y}_{C} = y_{C} + 0.1445\, y_{HP}$$

The total manufacturing sector of Chattisgarh is followed by Madya Pradesh, whose input and output losses are,

$$0.2107\ x_{HP}$$

$$0.2107\ y_{HP}$$

The total manufacturing sector of Jharkhand closely competed with Madya Pradesh, whose estimated input and output losses are 23% times the input and output vector of Himachal Pradesh. Its input and output targets are,

$$\hat{x}_{JA} = x_{JA} - 0.2309\, x_{HP}$$

$$\hat{y}_{JA} = y_{JA} + 0.2309\, y_{HP}$$

The total manufacturing sector of Odisha suffered input and output losses as shown below:

$$0.3359\ x_{HP}$$

$$0.3359\ y_{HP}$$

For the total manufacturing sectors of Dadra & Nagar Haveli (DNH), Delhi and Assam the efficient peer is Goa in the direction of the production plan of which inputs are contracted and outputs are expanded to reach the pessimistic frontier. The efficient targets for these states are,

(i) $$\hat{x}_{DNH} = x_{DNH} - 0.2534\, x_{Goa}$$

$$\hat{y}_{DNH} = y_{DNH} + 0.2534\, y_{Goa}$$

(ii) $$\hat{x}_{Assam} = x_{Assam} - 0.3467\, x_{Goa}$$

$$\hat{y}_{Assam} = y_{Assam} + 0.3467\, y_{Goa}$$

(iii) $$\hat{x}_{Delhi} = x_{Delhi} - 0.3917\, x_{Goa}$$

$$\hat{y}_{Delhi} = y_{Delhi} + 0.3917\, y_{Goa}$$

Had the total manufacturing sector of Kerala been directional distance efficient in the direction of Diu & Damon (DD), the inputs and outputs it could have saved are,

$$0.2325\ x_{DD}$$

$$0.2325\ y_{DD}$$

The total manufacturing sector of Sikkim serves as efficient peer to the total manufacturing sectors of Jammu & Kashmere, Bihar and Puduchery. Their efficient input and output targets are,

(i) $$\hat{x}_{JK} = x_{JK} - 0.1094\, x_{Sik}$$

$$\hat{y}_{JK} = y_{JK} + 0.1094\, y_{Sik}$$

(ii) $$\hat{x}_{Bihar} = x_{Bihar} - 0.6177\, x_{Sik}$$

$$\hat{y}_{Bihar} = y_{Bihar} + 0.6177\, y_{Sik}$$

(iii) $$\hat{x}_{PUD} = x_{PUD} - 0.7076\, x_{Sik}$$

$$\hat{y}_{PUD} = y_{PUD} + 0.7076\, y_{Sik}$$

For those manufacturing sectors that are not assessed efficient by both the optimistic and pessimistic view points, the efficiencies can be expressed as intervals.

1. Uttar Pradesh:

$$0 \le \beta_{FDH} \le 0.1338$$

2. Haryana:

$$0 \le \beta_{FDH} \le 0.1247$$

3. Rajasthan:

$$0 \le \beta_{FDH} \le 0.3206$$

4. Telangana:

$$0 \le \beta_{FDH} \le 0.3484$$

5. Andhra Pradesh:

$$0 \le \beta_{FDH} \le 0.4630$$

6. West Bengal:

$0 \le \beta_{FDH} \le 0.4978$

7. Madya Pradesh:

$0 \le \beta_{FDH} \le 0.2107$

8. Jharkhand:

$0 \le \beta_{FDH} \le 0.2309$

9. Odisha:

$0 \le \beta_{FDH} \le 0.3359$

10. Chattisgarh:

$0 \le \beta_{FDH} \le 0.1445$

11. Kerala:

$0 \le \beta_{FDH} \le 0.2325$

12. Dadra and Nagar Haveli:

$0.0482 \le \beta_{FDH} \le 0.2534$

13. Delhi:

$0.0472 \le \beta_{FDH} \le 0.3917$

14. Assam:

$0.2744 \le \beta_{FDH} \le 0.3467$

15. Jammu & Kashmere:

$0 \le \beta_{FDH} \le 0.1094$

16. Bihar:

$0.4289 \le \beta_{FDH} \le 0.6177$

17. Puducheri:

$0.5633 \le \beta_{FDH} \le 0.7076$

8. CONCLUS IONS

On input and output variables, sometimes Crisp data are not available, but, never-the-less upper and lower bound values are available for these variables. In such cases directional *efficiency* can also be expressed in bounds. In this case we envisage two FDH - frontiers called the optimistic and pessimistic frontiers. If the inefficient production plan is directed on to the optimistic frontier, the under lying assumptions are that the inefficient producer applies inputs at lower bounds but produce outputs at upper bounds. But, his rivals employ inputs at upper bounds but produces outputs at lower bounds. When the inefficient plan is moved forward to reach pessimistic frontier in an appropriate direction we assume it as the producer whose efficiency is under evaluation employs inputs at upper bounds but produces outputs at lower bounds. His rivals implement inputs at lower bounds but produce outputs at upper bounds.

The directional distance problem solved under pessimistic approach is as follows:

$$\beta^{P}_{FDH} = Max\,\beta$$

such that
$$x^{L}_{k} \le x^{U}_{0} - \beta x^{L}_{k} \qquad \text{...... (8.1)}$$
$$y^{U}_{k} \ge y^{L}_{0} + \beta y^{U}_{k}$$
$$k \in R(x_0, y_0; VRS)$$

$$\beta^{P}_{FDH} = \underset{k}{Max}\, Min\left\{ \underset{i}{Min}\left(\frac{x^{U}_{i0}}{x^{L}_{ik}}\right) - 1,\ 1 - Max\left(\frac{y^{L}_{r0}}{y^{U}_{rk}}\right) \right\} \qquad \text{...... (8.2)}$$

The upper bounds efficiency distribution is as follows:

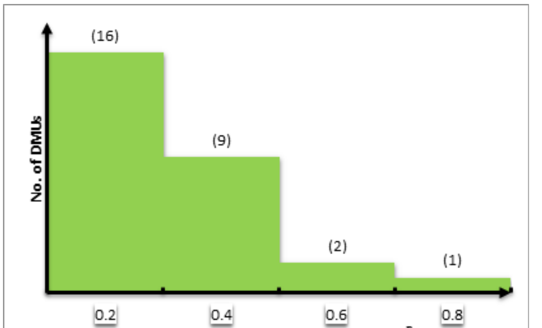

Figure (8.1) (Upper bound) β^{P}_{FDH}

There are 16 decision making units experienced marginal input and output losses, below 20 percent of the inputs and outputs of their efficient peers due to free disposability of inputs and outputs. Nine total manufacturing sectors experienced input and outputs losses, more than 20 percent but less than 40 percent of the inputs and outputs of their peers. The freely disposed off inputs and potential outputs of the two total manufacturing sectors are found to be more than 40 percent but less than 60 percent of their efficient peers. One Indian state freely disposed off inputs and potential outputs are more than 60 percent but less than 80 percent of the inputs and outputs of its efficient peer. Exponential distribution appropriate s the efficiency distribution.

$$f(z) = 6.747\ e^{-6.747z},\ 0 \le z < \infty$$

The calculated value of Chi Square is, 0.0000554.

The null hypothesis can not be rejected. The FDH upper bounds of directional efficiency follows exponential distribution.

The directional distance problem solved for lower bounds is as follows:

$$\beta^{O}_{FDH} = Max\beta$$

such that $x^{L}_{0} - \beta x^{U}_{k} \geq x^{U}_{k}$

$$y^{L}_{0} + \beta y^{U}_{k} \leq y^{U}_{k}$$

$$k \in R$$

$$\beta^{O}_{FDH} = \underset{k}{Max}\beta_k = \underset{k}{Max}\left\{ \underset{i}{Min}\left(\frac{x^{L}_{i0}}{x^{U}_{ik}}\right) - 1, 1 - \underset{r}{Max}\left(\frac{y^{U}_{r0}}{y^{L}_{rk}}\right) \right\} \quad \text{...... (8.3)}$$

Out of 28, twenty total manufacturing sectors have not lost any inputs or outputs due to free disposability, while their efficiencies are measured relative to optimistic frontier. Two states are found experiencing input and output losses. Three total manufacturing sectors suffered from significant input and output losses.

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