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Statistics

COST EFFICIENCY AND SHORTEST COST TARGETS

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ABSTRACT Shortest targets are investigated in Data Envelopment Analysis, as these provide encouraging goals to inefficient decision making units. This study enquires closest cost targets to the total manufacturing sectors of major Indian States.

KEYWORDS : Data Envelopment Analysis, Directional Distance Functions, Free Disposable Hull, Shortest targets.

1. INTRODUCTION:

Efficiency measurement dates back to MJ. Farrell (1957), who suggested a method to empirically evaluate efficiencies of firms in production activity. Shephard (1970) introduced mathematical rigor based distance functions to evaluate input, output and graph efficiencies of production unit. Shephard's Lemma connects production with cost, that gave rise to evaluation of economic efficiency of decision making units. However, the most powerful methodology for assessment of efficiency was introduced by Charnes, Cooper and Rhodes (1978), who replaced the unknown production possibility set and its outer boundary by a known one that is based on the axioms of inclusion, free disposability, ray unboundedness and minimum extrapolation, that involves a sample of firms which combine similar inputs to produce similar outputs. The production possibility set of CCR is convex cone for which the interior activity can be compared to a linear combination of frontier activities. CCR production frontier assumes that returns to scale are constant. Often economic data are subjected to returns to scale, which may be constant, decreasing or increasing in different input ranges of input domain. The deficiency of CCR formulation was removed by Bankar, Charnes and Cooper (1984) who proposed a technology set that is based on the axioms of inclusion, free disposability, convexity and minimum extrapolation. An interior activity can now be compared to a convex combination of frontier activities.

The activities so emerged define virtual DMU, which is mostly, an unobserved DMU. The CCR and BCC efficiency measures can be obtained solving linear programming problems resulting in radial input / output efficiency measures.

Fare (1975) formulated an asymmetric input technical efficiency measure to measure input technical efficiency.

Fare and Lovell (1978) sought dimension wise reduction of inputs, consequently obtained a non-radial measure of efficiency.

Briec *et al.,* (2011) extended Fare and Lovell model, formulated an objective function involving component, similar to geometric mean.

The radial measures serve to measure efficiency in short run perspective. In short run, due to the rigidity of the technology for input / output substitution, input vector of the inefficient DMU and efficient DMU with which inefficient activity is compared, possess the same input mix. Along the ray from the origin at every input vector, input mix remains to be the same, consequently, the technique of production remains to be the same. Thus, technical efficiency refers to radial measures. "Radiality seems to be a reasonable proxy for similarity, because all the firms on the same ray share the same combination of inputs" (Gonzalez, Alvarez, 2001). Radial efficiency measures select a priori direction that ignores input substitution or output transformation, the information of which is embedded in input / output isoquants.

Fare (1975) formulated Hyperbolic Graph efficiency measure that expands outputs and contracts inputs simultaneously, where the metric of input reduction is inverse of the metric of output expansion. The path pursued by the inefficient firm to reach the frontier is hyperbola. The problem is proposed as a non-linear programming problem, that can be transformed into a linear programming problem by Taylor's series expansion.

Another, very widely used efficiency measure is Russell's / non-radial measure of efficiency that seeks dimension specific reduction of inputs and / or dimension specific expansion of outputs.

Chamber *et al.,* (1998) formulated directional distance functions, which are an important class, but sensitive to the choice of direction of input contraction and / or output expansion. Farrell's (Shephard's) radial distance measures can be obtained as special cases of directional distance functions. Unlike it is seen in radial measures, a decision making unit attains full efficiency if the value of directional distance function is zero. The coordinates of the DMU operating in the interior of technology set can be projected onto the frontier in any feasible direction.

Fare *et al.,* (2010) produced slack based directional distance functions and show Tone's slack based efficiency measure is a special case of directional distance measure.

2. FREE DISPOSABLE HULL

For a firm or farm identification of observed peer is more important than identification of a virtual peer that is mostly unobservable for both following and comparison. The follower and his efficient peer shall face the same returns to scale and employ the same technique in short run comparisons. Identification of a single efficient peer is not possible in convex technologies.

The most popular non-convex technology is Free Disposable Hull (FDH), popularized by Tulkens (1993). FDH is a technology set based on the axioms of inclusion and free disposability and minimum extrapolation. If T_{DEA} and T_{FDH} are production possibility sets of DEA and FDH, then,

$$T_{FDH} \subseteq T_{DEA}$$

Consequently, the technical efficiencies derived comparing the inefficient unit with a frontier firm are as follows:

Input Technical Efficiency : $\lambda_{FDH} \geq \lambda_{DEA}$

Output Technical Efficiency : $\theta_{FDH} \leq \theta_{DEA}$

Cost Efficiency : $CE_{FDH} \geq CE_{DEA}$

3. SHORT EST TARGETS

Distance functions while evaluated, not only provide efficiency of individual decision making unit, but also provide bench marks to it if it is an inefficient firm. Large targets suggest structural reorganization, the act of which the inefficient firm hesitates to pursue. Further the farthest efficient firms are dissimilar in size and the inefficient firm may not scale up its activities to reach the coordinates of its efficient peer. For adjustment of its activity an inefficient decision making unit requires its efficient peer closest in proximity.

(a) Frei and Harker (1999) proposed least projection algorithm to find shortest projection from an assessed DMU to the efficient facet. This facet is identified for the DMU under assessment by solving suitably formulated multiplier – linear programming problems.

(b) Silva Portela (2003) proposed closest targets in non-oriented approach, the technology sets being convex and non-convex.

(c) Tulkens (1993), Deprin *et al.,* (1984) developed free disposable technology structured on the axioms of inclusion, free disposability of inputs and outputs and minimum extraploation for sampled decision making units. FDH is non-convex production possibility set. Tulkens produced a numerical algorithm that provides closed form expressions to evaluate, input and output technical efficiency of dominated (inefficient) firm. His methodology divides the production plans into three disjoint sets consisting of dominated firms, non-dominated (efficient) firms and neither dominating nor dominated firms. The FDH frontier is spun by the input and output vectors of dominating firms. The inefficient firm's input and output vector are projected on to the FDH frontier depending upon the orientation. The radial contraction of inputs and expansion of outputs leave positive slacks in many occasions, implying that the radial targets fall on the weak efficient boundary of input / output sets, implying that the projections to land on efficient subset of input / output set, requires further contraction of one or more inputs and expansion of one or more outputs.

Input orientation applied suitably on FDH technology set avoids slacks, if Directional Distance approach is followed choosing the directions of dominating decision making units which operate in the economic region of inefficient production plan under evaluation. Tulkens argued in favour of replications which expand the set of dominating production plans.

(d) Briec *et al.,* (2004) to model FDH Production possibility set, consequently to obtain FDH production frontier for efficiency evaluation of inefficient decision making units provided methodology to construct scaled better set, the production plans of which dominate the inefficient production plans, posterior to appropriate scaling. The members of the scaled better set, scaled appropriately spin the variable, constant, non-increasing and non-decreasing returns to scale FDH production frontiers.

(e) Chung et.al (1997) chose an endogeneous direction to measure directional efficiency of inefficient decision making units, the direction being

$$\left(g_x, g_y\right) = \left(x_0, y_0\right)$$

where $\left(x_0, y_0\right)$ is the inefficient production plan under evaluation. This study chooses all feasible directions,

$$\left(g_x^k, g_y^k\right) = \left(\delta_k x_k, \delta_k y_k\right)$$

such that $\left(x_k, y_k\right)$ belongs to the 'scaled better set' and chooses the least distance to determine k, consequently the closest targets.

Scaled better set:

$$B\left(x_0, y_0, \delta \in \Gamma\right) = \left\{\left(x_k, y_k\right) : \delta_k x_k \le x_0, \delta_k y_k \ge y_0, \delta \in \Gamma\right\}$$

4. RETURNS TO SCALE:

Returns to scale are surface property of the production possibility set. If $\left(x_0, y_0\right)$ is the input and output vector of a firm operating interior to the technology set, depending on the flexibility of its production plan, it needs to adjust its input and / or outputs to reach the surface of the technology set convex or non-convex. Returns to scale at input and output orientation projection points need not be the same.

In DEA frame work for taxonomy of returns to scale, the observed inefficient firm's input and / or output vectors are projected on to frontiers of technology set admitting constant, non-increasing and non-decreasing returns to scale. Let the respective production possibility sets be denoted by,

$$T^{DEA-CRS}, \; T^{DEA-NIRS} \text{ and } T^{DEA-NDRS}$$

Clearly, we have, $T^{DEA-NIRS} \subseteq T^{DEA-CRS}$

$$T^{DEA-NDRS} \subseteq T^{DEA-CRS}$$

The technology set admitting variable returns to scale be obtained as intersection as follows:

$$T^{DEA-VRS} = T^{DEA-NDRS} \cap T^{DEA-NIRS}$$

Clearly, $T^{DEA-VRS} \subseteq T^{DEA-NDRS}$

$$T^{DEA-VRS} \subseteq T^{DEA-NIRS}$$

The above inclusion properties hold good for FDH technology also. The difference between DEA and FDH technology is that the former satisfies convexity and the later does not. Consequently, we have,

$T_{FDH} \subseteq T_{DEA}$ for all returns to scale, which implies that in general FDH technology provides closer targets than DEA technology. Redundancy or retention of convexity is an empirical exercise.

Briec *et al.,* (2004) proposed a non-linear programming problem that separates returns to scale from convexity. The problem is stated as follows for input orientation,

$$DF_i\left(y_0, x_0, \delta \in \Gamma\right) = Min\, \lambda$$

s.t $$\sum_{j=1}^{n}\lambda_j\delta x_{ij} \le \lambda x_{i0},\ i\in M$$

$$\sum_{i=1}^{n}\lambda_j\delta y_{rj} \ge y_{r0},\ r\in S$$

$$\sum_{j=1}^{n}\lambda_j = 1$$

$$\lambda_j \ge 0$$

$\delta > 0$, $0<\delta\le 1$ and $\delta\ge 1$, respectively model constant, non-increasing and non-decreasing returns to scale. Allowing ray expansion and contraction of output and input vectors that are observed, a DEA frontier admitting constant returns to scale can be spanned. Allowing only ray input and output contraction, the non-increasing returns to scale frontier can be spanned. A non-decreasing returns to scale frontier can be spanned by allowing only ray input contraction and output expansion.

The above problem can be extended to FDH technology, allowing $\lambda_j\in\{0,1\},\ j\in N$. The above problem becomes a non-linear mixed integer (NLMI) programming problem. The taxonomy of returns to scale of DEA and FDH technology is the same.

5. VECTOR DOMINANCE (Briec et.al, 2004)

Let (x_0, y_0) and (x, y) be two production plans, where $x, x_0\in R_m^+$ be input vectors and $y, y_0\in R_s^+$ be output vectors. The production plan (x_0, y_0) is said to be dominated by the production plan (x, y) if,

$$x\le x_0 \text{ and } y\ge y_0$$

where $x\le x_0$ implies atleast one component of x is less than the corresponding component of x_0 and $y\ge y_0$ implies atleast one component y is larger than the corresponding component of y_0. Non-dominated production plans are efficient while dominated production plans are inefficient.

To classify a production plan according to its returns to scale an index set of better observations is constructed, rescaling input and output vectors under returns to scale specification. Briec *et al.*, (2004) named such a set as scaled better set.

$$B(x_0,y_0,\delta\in\Gamma)=\{(x_k,y_k):\delta x_k\le x_0,\ \delta y_k\ge y_0,\ \delta\in\Gamma\}$$

(i) $\delta\in\Gamma\Rightarrow\delta>0\Rightarrow CRS$

(ii) $\delta\in\Gamma\Rightarrow 0<\delta\le 1\Rightarrow NIRS$

(iii) $\delta\in\Gamma\Rightarrow\delta\ge 1\Rightarrow NDRS$

(iv) $\delta\in\Gamma\Rightarrow\delta=1\Rightarrow VRS$

(v) The scaled vectors of $B(x_0,y_0,\Gamma)$ generate the economic region of the input level set $L^{FDH}(y_0)$ denoted by, $L^{FDH}(y_0/\delta x_k\le x_0)$. The scaled vectors δx_k span the isoquant of the conditional FDH input level set. $\{\delta x_k\}$ constitute the efficient subset of $L^{FDH}(y_0/\delta x_k\le x_0)$, where $\delta\in\Gamma$.

For the construction of the scaled better set, one need not experiment with all the δ values belonging to the ranges postulated for different returns to scale. It is sufficient if one experiments with critical values of δ.

6. SHORTEST COST EFFICIENCY TARGETS:

If input prices are known, for each inefficient firm cost efficiency targets can be found. Divergence between factor minimal cost and experienced cost gives rise to cost inefficiency. Following Briec et.al (2004), cost efficiency can be expressed in closed form expression as follows:

(i) Let returns to scale be variable, so that $\delta=1$. Then cost efficiency in FDH frame work can be expressed as,

$$CE(\delta=1)=\frac{C(y_0,p)}{px_0}=\underset{x}{Min}\left\{\frac{px}{px_0}:x\in L^{FDH}(y_0,\delta=1)\right\}$$

$$=\underset{x}{Min}\left\{qx:x\in L^{FDH}(y_0,\delta=1)\right\}$$

where $q=\dfrac{p}{px_0}$

$$CE(\delta=1)=\underset{x}{Min}\left\{qx:x\in\bigcup_{y_k\ge y_0}[x:x_k\le x,\delta=1]\right\}$$

$$=\underset{x}{Min}\left\{qx:x\in Eff\bigcup_{y_k\ge y_0}[x:x_k\le x,\delta=1]\right\}$$

$$=\underset{x}{Min}\left\{qx:x\in\bigcup_{y_k\ge y_0}Eff[x:x_k\le x,\delta=1]\right\}$$

$$=\underset{x}{Min}\left\{qx:x\in\bigcup_{y_k\ge y_0}[x_k]\right\}$$

$$CE(\delta=1)=\underset{x_k}{Min}\{qx_k:y_k\ge y_0\}$$

(ii) Let returns to scale be constant,

$$\delta\ge 0$$

$$CE(\delta\ge 0)=\underset{\delta x_k}{Min}\{q\delta x_k:\delta y_k\ge y_0\}$$

choose $\delta_k = \underset{r}{Max}\left(\frac{y_{r0}}{y_{rk}}\right)$

$$CE\left(\delta \geq 0\right) = \underset{\delta_k x_k}{Min}\left\{\delta_k q x_k\right\}$$

$$= \underset{\delta_k x_k}{Min}\left\{\underset{r}{Max}\left(\frac{y_{r0}}{y_{rk}}\right) q x_k\right\}$$

(iii) Let returns to scale be non-increasing.

$$0 < \delta \leq 1$$

$$\Rightarrow \underset{r}{Max}\frac{y_{r0}}{y_{rk}} \leq 1$$

$$\Rightarrow \delta_k = \underset{r}{Max}\left(\frac{y_{r0}}{y_{rk}}\right)$$

$$CE\left(\delta \leq 1\right) = \underset{\delta_k x_k}{Min}\left\{\underset{r}{Max}\frac{y_{r0}}{y_{rk}} q x_k : \underset{r}{Max}\frac{y_{r0}}{y_{rk}} \leq 1\right\}$$

(iv) Let returns to scale be non-decreasing.

$$\delta \geq 1$$

choose

$$\delta_k = Max\left\{1, \underset{r}{Max}\frac{y_{r0}}{y_{rk}}\right\}$$

$$CE\left(\delta \geq 1\right) = \underset{\delta_k x_k}{Min}\left[Max\left\{1, \underset{r}{Max}\frac{y_{r0}}{y_{rk}}\right\} q x_k\right]$$

Combining all the four cases, we express cost efficiency as follows:

$$CE = \begin{cases} \underset{x_k}{Min}\left\{ q x_k : y_k \geq y_0, \delta_k = 1\right\} \\ \underset{\delta_k x_k}{Min}\left[\underset{r}{Max}\left(\frac{y_{r0}}{y_{rk}}\right) q x_k\right], \delta_k \geq 0 \\ \underset{\delta_k x_k}{Min}\left[\underset{r}{Max}\left(\frac{y_{r0}}{y_{rk}}\right) q x_k : Max\left(\frac{y_{r0}}{y_{rk}}\right) \leq 1\right], \delta_k \leq 1 \\ \underset{\delta_k x_k}{Min}\left[Max\left\{1, \underset{r}{Max}\frac{y_{r0}}{y_{rk}}\right\} q x_k\right], \delta_k \geq 1 \end{cases}$$

Numerically cost efficiency lies between zero and one. Such firms which operate close to unity are closely efficient, but those which operate close to zero are closely inefficient. Those which operate in the middle of zero-to-one scale perform average. Cost efficiency computations provide a single cost efficient peer to each cost inefficient firm. They also provide input targets to cost inefficient firms. The FDH-Cost Efficiency computations do not require linear programming problems to be solved as we do in DEA formulations. Simple arithmetic manipulations provide FDH estimates of Cost Efficiency. Further, the FDH cost targets penalize lesser of the morale of the inefficient firm than the convex cost targets. The cost efficiency computations of FDH technology are based on closed form solutions of cost efficiency problems.

7. DATA:

The numerical example worked out refers to the data collected from the bulletins of Annual Survey of Industries (ASI), for 2012-13. The variables of the study are (i) Fixed Capital, (ii) Total Persons Engaged, (iii) Total Emoluments, (iv) Net Value Added.

8. NUMERICAL RESULTS:

FDH - COST EFFICIENCY

S.No	Total Manufacturing Sector	CE(CRS)	CE(DRS)	CE(IRS)	RTS	PEER
1	Maharastra (MH)	1.0	1.0	1.0	CRS	--
2	Gujarat (GUJ)	0.5894	0.7384	0.5894	DRS	MH
3	Tamilnadu (TN)	0.6161	0.6161	0.7792	IRS	MH
4	Karnataka (KA)	0.5505	0.5505	0.6783	IRS	MH
5	Uttar Pradesh (UP)	0.5432	0.6812	0.5432	DRS	MH
6	Haryana (HA)	0.6952	0.8715	0.6959	DRS	UK
7	Uttarakhand (UK)	1.0	1.0	1.0	CRS	--
8	Rajasthan (RA)	0.6953	0.6953	0.8374	IRS	UK
9	Telangana (TEL)	0.5827	0.5827	0.6747	IRS	UK
10	Andhra Pradesh (AP)	0.3190	0.3190	0.3483	IRS	UK
11	West Bengal (WB)	0.3594	0.3594	0.3965	IRS	HP
12	Himachal Pradesh (HP)	1.0	1.0	1.0	CRS	--
13	Madhya Pradesh (MP)	0.3441	0.3441	0.3790	IRS	HP
14	Jharkhand (JHA)	0.6068	0.6068	0.6991	IRS	HP
15	Punjab (PUN)	0.5829	0.5829	0.7628	IRS	HP
16	Odisha (ODI)	0.2994	0.2994	0.3788	IRS	HP

The total manufacturing sectors above are arranged in descending order of their total value added. The total manufacturing sectors of Maharastra, Uttarakhand and Himachal Pradesh are cost efficient and attained constant returns to scale. Returns to scale are decreasing in total manufacturing sector of Gujarat, Uttar Pradesh and Haryana. In the remaining states returns to scale are found increasing.

For example, in one input and two output production a firm attains equilibrium, there by achieves cost efficiency when the ratio of marginal products equal to ratio of input prices. The total manufacturing sector of Gujarat admits decreasing returns to scale. To achieve cost efficiency, the technique of the total manufacturing sector of Maharastra shall be the technique of Gujarat. Since each input combination of the isoquant of input set refers to a technique, the total manufacturing sector of Gujarat shall choose its technique such that its marginal rate of technical substitution is identical with that of the total manufacturing sector of Maharastra. The cost efficiency of total manufacturing sector of Gujarat is 0.7384, implying 26 percent of input losses. It admits decreasing returns to scale. Any firm is said to be scale efficient if and only if it admits constant returns to scale. The cost efficient peer of Gujarat is Maharastra, for which returns to scale are constant. To attain scale efficiency total manufacturing sector of Gujarat shall contract its input suitable to attain cost efficiency (CRS) and there by scale efficiency.

9. CONCLUSION

Among the 16 states, Maharastra, Uttarakhand and Himachal Pradesh are cost efficient and their projections land on the constant returns to scale FDH production frontier. Returns to scale are found decreasing in the total manufacturing sectors of Gujarat, Uttar Pradesh and Haryana. The remaining states enjoy increasing returns to scale.

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