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Mathematics

PORTFOLIO OPTIMIZATION USING MODIFIED INFORMATION RATIO

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ABSTRACT In this paper, we propose single-asset portfolio optimization approach based on modified information ratio. Portfolio performances are commonly assessed against a benchmark using information ratio. With the help of numerical example this paper shows why maximizing information ratio could be wrong policy. So this work overcomes the drawbacks of traditional information ratio and demonstrates the benefits of using modified information ratio.

KEYWORDS : Portfolio; Risk; Return; Optimization; Information ratio.

Introduction

In general a man earns and spends money. Sometimes, he may have extra money than his expenditures at other times, he may want to but more than he can offer. Such kind of imbalances will lead him either to borrow or to save excess money. With his saving, he can do any of two things; one option is to bury his excess money until his expenditure exceeds present income. In this case, when he will some amount he saved. Whereas other option is to give up the possession of his exceeds money to get greater amount of money in future. This type of trade-off is called investment. In short, investment is what someone do with the excess money to increase it in future. A collection of investments all owned by the some individual or organization form portfolio. Portfolio management is the art of selecting suitable investment plan (asset allocation) as per investment financial requirements. Portfolio optimization is determination of weights of various financial assets in the portfolio in a way, that is most suitable to the investor's objective. During today's economic stress around the world, it would be beneficial to have some insight into the tools that help investor's learn about the riskiness of their portfolios.

In portfolio theory several methods have been introduced to measure risk such as variance, mean-absolute-deviation, value-at-risk, Conditional value-at-risk, Sharpe Ratio etc. It is important to decide which risk measure should be taken into account. In 1952, the University of California's noble prize winner economist(Markowitz {3}) provided the Modern Portfolio Theory to achieve optimal portfolio by using variance as risk measure in his pioneer paper on Portfolio selection. With this type we have variance model of (Kan and Smith {5}) and mean absolute deviation model of(Liu {6}). In 1964 (William Sharpe {8}) introduced Capital Asset Pricing Model (CAPM) that defined risk as volatility relative to market. The first direct challenge to the CAPM and related theories come, when (Stephen Ross {7}) developed the Arbitrage Pricing Theory (APT), using the fundamental Concept that various assets with the same exposure to risk have to be priced the same by the market. After that, extensive research was conducted by (Fama {1}) and (Samuelson {4}) on multi-factor models. Then, Efficient Market Hypothesis (EMH) was developed. Perhaps the best overview of this post-modern view of portfolio analysis is given by (Grinold and Kahn {2}). The idea of Markwitz's model was still being used indirectly in the above mentioned models. In fact, the information ratio is built on Markowitz's mean-variance analysis, according to which, mean and variance are satisfactory measures for characterizing a portfolio. The information ratio is a single number that summarizes the mean-variance properties of a portfolio. It is similar to Sharpe ratio as it measures excess return per unit of risk. The paper is structured in the following way. For better understanding of the concept, Section 2 introduces basic definition of Information ratio and its application to real world situations with the help of numerical example . Section 3 explains negative impacts on traditional information ratio, and shows a way to improve traditional Information ratio. The definition and benefits of modified information ratio are also explained in Section 4. Concluding remarks are given in Section 5.

2. **Definition of Information Ratio (IR)**

A measure of portfolio's performance against return and risk relative to a benchmark is called information ratio. In other words, it's a ratio of portfolio's return above the return of a benchmark to the standard deviation of those excess returns.

$$\text{IR} = \frac{E(R_P - R_B)}{\sigma_{P-B}}$$

Where R_P and R_B represents return of the portfolio and return of the benchmark for time respectively. The term σ_{P-B} is the variability (standard deviation) of the difference in returns between the portfolio and the benchmark. The portfolio's excess return R_P-R_B is also known as active return and the Volatility of excess return σ_{P-B} is known as active risk.

The information ratio is highly dependent on the chosen benchmark and the time period under measurement. While composing two assets with the some volatility, the assets with larger return are chosen. Similarly, when composing two assets with some return, the one with lesser volatility (risk) is preferred. But when one asset has larger return as well as larger risk than another asset, the choice depends on the investor's objective. In this case, the rational investor must judge whether the additional return is worth the additional risk. A portfolio having high active return and high active risk than another portfolio, does not mean high information ratio. For example, let portfolio x has mean return (R_X) 12% and active risk (σ_{X-B}) 8% and portfolio Y has mean return (R_Y) 10% and active risk (σ_{Y-B}) 4%. Assume that benchmark mean return (R_B)is 2%.

Information Ratio of portfolio X

$$\text{IR} = \frac{R_X - R_B}{\sigma_{X-B}} = \frac{12-2}{8} = \frac{10}{8} = 1.25$$

Information Ratio of portfolio Y

$$\text{IR} = \frac{R_Y - R_B}{\sigma_{Y-B}} = \frac{10-2}{4} = \frac{8}{4} = 2.$$

Portfolio Y has lower returned but higher IR. A high IR means an investor can achieve higher return by taking an additional risk.

3. **Negative Impacts on Information Ratio**

Negative excess (active) returns can have negative impact on the information ratio. For example let portfolio x has active return of -2.7% and active risk of 4.2% and portfolio Y has active return of -6.7% and active risk of 12.5%.

Information Ratio of portfolio X

$$\text{IR} = \frac{R_X - R_B}{\sigma_{X-B}} = \frac{-2.7}{4.2} = -0.64$$

Information Ratio of portfolio Y

$$\text{IR} = \frac{R_Y - R_B}{\sigma_{Y-B}} = \frac{-6.7}{12.5} = -0.53$$

Here since portfolio Y has higher information ratio, one would conclude that portfolio Y is better than portfolio x. But portfolio X has higher active return and lower active risk, which means portfolio is more promising one. As we can see that there is a serious flow with information ratio when the excess returns are negative.

In order to overcome this drawback, a modified information ratio is introduced in this paper.

4. Modified Information Ratio and Benefits of Modified Information Ratio

Ratio of portfolio's mean return above the return of a benchmark to the semi variance of those excess returns is defined as modified information ratio.

$$\text{Modified Information Ratio} = \frac{\text{Mean (ER)}}{\text{Semi Variance (ER)}}$$

$$\text{Where ER} = R_P - R_B$$

$$\text{Semi Variance(ER)} = \frac{1}{m}\sum_{\text{ER} < \text{Mean (ER)}}(\text{Mean (ER)} - \text{ER})^2$$

While comparing two portfolios X and Y, we find that if excess returns ER are positive, using either formula of information ratio, will produce the same result. But when excess returns are negative, modified information ratio is preferable as shown in the calculations:

Table 1: Comparison of IR and Modified IR

		PORTFOLIO X		
Return of Portfolio (X) R_X	Return of Benchmark R_B	Excess Return (ER)= R_X-R_B	Mean Excess Return (MER)-Excess Return (ER)	$(MER-ER)^2$
-30	4	-34	8.75	76.5625
-23	5	-28	2.75	7.5625
-2	7	-9	-16.25	264.0625
-21	9	-30	4.75	22.5625
Mean Excess Return (MER)	-25.25			
Semivariance (ER)	35.5625			
Standard Deviation (ER)	11.1168041			
Information Ratio (IR)	-2.271336238			
Modified Information Ratio (MIR)	-0.710017575			
		PORTFOLIO Y		
Return of Portfolio (Y) R_Y	Return of Benchmark R_B	Excess Return (ER)= R_Y-R_B	Mean Excess Return (MER)-Excess Return (ER)	$(MER-ER)^2$
-12	4	-16	-5.5	30.25
-27	5	-32	10.5	110.25
-12	7	-19	-2.5	6.25
-10	9	-19	-2.5	6.25
Mean Excess Return (MER)	-21.5			
Semivariance (ER)	110.25			
Standard Deviation(ER)	7.141428429			
Information Ratio (IR)	-3.010602181			
Modified Information Ratio (MIR)	-0.195011338			

In above table, portfolio Y has higher mean excess return and lower standard deviation of excess return than portfolio X. But still information ratio of portfolio X is greater than that of portfolio Y. Thus considering IR (information ratio) when excess returns are negative does not lead to a consistent choice. Whereas modified information ratio gives correct results.

5. Conclusion

The paper conclude that the modified information ratio enable funds to be ranked correctly. The empirical study shows that the modified information ratios are reliable measures of fund performance. The four factors influence the quality of the modified information ratio are data frequency, non-normality of fund returns, horizon time and benchmark selection. It should be noted that proposed framework is valid for funds with non-symmetric returns.

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