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Mathematics

MULTI CRITERIA DECISION MAKING MODEL BASED ON THE VALUE OF FUZZY NUMBERS – MULTIPLE DECISION MAKERS

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ABSTRACT We propose new multi criteria fuzzy decision making model based on the value of fuzzy numbers which in turn will be very helpful in developing fuzzy expert systems. Here arithmetic mean operation of fuzzy numbers is used for compiling experts' judgments. We also propose a method for finding the weights of decision makers.

KEYWORDS : Fuzzy numbers, Arithmetic mean, Value of fuzzy numbers, Multi Criteria Decision Making Model

Introduction

Most of the real life problems are complex in nature because of the indistinctness and impreciseness of the available data. In 1970 Bellman and Zadeh proposed the concept of fuzzy sets and fuzzy models to effectively handle these imprecise data which help us to avoid information loss through computing with words. To solve such real world problems, we can develop fuzzy expert systems by seeking the help of experts who have knowledge in that particular area. There may be many factors that influence a certain problem. While developing expert system, one has to rank these factors based on the experts' judgements. Decision making is the process of choosing the best alternatives for the accomplishment of goals. The need for decision arises when inconsistent events occur. To make sense of these events, the decision maker must put the events in the proper framework so as to give them meaning, which allows him to draw upon previous experience to decide what to do. If it is a new situation, the decision maker can make an action plan that deals with its exceptionality. Most of the decision problems are solved without a complete search of information. When the number of aspects of a decision situation is constant, an increase in the number alternatives leads to a greater number of investigated aspects [3]. In a real world Multi Criteria Decision Making situation, decision makers have different acquaintance, expertise and skills and hence the weightage of each decision maker against an attribute may not be equal. If we assign same weightage to all decision makers, we may not have accurate solutions. Determining decision makers' weights is a vital part of decision making process. In decision making process most often either we give equal weightage to all decision makers or the group leader assigns a weight to each Decision maker which may not be justified. Evaluating the importance of each decision maker is one of the significant steps in decision making process. Each decision maker has his own suggestions, approaches, inspirations and individualities. In aggregation stage, the judgments of different decision makers are combined to get final weightage of each factor. But at the final decision making stage we can reach at more accurate result if we take into account the weights of decision makers also.

Since 1963, many methods have been developed to determine the weights of decision makers. Hojjat Mianabadi et al [1] presents a new method to assess the relative weights of decision makers by integrating subjective preferences of group leader and assessments of decision makers by other members of the team simultaneously. In this method it is assumed that each decision maker has prior information on the proficiency and rationality of other members of the group. But in real world problems, it may not be reasonable to ask one person in the group to assign weights to other persons of the group according to his respect on their expertise. Because group members may feel aversion to reveal the weight of other group members as it could lead to hard feelings within the group. Thus experts' weights are calculated based on the strength of the differences in the opinion expressed by other decision makers in the group. Taking account of all these, a new method have been proposed in this paper.

The Arithmetic mean operation of TrFNs is explained here, which will be more advantageous in ranking procedure, as this method is easier to compile the variety of experts' judgements. Then a new ranking method based on the values of the fuzzy numbers is explained. This paper is organized as follows: Section 1 depicts the method of finding Arithmetic Mean of TrFNs; Section 2 describes the value of fuzzy numbers; Section 3 explains the new decision making model with illustrations and Section 4 concludes the work.

1. Arithmetic Mean Operation of Trapezoidal Fuzzy Numbers (TrFNs) [2]

Consider the Trapezoidal Fuzzy Numbers:

$A_1=(a_1,a_2,a_3,a_4)$, $A_2=(b_1,b_2,b_3,b_4)$,…, $A_n=(n_1,n_2,n_3,n_4)$

with membership functions,

$$\mu_{A_1}(x_1) = \begin{cases} 0\ , & x_1 \le a_1 \\ \dfrac{x_1 - a_1}{a_2 - a_1}, & a_1 \le x_1 \le a_2 \\ 1\ , & a_2 \le x_1 \le a_3 \\ \dfrac{a_4 - x_1}{a_4 - a_3}, & a_3 \le x_1 \le a_4 \\ 0\ , & x_1 \ge a_4 \end{cases}$$

$$\mu_{A_2}(x_2) = \begin{cases} 0\ , & x_2 \le b_1 \\ \dfrac{x_2 - b_1}{b_2 - b_1}, & b_1 \le x_2 \le b_2 \\ 1\ , & b_2 \le x_2 \le b_3 \\ \dfrac{b_4 - x_2}{b_4 - b_3}, & b_3 \le x_2 \le b_4 \\ 0\ , & x_2 \ge b_4 \end{cases}$$

$$\mu_{A_n}(x_n) = \begin{cases} 0\ , & x_n \le n_1 \\ \dfrac{x_n - n_1}{n_2 - n_1}, & n_1 \le x_n \le n_2 \\ 1\ , & n_2 \le x_n \le n_3 \\ \dfrac{n_4 - x_n}{n_4 - n_3}, & n_3 \le x_n \le n_4 \\ 0\ , & x_n \ge n_4 \end{cases}$$

Or,

$$\mu_{A_1}(x_1) = max\left[min\left(\frac{x_1 - a_1}{a_2 - a_1}, 1, \frac{a_4 - x_1}{a_4 - a_3}\right), 0\right]$$

$$\mu_{A_2}(x_2) = max\left[min\left(\frac{x_2 - b_1}{b_2 - b_1}, 1, \frac{b_4 - x_2}{b_4 - b_3}\right), 0\right]$$

$$\mu_{A_n}(x_n) = max\left[min\left(\frac{x_n - n_1}{n_2 - n_1}, 1, \frac{n_4 - x_n}{n_4 - n_3}\right), 0\right]$$

The α- cuts of these fuzzy numbers are given by:

$$A_1(\alpha) = [A_{1L}(\alpha), A_{1U}(\alpha)] = [a_1 + \alpha(a_2 - a_1),\ a_4 - \alpha(a_4 - a_3)]$$

$$A_2(\alpha) = [A_{2L}(\alpha), A_{2U}(\alpha)] = [b_1 + \alpha(b_2 - b_1),\ b_4 - \alpha(b_4 - b_3)]$$

....

....

$$A_n(\alpha) = [A_{nL}(\alpha), A_{nU}(\alpha)] = [n_1 + \alpha(n_2 - n_1),\ n_4 - \alpha(n_4 - n_3)]$$

Then we define the Arithmetic Mean of these fuzzy numbers as follows:

Let $A_V = \frac{A_1 + A_2 + \cdots + A_n}{n} = \left(\frac{a_1+b_1+\cdots+n_1}{n}, \frac{a_2+b_2+\cdots+n_2}{n}, \frac{a_3+b_3+\cdots+n_3}{n}, \frac{a_4+b_4+\cdots+n_4}{n}\right)$

with membership function,

$$\mu_{A_V}(X) = \begin{cases} sup\left[min\left(\frac{x_1 - a_1}{a_2 - a_1}, \frac{x_2 - b_1}{b_2 - b_1}, \ldots, \frac{x_n - n_1}{n_2 - n_1}\right) : \frac{x_1 + x_2 + \cdots + x_n}{n} = X\right] & if \quad a_1 \le x_1 \le a_2, b_1 \le x_2 \le b_2, \ldots, n_1 \le x_n \le n_2 \\ 1 & if \quad a_2 \le x_1 \le a_3, b_2 \le x_2 \le b_3, \ldots, n_2 \le x_n \le n_3 \\ sup\left[min\left(\frac{a_4 - x_1}{a_4 - a_3}, \frac{b_4 - x_2}{b_4 - b_3}, \ldots, \frac{n_4 - x_n}{n_4 - n_3},\right) : \frac{x_1 + x_2 + \cdots + x_n}{n} = X\right] & if \quad a_3 \le x_1 \le a_4, b_3 \le x_2 \le b_4, \ldots, n_3 \le x_n \le n_4 \end{cases}$$

That is,

$$\mu_{A_V}(X) = \begin{cases} \dfrac{X - \left(\frac{a_1 + b_1 + \cdots + n_1}{n}\right)}{\left(\frac{a_2 + b_2 + \cdots + n_2}{n}\right) - \left(\frac{a_1 + b_1 + \cdots + n_1}{n}\right)} & if \left(\frac{a_1 + b_1 + \cdots + n_1}{n}\right) \le X \le \left(\frac{a_2 + b_2 + \cdots + n_2}{n}\right) \\ 1 & if \left(\frac{a_2 + b_2 + \cdots + n_2}{n}\right) \le X \le \left(\frac{a_3 + b_3 + \cdots + n_3}{n}\right) \\ \dfrac{\left(\frac{a_4 + b_4 + \cdots + n_4}{n}\right) - X}{\left(\frac{a_4 + b_4 + \cdots + n_4}{n}\right) - \left(\frac{a_3 + b_3 + \cdots + n_3}{n}\right)} & if \left(\frac{a_3 + b_3 + \cdots + n_3}{n}\right) \le X \le \left(\frac{a_4 + b_4 + \cdots + n_4}{n}\right) \\ 0 & otherwise \end{cases}$$

The α- cut of A_V is given as:

$$A_V(\alpha) = [A_{VL}(\alpha),\ A_{VU}(\alpha)]$$

$$= \left[\left\{\left(\frac{a_1 + b_1 + \cdots + n_1}{n}\right) + \alpha\left(\frac{a_2 + b_2 + \cdots + n_2}{n} - \frac{a_1 + b_1 + \cdots + n_1}{n}\right)\right\}, \left\{\left(\frac{a_4 + b_4 + \cdots + n_4}{n}\right) - \alpha\left(\frac{a_4 + b_4 + \cdots + n_4}{n} - \frac{a_3 + b_3 + \cdots + n_3}{n}\right)\right\}\right]$$

Note: Similarly we can define arithmetic mean operation for triangular fuzzy numbers.

2. Value of Fuzzy Numbers [2]

Proposition 1: The value of a Trapezoidal fuzzy number A=(a,b,c,d) is given by

$$val(A) = \frac{a}{6} + \frac{b}{3} + \frac{c}{3} + \frac{d}{6}$$

Proof:

The $\alpha - cut$ of the Trapezoidal fuzzy number $A = (a, b, c, d)$ is given by

$$A(\alpha) = [A_L(\alpha), A_U(\alpha)] = [a + \alpha(b - a),\ d - \alpha(d - c)]$$

Then the value of the fuzzy number $A = (a, b, c, d)$ is denoted as $val(A)$ and is defined by

$$val(A) = \int_0^1 \alpha\, [A_U(\alpha) + A_L(\alpha)]d\alpha$$
$$= \int_0^1 \alpha\, [d - \alpha(d - c) + a + \alpha(b - a)]d\alpha$$
$$= \frac{a}{6} + \frac{b}{3} + \frac{c}{3} + \frac{d}{6}$$

Proposition 2: The value of a Triangular fuzzy number B=(a,b,c) is given by

Proof:

The $\alpha - cut$ of the Triangular fuzzy number $B = (a, b, c)$ is given by

$$B(\alpha) = [B_L(\alpha), B_U(\alpha)] = [a + \alpha(b - a),\ c + \alpha(b - c)]$$

$$val(B) = \int_0^1 \alpha\, [A_U(\alpha) + A_L(\alpha)]d\alpha$$
$$= \int_0^1 \alpha\, [c + \alpha(b - c) + a + \alpha(b - a)]d\alpha$$
$$= \frac{a}{6} + \frac{2b}{3} + \frac{c}{6}$$

Theorem 2.1

If $A_1=(a_1,a_2,a_3,a_4)$ and $A_2=(b_1,b_2,b_3,b_4)$ are two trapezoidal fuzzy numbers such that $A_1=A_2$, then $val(A_1)=val(A_2)$.

Proof:

Let us assume that $A_1 \neq A_2$ with $val(A_1)=val(A_2)$.

Without loss of generality, we have $a_1=b_1, a_2=b_2$, $a_3<b_3$ and $a_4<b_4$

$val(A_1)= val(A_2) \Rightarrow val(A_1)-val(A_2)=0$

$$\Rightarrow \frac{a_1}{6} + \frac{a_2}{3} + \frac{a_3}{3} + \frac{a_4}{6} - \frac{b_1}{6} - \frac{b_2}{3} - \frac{b_3}{3} - \frac{b_4}{6} = 0$$

$$\Rightarrow a_3 - 2b_3 + a_4 - 2b_4 = 0$$

a negative number + a negative number = 0 which is a contradiction

Therefore $A_1=A_2$.

Theorem 2.2

If $A_1=(a_1,a_2,a_3)$ and $A_2=(b_1,b_2,b_3)$ are two triangular fuzzy numbers, then $A_1=A_2 \Leftrightarrow val(A_1)=val(A_2)$.

Proof:

Proof of the first part is similar to the proof of Theorem 3.1.1

ie; $A_1=A_2 \Rightarrow val(A_1)=val(A_2)$.

Conversely,

Let us assume that $val(A_1)=val(A_2)$

$$\Rightarrow \frac{a_1}{6} + \frac{2a_2}{3} + \frac{a_3}{6} = \frac{b_1}{6} + \frac{2\,b_2}{3} + \frac{b_3}{6}$$

$$\Rightarrow a_1 = b_1,\ a_2 = b_2,\ a_3 = b_3 \text{ (equating similar terms)}$$

$$\Rightarrow A_1 = A_2$$

Result 1 : [2]

We determine the ranking of A_i and A_j as follows:

(i) $val(A_i) > val(A_j) \Rightarrow A_i > A_j$

(ii) $val(A_i) < val(A_j) \Rightarrow A_i < A_j$

(iii) $val(A_i) = val(A_j) \Rightarrow A_i \sim A_j$

Result 2 : [2] The weight of the alternative A_i over all other (n-1) alternatives Aj is given by

The weight of the alternative A_i over all other (n-1) alternatives A_j is given by

$$weight(A_i) = \sum_{j=1,j\neq i}^{n} v(A_i, A_j)$$

Note:

(i) $0 \le v(A_i, A_j) \le 1$

(ii) $v(A_i, A_i) = 0.5$

(iii) $v(A_i, A_j) + v(A_j, A_i) = 1; i, j = 1,2, \ldots, n; i \neq j$

(iv) $\sum_{i=1}^{n} weight(A_i) = 1$

Proposed De cision Making Model

We propose a Multi Criteria Decision Making Model for a decision making situation with multiple decision makers. We discuss the model when the weights of the criteria are known. We define Score for each criteria based on the linguistic terms used for decision making and weights of the criteria. Based on the score obtained we can rank the alternatives.

Here we propose a simple and easiest method for multi criteria decision making when there are multiple decision makers. Suppose we need to rank m alternatives $A_1, A_2, .., A_m$ based on n criteria $C_1, C_2, .., C_n$. Let there are k decision makers.

Let d_ijk denotes the degree assigned by the k^th decision maker that the alternative Ai satisfies criteria Cj. dijk can be chosen based on some standard fuzzy scaling. Let w_j be the weight of the criteria C_j for $_j$=1,2,…,n which are calculated based on the experts' judgments following the method explained in section 2.

Let the weight of the decision makers $D_1, D_2, \ldots, _{Dk}$ are $W_{D1}, W_{D2}, \ldots, W_{Dk}$.

Each decision maker finds the score Score(A_{ik}) of each alternative A_i and then final score of the alternative A_i is obtained using the formula

$$S(A_j) = \sum_{i=1}^{k} \left[\frac{W_{D_i} * Score(A_{ij})}{k} \right]$$

3.1 Proposed Method for finding Weights of Decision Makers

In a real world problem, group leader may have different estimations about relative weights of decision makers. Assuming R_i as the relative weight of i^{th} decision maker assigned by the group leader and a_j as the normalised weight assigned by other decision makers, we calculate the final weight of each decision maker using the following formula.

$$D_j = \alpha R_j + \beta\, a_j,\ j = 1,2,\ldots,k$$

where α and β are the weight of respect of group leader's opinion and other decision makers' opinion.

$$\alpha, \beta \in [0,1] \text{ such that } \alpha + \beta = 1.$$

Here in this method we give equal respect to both of them. So α=β=0.5

In real world problems it is comfortable to give opinions in linguistic terms such as excellent, very good, good etc. These linguistic terms are fuzzified using some prescribed scaling and Rj, j=1,2,…,k is calculated as the value of these fuzzy numbers. Each of the decision makers is asked to give rating in linguistic terms for all other decision makers. We fuzzify these linguistic terms and a single rating is obtained as the arithmetic mean of all these fuzzy numbers. Then we calculate vj (j=1,2,…,k) as the value of this single judgment fuzzy number and aj is calculated using the formula

$$a_j = \frac{v_j}{v_1 + v_2 + \cdots + v_{j-1} + v_{j+1} + \cdots + v_k}$$

Illustration

Suppose there are k decision makers $D_1, D_2, \ldots, D_k$ and one of them is selected as group leader. Suppose Dk is selected as group leader. Then Dk assesses all other k-$_1$ decision makers and the judgments will be in linguistic terms. These linguistic terms are fuzzified using some prescribed scaling such as the trapezoidal fuzzy numbers

$$(l_1, m_1, n_1, u_1), (l_2, m_2, n_2, u_2), \ldots, (l_{k-1}, m_{k-1}, n_{k-1}, u_{k-1}).$$

Then

$$R_1 = val(l_1, m_1, n_1, u_1) = \frac{l_1}{6} + \frac{m_1}{3} + \frac{n_1}{3} + \frac{u_1}{6}$$

$$R_2 = val(l_2, m_2, n_2, u_2) = \frac{l_2}{6} + \frac{m_2}{3} + \frac{n_2}{3} + \frac{u_2}{6}$$

… … …. … … … … …

$$R_{k-1} = val(l_{k-1}, m_{k-1}, n_{k-1}, u_{k-1}) = \frac{l_{k-1}}{6} + \frac{m_{k-1}}{3} + \frac{n_{k-1}}{3} + \frac{u_{k-1}}{6}$$

$$R_k = max\ (R_1,\ R_2, \ldots,\ val(B) = \frac{a}{6} + \frac{2b}{3} + \frac{c}{6}$$

Then each of the decision makers is asked to assess all other individuals on the group. For each decision maker we get k-1 judgments which are compiled to have a single judgment using arithmetic mean of fuzzy numbers. By considering the values of these single judgment fuzzy numbers we find $v_1, v_2, \ldots vk$.

Then

$$a_1 = \frac{v_1}{v_2 + v_3 + v_4 + \cdots + v_k}$$

$$a_2 = \frac{v_2}{v_1 + v_3 + v_4 + \cdots + v_k}$$

… … … … …

$$a_k = \frac{v_1}{v_1 + v_2 + v_3 + \cdots + v_{k-1}}$$

As we are giving same respect for the opinion of both group leader and other individuals in the group, we take α= β=0.5

So W_{D1})=0.5R_1+0.5$_{a1}$, W_{D2})= 0.5R_2+0.5$_{a2}$, ...,W_{Dk}=0.5Rk+0.5k

Conclusion

In multi criteria decision making situations, to find the weights of all factors influencing the objective of the problem we can use value of the fuzzy numbers as explained in section 2. We can use Arithmetic mean of fuzzy numbers for compiling experts' judgments. We apply this method easily and effectively as it doesn't need much computational effort. In the proposed decision making model, we are considering the weights of decision makers also since the proficiency, experience and knowledge of various decision makers' are not the same.

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