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Mathematics

FINDING THE SUM OF 2 SQUARE NUMBERS THROUGH PATTERN OBSERVATION AND ANALYSIS

Utsav Datta U.G Student. Department of Engineering Amity University Kolkata

ABSTRACT The objective of this theoretical mathematics paper is to find an equation for computing the sum of 2 square numbers. In this paper we take examples of small numbers and with the help of the examples, analyse the patterns between the sums of the square numbers. By analysis the pattern we come to an equation which successfully finds the sum of the square numbers. After observing the patterns, we conclude through the paper that -the sum of 2 square numbers can be written as 2 times the square of the smaller number plus 'd' times the smaller number plus 'd' times the bigger number. Where 'd' is the difference between the numbers.

KEYWORDS : analysis, pattern, relation between numbers, sum of square numbers, series.

INTRODUCTION –

The objective of this paper is simple. We study sums of different square numbers and through pattern analysis try to come to a single formula through which we can find the sum of two different square numbers. The objective is not to find a simpler way to calculate a^2+b^2, but a different way to look at it.

We will use different examples and use the examples to find similarities between the sums of different square numbers. Using the similarities and the observed pattern we will come to the conclusion.

The calculations are done in 5 different parts. In the first part we observe the pattern in the sum of 2 consecutive square numbers, like 2^2+3^2. Then we skip one number and add the squares, like 3^2+5^2. Then we find the sum by skipping 2 numbers and adding the squares, like 3^2+6^2, then we do the same by skipping 3 numbers and so on. By doing so we find a pattern between the sums and by observing the patterns we conclude a relation for a^2+b^2.

Note – The numbers we use in the examples are completely arbitrary.

THE ANALY SIS PART -

<u>SUM OF TWO CONSECUTIVE SQUARE NUMBERS –</u>

If we look at $1^2+2^2=5$ -i

Then $2^2+3^2=13$ -ii

$3^2+4^2=25$ -iii

$4^2+5^2=41$ -iv

And so on…

Know if we subtract the sum of (i) from (ii), the sum of (ii) from (iii), the sum of (iii) from (iv) and so on we get

13-5 = 8

25-13 = 12

41-25 = 16

And so on. Through this pattern we can observe that the *differencebetween the sum increases by 4.*

Therefore, we can write the sum of two consecutive square numbers in terms of previous two consecutive square numbers. By –

$a^2+b^2=(a-1)^2+(b-1)^2+4a$ - (v)

We can use the equation (v) in a recursive manner and generalize through an example –

$$6^2+7^2=5^2+6^2+4(6)$$

Further 5^2+6^2 can be broken down into $4^2+5^2+4(5)$

Therefore $6^2+7^2=4^2+5^2+4(5)+4(6)$

$=3^2+4^2+4(4)+4(5)+4(6)$

$=2^2+3^2+4(3)+4(4)+4(5)+4(6)$

$=1^2+2^2+4(2)+4(3)+4(4)+4(5)+4(6)$

$=1^2+2^2+4(2+3+4+5+6)$

Taking (2 + 3 + 4 + 5 + 6) as x, we can find the value of x by $x = \frac{n(n+1)}{2} - 1$ - (vi)

In equation (vi), n = a. If a is the smaller number in $a^2 + b^2$ i.e. a < b.

From equation (vi) we get $x = \frac{(n+2)(n-1)}{2}$

Putting the value of x in [$1^2 + 2^2 + 4x$] we get $\Rightarrow 5 + 2(n+2)(n-1)$ - (vii)

Therefore, $n^2 + (n+1)^2 = 5 + 2(n+2)(n-1)$

Now we calculate the sum by skipping one number and then adding the squares of the numbers. That is $n^2 + (n+2)^2$.

If we look at $2^2 + 4^2$ and we put n = 2 and then compute the sum by using the formula – (vii) we get 13, which is actually 7 short from the actual answer. This 7 can be brought from 3 + 4.

3 → the number between 2 and 4

4 → the bigger number.

Looking at another example for analysing – $5^2 + 7^2$

Putting $n = 5$ and computing using the equation (vii) we get 61, which is 13 short from the actual answer. This 13 can be brought from 6 + 7

6 → number between 5 and 7

7 → the bigger number

For generalizing the above pattern, we take equation (vii) and add [(n + 1) + y] to it where y = the bigger number.

We get –

$5 + 2(n+2)(n-1) + (n+1) + y$

On simplification we get –

$2 + 2n^2 + 3n + y$ - (viii)

Therefore $n^2 + (n+2)^2 = 2 + 2n^2 + 3n + y$

Now we analyse the sum of square numbers where the difference between the numbers is 3. That is, we skip 2 numbers between the numbers. Therefore, we study the pattern for $n^2 + (n+3)^2$

If we look at $4^2 + 7^2$

We put n = 4 and y = 7 and we compute the sum by using the equation (viii) we get 53, which is 12 short from the actual right answer.

This 12 can be brought from (7-1)2; which is basically $2(y-1)$, this pattern is observed for any number 'n' for the sum of [$n^2 + (n+3)^2$]

Therefore, we add $2(y-1)$ to the equation (viii) and simplify to get

$2n^2 + 3n + 2 + y + 2(y-1)$

$\Rightarrow$ $2n^2 + 3n + 3y$ - (ix)

Where 'n' = smaller number and 'y' = the bigger number.

If we see at $2^2 + 5^2$ and put n=2 and y=5 and compute the sum using equation (ix), we get 29. Which is correct.

Now we calculate the sum of square numbers where the difference between the numbers is 4. That is, we skip 3 numbers betwee the numbers. Therefore, we study the pattern for $n^2 + (n+4)^2$

We start with the formula (ix) and for any number 'n' which follows the pattern we check the answer.

If we take n = 2 and y = 6 and compute the sum using the formula (ix) we get the answer as 32 instead the actual answer which is 40. So, we are 8 short.

If we take n = 3 and y = 7 and compute the sum using the formula (ix) we get the answer as 48 instead of 58. So, we are 10 short in this case.

Like this if we compute for n = 4 and y = 8 we are 12 short than the actual answer. Therefore, we can see –

$2^2 + 6^2 =$ formula answer + 8

$3^2 + 7^2$ = formula answer + 10

$4^2 + 8^2$ = formula answer + 12

$5^2 + 9^2$ = formula answer + 14 and so on…

Therefore, if we look at the pattern we can see that we are falling short by exactly $(n+y)$ where n = smaller number and y = the bigger number.

Therefore adding (n + y) to the equation (ix) we get [$2n^2 + 3n + 3y + n + y$] which is

$2n^2 + 4n + 4y$ - (x)

Therefore $n^2 + (n+4)^2 = 2n^2 + 4n + 4y$

Now we look at the sum of square numbers where the difference between the numbers is 5. That is, we skip 4 numbers between the numbers. Therefore, we study the sum for $n^2 + (n+5)^2$

If we take n=2 and y=7 and compute the sum using the formula (x) we get 44, which is 9 short from the actual answer of $2^2 + 7^2$.

If we take n=3 and y=8 and compute the sum using the formula (x) we get 62, which is 11 short from the actual answer of $3^2 + 8^2$.

Therefore, we can see and conclude that we are falling short by (n + y).

Therefore, we add (n + y) to the equation (x) and we get

$2n^2 + 5n + 5y$ - (xi)

Therefore, we get

$n^2 + (n+5)^2 = 2n^2 + 5n + 5y$

Therefore, from the equations (ix), (x) and (xi) we can see a pattern which is

$n^2 + (n+3)^2 = 2n^2 + 3n + 3y$ - (ix)

$n^2 + (n+4)^2 = 2n^2 + 4n + 4y$ - (x)

$n^2 + (n+5)^2 = 2n^2 + 5n + 5y$ - (xi)

And so on…

Therefore, we conclude –

The sum of squares of 2 numbers, where 'd' be the difference between the numbers is 2 times the square of the smaller number plus 'd' timesthe smallenumber plus 'd' times the larger number.

MATHEMETICALLY

Sum = $2n^2 + dn + dy$ - (xii)

Where n = smaller number, d = the difference between the two numbers and y = the bigger number.

PROVING THAT $\boldsymbol{n^2 + y^2 = 2n^2 + dn + dy}$

$\Rightarrow 2n^2 + dn + dy$

Since $d = (y - n)$

$\Rightarrow 2n^2 + (y-n)n + (y-n)y$

$\Rightarrow 2n^2 + ny - n^2 + y^2 - ny$

$\Rightarrow 2n^2 - n^2 + y^2$

$\Rightarrow n^2 + y^2$

CONCLUSION –

Through analysing different examples of the sum of square numbers we get the formula (xii), which shows that the sum of 2 square numbers can be written as 2 times the square of the smaller number plus 'd' times the smaller number plus 'd' times the bigger number. Where 'd' is the difference between the numbers.