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A FUZZY OPTIMIZATION IN TRANSPORTATION PROBLEM BASED ON POSSIBILITY THEORY

KEYWORDS: Fuzzy numbers, Fuzzy Transportation Problem Possibility Theory, Modality approach.

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ABSTRACT *This paper considers a solution of Transport Problem with Fuzzy environment based on Possibility Theory. Possibility Theory and consistent with decision makers subjectiveness and practical requirements, the fuzzy transportation problem is transformed to a crisp linear transportation problem by defuzzifying fuzzy constraints and objectives with application of fractile and modality approach. At the end a numerical example is presented to illustrate how possibility theory works.*

1. INTRODUCTION

Transportation is an essential part in our day life, social, industrial and economical process that encounters to the increasing level of vehicle which leads to increasing demand and deterioration of transport infrastructure as well as others. So, transportation problem is essential to deal with and required decision alternatives from experts such as rehabilitation and maintenance, transport planning, signed controls and etc. mathematically, the Transportation Problem is a special case of linear programs problem which arises in many practical application. A Fuzzy Transportation Problem in which the transports costs parameters supply and demand quantities and fuzzy quantities.

In practice, there are many situations where it is impossible to get precise data for uncertainty of information.
Most previous research in Fuzzy Transportation problem assumes that all parameters are fuzzy numbers where the representation is other normal or abnormal, triangular or trapezoidal.

The possibility theory provides a feasible and scientific method to describe the decision makers attitudes about fuzziness. For this complexity, we proposed Possibility Theory method to solve the fuzzy transportation problem. We will apply Possibility Theory to formalize the Fuzzy Transportation Problem by introducing fractile and modality approach.

2. PRELIMINARIES

2.1 Definition

Fuzzy Number

A generalized fuzzy number is determined by a membership-function mapping elements of a domain space or universe of discourse Z to the unit interval [0,1]

(i.e) $\tilde{A} = \{(z, \mu_{\bar{A}}(z)); z \in Z\}$

Here is a mapping called the degree of membership function of the fuzzy set $\bar{!}$ and $\mu_{\bar{A}}(z)$ is called membership value of $z \in Z$ in the fuzzy set $\bar{A}$. The membership grades are often represented by real numbers ranging from [0,1].

2.2 Definition: -cut of a Fuzzy Number.

The -cut of a fuzzy number $\bar{A}(z)$ is defined as

$$\bar{A}(\alpha) = \{z; \mu(z) \geq \alpha, \alpha \in [0,1]\}$$

2.3 Definition: Trapezoidal Fuzzy Number

For a Trapezoidal Fuzzy Number $\bar{A}(z)$ it can be represented by $\bar{A}(a_1, a_2, a_3, a_4; 1)$ with membership function μ(z) is given by,

$$\mu(z) = \begin{cases} \frac{z-a_1}{a_2-a_1}, & a_1 \leq z \leq a_2 \\ 1, & a_2 \leq z \leq a_2 \\ \frac{a_4-z}{a_4-a_3}, & a_3 \leq z \leq a_4 \\ 0, & \text{otherwise} \end{cases}$$

2.4 Possibility Theory

The Possibility Theory was introduced by Negoita and Zadeh [4], inspired by Gainer [5] Possibility Theory is study of an uncertainly theory devoted to solve incomplete information. As such it complements Probability Theory. It differs from the probability by using of a pair of dual set functions (possibility and necessity measures) instead of only one. This fact makes it easier to capture partial ignorance. Further, it is not additional and makes sense on ordinal structures. In Zadesh's point of view, possibility distributions were meant to provide a graded semantics to natural languages statements. However possibility and necessary measures can be also being the basis of a full-pledged representation of partial belief that similar to probability. Then, it can be seen as a coarse non-numerical version of probability theory or as a frame work far reasoning with imprecise probabilities.

If $\tilde{A}$ and $\tilde{B}$ are fuzzy sets and a Fuzzy event $\lambda \in \tilde{A}$, the possibility and necessity of $\lambda \in$ are defined as

$$M_{\bar{A}}(\bar{B}) = Sup \min \{\mu_{\bar{A}}(r), \mu_{\bar{B}}(r)\}$$

$$N_{\bar{A}}(\bar{B}) = Inf \max \{1-\mu_{\bar{A}}(r), \mu_{\bar{B}}(r)\}$$

If $\bar{a}$ and $\bar{b}$ are two fuzzy numbers and their membership functions $\mu_{\bar{a}}(x)$ and $\mu_{\bar{b}}(x)$ respectively, the measure of possibility and necessity are defined as.

$$Ps(\bar{a}*\bar{b}) = Sup\{\min \mu_{\bar{a}}(x), \mu_{\bar{b}}(y) / x, y \in R, x*y\}$$

$$Ns(\bar{a}*\bar{b}) = Inf\{\max(1-\mu_{\bar{a}}(x), \mu_{\bar{b}}(y) / x, y \in R, x*y\}$$

If $\bar{b}$ is a crisp number, then we can derive that

$$Ps(\bar{a} \leq \bar{b}) = Sup\{\mu_{\bar{a}}(x) / x \in R, x \leq b\}$$

$$Ns(\bar{a} \leq \bar{b}) = Sup\{(1-\mu_{\bar{a}}(x) / x \in R, x \leq b\}$$

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Where Ps, Ns are possibility, necessity respectively.

3. Mathematical Model of Fuzzy Transportation Problem
Mathematically a Transportation Problem can be stated as follows.

$$Minimize\ Z = \sum_{i=1}^{m} \sum_{j=1}^{m} \overline{C}_{ij}\ x_{ij} \qquad (1)$$

Subject to

$$\left.\begin{array}{ll} \sum_{j=1}^{m} x_{ij} = \tilde{a}_i, & i = 1,2,....m \\ \sum_{i=1}^{m} x_{ij} = \tilde{b}_i, & j = 1,2,....m \\ x_{ij} \geq 0 & \end{array}\right\} \qquad (2)$$

where Cij the cost of transportation of an unit from the ith source to the jth destination and the quantity xij is to be some positive integer or zero, which is to be transported from the ith origin jth destination. An obvious necessary and sufficient condition for the linear programming problem given in (1) to have a solution is that

$$\sum_{i=1}^{n} \overline{a}_j \sum_{j=1}^{m} \overline{b}_j \qquad (3)$$

The transportation costs $\overline{C}_{ij}$, supply $\overline{a}_i$ and $\overline{b}_j$ quantities are fuzzy quantities.

Here we are Trapezoidal number to represent the fuzziness of parameters in FTP. (ie) we assume that

$\overline{C}_{ij}$ = (Cij1, Cij2; Lij1, Rij), i = 1, 2, ...m j = 1, 2, ...n

$\overline{a}_i$ = (ai1, ai2; Li, Ri), i = 1, 2, ...m

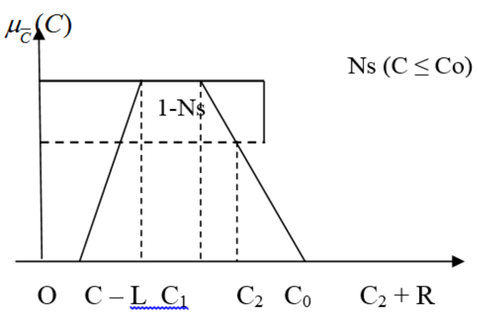
$\overline{b}_j$ = (bj1, bj2; Lj, Rj), j = 1, 2, ...n

The possibility and necessity measure are adopted as ranking index.

3.1 Modality Approach
A modality optimization model correspondents to the minimum-risk approach to a stochastic programming problem. The minimum-risk approach is called the maximum probability approach on the aspiration criterion approach. A modality optimization approach, the decision-maker puts more importance on the certainly degree comparing to the fractile approach.

3.2 Necessity Optimization
From the point of necessity and modality approach, the decision maker in FTP expects the total cost should not be greater a given level Co.

(ie) Max Ns ($\overline{C} \leq$ Co) (4)



$$N_s(\overline{C} \leq C_o) = \frac{C_o - C_2}{R}$$

and hence, the objective FTP can be converted to

$$Min \frac{C_o - C_s}{R} = \frac{\sum_{i=i}^{m} \sum_{j=i}^{n} C_{ij} x_{ij} - Co}{\sum_{i=i}^{m} \sum_{j=i}^{n} R_{ij} x_{ij}} \qquad (5)$$

Subject to

$$a_{i,} - (1-\alpha_i)L_i \leq \sum_{j=1}^{n} x_{ij} \leq a^{R}_{i\alpha j} = a_{i2} + (1-\alpha_i)R_i \quad i = 1,2,...m \qquad (6)$$

$$b_{j,} - (1-\beta_i)L_i = b^{L}_{\beta j} \leq \sum_{i=1}^{m} x_{ij} \leq b^{R}_{j\beta} = b_i 2 + (1-\beta_j)R_{ji} \quad i = 1,2,..n \qquad (7)$$

This is a fractional programming problem which can be transformed to a linear programming problem by the substitution.

$$\dot{y} = \frac{1}{\sum_{i=i}^{m} \sum_{j=i}^{n} R_{ij} x_{ij}}, \quad Z^{(i)}_{ij} = x_{ij} \quad \forall i,j$$

Now we obtain the revised model as follows.

$$Min \sum_{i=i}^{m} \sum_{j=i}^{n} C_{ij2} Z^{(i)}_{ij} - Co\ \dot{y}$$

Subject to

$$\sum_{j=i}^{n} Z^{(1)}_{ij} - \left[a_{i1} - (1-\alpha_i)L_i / \dot{y} \geq 0,\right.$$

$$\sum_{j=i}^{n} Z^{(1)}_{ij} - \left[a_{i2} + (1-\alpha_i)R_i / \dot{y} \geq 0\right.$$

$$\sum_{i=1}^{m} Z^{(1)}_{ij} - \left[b_{j1} - (1-\beta_j)L_j / \dot{y} \geq 0\right.$$

$$\sum_{i=1}^{m} Z^{(1)}_{ij} - \left[b_{j1} + (1-\beta_j)R_i / \dot{y} \geq 0\right.$$

$$\sum_{i=1}^{m} \sum_{j=1}^{n} R_{ij} Z_{ij} = 1$$

$$y \geq 0, Z^{(1)}_{ij} \geq 0, \quad i = 1,2.....m; \quad j = 1, 2 ....n$$

(9)

3.2 Possibility Optimization

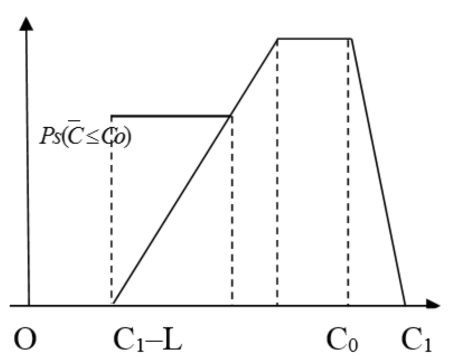


Fig.2

From the point of possibility, the decision-maker expects the total cost should not be greater than a given level Co.

=(ie) Max. Ps ($\overline{C} \leq$ Co) (10)

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From the fig. 2, we obtain the revised model for possibility as follows.

$$P_s(\overline{C} \leq C_o) = \frac{C_o - C_1 + L}{L}$$

Here, the objective function of FTP can be transformed to

$$Min \frac{C_o - C_1 + L}{L} = \min \frac{\sum_{i=i}^{m} \sum_{j=i}^{n} C_{ij} 1 x_{ij} - Co}{\sum_{i=i}^{m} \sum_{j=i}^{n} L_{ij} x_{ij}} \qquad (11)$$

(ie) now the FTP transformed to

$$Min \ \ C = \frac{\sum_{i=i}^{m} \sum_{j=i}^{n} C_{ij} 1 x_{ij} - Co}{\sum_{i=i}^{m} \sum_{j=i}^{n} L_{ij} x_{ij}} \qquad (12)$$

subject to the constraints (6) & (7)

Again it is a fractional programming problem which can be transformed to a linear programming by taking the substitution problem.

$$\ddot{y} = \frac{1}{\sum_{i=i}^{m} \sum_{j=i}^{n} L_{ij} x_{ij}}, \quad Z_{ij}^{(2)} = x_{ij} \quad \forall \text{i,j}$$

Now, we obtain the revised model for possibility optimization as follows.

$$Min \sum_{i=i}^{m} \sum_{j=i}^{n} C_{ij} 1 Z^{(2)}{}_{ij} - Co\,\ddot{y} \qquad (13)$$

Subject to

$$\sum_{j2i}^{n} Z_{ij}^{(2)} - \left[a_{i1} - (1-\alpha_i) L_i \,/\, \ddot{y} \geq 0 \right.$$

$$\sum_{j2i}^{n} Z_{ij}^{(2)} - \left[a_{i2} + (1-\alpha_i) R_i \,/\, \ddot{y} \geq 0 \right.$$

$$\sum_{i21}^{m} Z_{ij}^{(2)} - \left[b_j 1 - (1-\beta_i) L_j \,/\, \ddot{y} \geq 0 \right.$$

$$\sum_{i21}^{m} Z_{ij}^{(2)} - \left[b_j 1 + (1-\beta_j) R_i \,/\, \ddot{y} \geq 0 \right.$$

$$\sum_{i=1}^{m} \sum_{j=1}^{n} L_{ij} Z_{ij}^{(2)} = 1$$

$$\ddot{y} \geq 0, \quad \text{i} = 1,2....\text{m}; \quad \text{j}= 1, 2 ......\text{n} \qquad (14)$$

By combining the optimization of possibility and necessity modality approach we proposed in fuzzify FTP in two crisp linear programming problems.

During the transformation process, some parameters are introduced to classify the decision-maker's subjectiveness about fuzziness, which makes the solutions more practical. As for constraints, we introduced α i and βj to reflect the decision-maker's requirements on the exient how the constraint is satisfied in the view of possibility with respect to the objective function, in the modality approach.

While in the modality approach, Cij was introduced as the upper bound of the total cost that decision-maker expected. In brief these parameters reflect the decision-maker's attitude to the fuzziness as FTP from the view of possibility or necessity.

Table - 1

	B1	B2	B3	B4	B5	Supply
A1	(8,9,3,2)	(2,3,1,5)	(2,5,1,6)	(10,16,4,8)	(7,12,5,5)	(22,28,20,30)
A2	(3,6,1,8)	(8,11,4,3)	(12,16,5,8)	(3,5,1,8)	(2,6,1,7)	(28,35,25,36)
A3	(13,15,5,8)	(3,8,1,10)	(10,18,4,9)	(4,10,1,1)	(12,20,5,2)	(16,24,12,28)
A4	(18,23,3,6)	(12,18,5,9)	(11,15,4,8)	(30,36,6,4)	(8,9,2,1)	(28,36,25,40)
Demand	(20,30,12,30)	(22,32,15,30)	(32,40,25,40)	(12,28,6,18)	(28,36,18,40)	

4. Numerical Illustration

In this numerical example, the models solved by using Lingo – software, so the solving process is omitted for simplification. The description of the transportation problem is standard tables, where the central part is the cost Cij, the column 'supply' are 'ai' and the row demand are bj while in the solution tables, and the central part is the quantities of transportation form Ai to Bj.

Let us consider four origins A1, A2, A3, A4 and the five destinations B1, B2, B3, B4, B5 since the supply and demand and cost of units are assumed to be fuzzy Trapezoidal numbers. By adding up the 'origins' supply and five destinations in 'demand' in table 1 the total supply and total demand are fuzzy trapezoidal numbers (94, 123, 82, 134) and (114, 166, 76, 158) depicted. By solving this FTP we get the values i, βj γ and Co as follows

ai = (0.85, 0.90, 0.80), βj = (.75, .90, .95, .85, .80)

γ = 0.95, 0.05 Co = .000

5. Conclusion

In this article, we have analyzed an optimal solution for fuzzy Transportation problem by using modality approach. By using the fuzzy transportation problem is transformed into two types of crisp linear programming problem. In the process of transformation some parameters are introduced to reflect decision-makers attitude to the uncertainty or fuzziness. The proposed methods suitable for fuzzy triangular numbers also. It is applicable for all type of Transportation Problems with normal or abnormal data.

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