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SPECIFIC STUDY ON GENERALISED IRRESOLUTE FUNCTION IN TOPOLOGICAL SPACES

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ABSTRACT The aim of this paper is to introduce and study the classes of g*Λ irresolute function and contra g*Λ irresolute function, and their different notions

KEYWORDS : g*Λ irresolute functions and g*Λ compactness.

1 INTRODUCTION

Levine[11] introduced the concepts of semi-open sets and semi-continuous in a topological space and investigated some of their properties. Strong forms of stronger and weaker forms of continuous map have been introduced and inves- tigated by several mathematicians. In 2007 M.Caldas,S.Jafari and T.Navalagi [10] introduced the concept of λ irresolute maps. The notion of irresolute functions [7] was introduced and investigated by M.Caldas in 2000.

Here we investigate some of their fundamental properties and the connec- tions between these maps and other existing topological maps are studied.

Throughout this paper (X,τ), (Y,σ) and (Z,η) (or simply X, Y and Z) will always denote topological spaces on which no separation axioms are as- sumed unless explicitly stated. Int(A),Cl(A),Intλ (A), Clλ (A),g*ΛCl(A) and g*ΛInt(A) denote the interior of A, closure of A, lambda interior of A, lambda closure of A, g* Lambda closure of A and g* Lambda Interior of A respectively.

2 Preliminary Definitions

Definition 2.2 A topological space(X,τ) is said to be

1. a generalized closed [15] if Cl(A) ⊂ U, whenever A ⊂ U and U is open in X.

2. a subset A of a space X is called λ-closed [6] if A = B∩C, where B is a Λ-set and C is a closed set.

3. a subset A of X is said to be a g*Λ closed set [21] if Cl_λ (A)⊂U whenever A⊂U, where U is semi open in X.

The complement of above closed sets are called its respective open sets.

The g*Λ closure (respectively closure, λ closure) of a subset A of X denoted by g*ΛCl(A),(Cl(A),Clλ A)is the intersection of all g*Λ closed sets (closed sets, λ closed sets) containing A.

Lemma 2.3 [3]

1. Every Λ-set is a λ-closed set,

2. Every open and closed sets are λ-closed sets.

Proposition 2.4 [21] In a topological space (X,τ),the following properties hold:

1. Every closed set is g*Λ closed,

2. Every open set is g*Λ closed,

3. Every λ closed(λ open) set is g*Λ closed(g*Λ open),

4. Union (intersection)of g*Λ closed (g*Λ open) sets is not g*Λ closed(g*Λ
open),

5. In T^1 space every g*Λ closed set (g*Λ open) is λ closed(λ open),

6. In Partition space every g*Λ closed(g*Λ open) set is g closed(g open),

7. In a door space every subset is g*Λ closed (g*Λ open), and

8. In T1/2 space every subset is g*Λ closed(g*Λ open).

Definition 2.5

A function f:(X,τ)−→(Y,σ) is called

1. [18] irresolute if for any semi open set S of (Y,σ),f^{-1}(S) is semi open in (X,τ),

2. [1] gc irresolute if the inverse images of g closed sets in(Y,σ) are g closed in (X,τ),

3. [10] λ irresolute if the inverse image of λ open sets in Y are λ open in (X,τ),

3 SPECIAL FACTS ON g*Λ IRRESOLUTE MAP

Definition 3.1

1. A map f:(X,τ)−→(Y,σ) is called g*Λ irresolute map if the inverse image of each g*Λ closed set in Y is g*Λ closed in X.

2. A map f:(X,τ)−→(Y,σ) is called contra g*Λ irresolute map if the inverse image of each g*Λ closed set in Y is g*Λ open in X.

Definition 3.2

A topological space X is said to be

1. g*Λ T_0 (resp g*Λ- T_1) if for x ,y ∈ X such that x = y there exist a g*Λ- open set U of X containing x but not y or (resp and) a g*Λ-open set V of X containing y but not x.

2. g*Λ-Urysohn if for x , y ∈ X such that x = y there exist a g*Λ-open set U of X containing x and a g*Λ-open set V of X containing y such that g*ΛCl(U)∩ g*ΛCl(V)= ∅

3. g*Λ normal if each pair of non empty disjoint closed sets can be separated by disjoint g*Λ open sets.

4. ultra normal if each pair of nonempty disjoint closed sets can be separated by disjoint clopen sets.

5. g*Λ Hausdroff or g*Λ T2 if for each pair of distinct points x and y in X there exist g*Λ open subsets U and V of X containing x and y respectively, such that U∩V = ∅.

6. g*Λ-ultra Hausdroff if for each pair of distinct points x and y in X there exist g*Λ clopen subsets U and V of X containing x and y respectively, such that U∩V = ∅.

Definition 3.3
A function f:(X,τ)−→(Y,σ) is said to be

1. g*Λ irresolute if for each x ∈ X and each V ∈ g*ΛO(Y,f(x)), there exists U ∈g*ΛO(X,x) such that f(U)⊆ V. Equivalently if the inverse image of each g*Λ open set in Y is g*Λ open in X.

2. Quasi g*Λ irresolute if for each x∈ X and each V∈g*ΛO(Y,f(x)), there exists U∈g*ΛO(X,x) such that f(U)⊆ g*ΛCl(V).

Theorem 3.4 Every g*Λ-irresolute function f:X−→Y is quasi- g*Λ-irresolute.

Proof is very clear as for any set V⊂g*ΛCl(V).

Theorem 3.5 If Y is g*Λ-T_2 and f:X−→Y is g*Λ-irresolute injection then X is g*Λ-T_2.

Proof: Since f is injective,for any pair of distinct points x,y∈X,f(x)=f(y). As Y is g*Λ-T_2 there exists U∈g*ΛO(Y,f(x)) and V∈g*ΛO(Y,f(y)) such that U∩V = ∅ AS f is injective it follows that f^{-1}(U))∩f^{-1}(V))=∅. Since f is g*Λ irresolute,by definition 3.24 there exists U_1 ∈ g*ΛO(X,x) and V1 ∈ g*ΛO(X,y) such that f(U_1)⊆U and f(V_1)⊆(V). It follows that U1 ⊆ f^{-1} (U) and V1 ⊆f−1(V). Hence we get U1∩V1⊆f^{-1}(U))∩f^{-1}(V))=∅. Thus X is g*Λ-T_2.

Theorem 3.6 If Y is g*Λ-Urysohn and f:X−→Y is quasi- g*Λ-irresolute in- jection then X is g*Λ-T_2 .

Proof: Since f is injective,for any pair of distinct points x,y ∈X, f(x)=f(y). As Y is g*Λ-Urysohn there exists U∈g*ΛO(Y,f(x)) and V∈g*ΛO(Y,f(y)) such that g*ΛCl(U)∩g*ΛCl(V) = ∅. Hence f^{-1}(g*ΛCl(U)) ∩ f^{-1}(g*ΛCl(V)) = ∅. Since f is quasi g*Λ irresolute, there exists U_1∈g*ΛO(X,x) and V_1 ∈g*ΛO(X,y) such that f(U1)⊆ g*ΛCl(U) and f(V1)⊆g*ΛCl(V). It follows that U1⊆f−1 (g*ΛCl(U)) and V_1 ⊆f^{-1} (g*ΛCl(V)). Hence we get U_1∩V_1⊆f^{-1}(g*ΛCl(U))∩f^{-1} (g*ΛCl(V))=∅. Thus X is g*Λ-T_2.

g*Λ-Urysohn spaces remains invariant under bijective M.g*Λ-open function as seen from the following theorem

Theorem 3.7 If a bijective function f:X−→Y is M.g*Λ open and X is g*Λ- urysohn then Y is g*Λ urysohn.

Proof: Let y_1,y_2 ∈ Y,y_1 =y_2 . Since f is bijective f^{-1} (y_1),f^{-1}(y_2)∈ X and f^{-1} (y_1)=f^{-1}(y_2). Then g*Λ-urysohn property of X gives the existence of g*Λ open set U containing f^{-1}(y_1) and a g*Λ open set V containing f^{-1}(y_2) such that g*ΛCl(U)∩g*ΛCl(V)= ∅. As g*ΛCl(U) is g*Λ closed set in X and f is M.g*Λ open and bijective f(g*ΛCl(U) is g*Λ closed in Y. U⊆ g*ΛCl(U) implies that f(U)⊆ f(g*ΛCl(U)). Since f(g*ΛCl(U) is g*Λ closed in Y, we get g*ΛCl(f(U))⊆ f(g*ΛCl(U)). Similarly we get g*ΛCl(f(V))⊆ f(g*ΛCl(V)). Therefore by injec- tive of f, we have g*ΛCl(f(U)) ∩ g*ΛCl(f(V)) ⊆ f(g*ΛCl(U)) ∩ f(g*ΛCl(V))= f[g*ΛCl(U) ∩ g*ΛCl(V)] = ∅. Thus M.g*Λ-openness gives the existence of two g*Λ open sets f(U) containing y_1 and f(V)containing y_2 such that g*Λ Cl(f(U))∩ g*ΛCl(f(V)) = ∅, which assures that Y is g*Λ-urysohn.

Conclusion:
Hausdorff space plays important role in the Baire category theorem in General topology. This paper, defined and discussed some more properties of g*Λ-Compactness, Furthermore, using these concepts we have developed some important Theorem. In future, we may implement these things into many research areas such as ideal topology, rough set topology, soft topology, fuzzy set topology, digital topology, etc.

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