



## Financial Time Series Prediction using Multi-Scale Neuro Fuzzy Function Approximation

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**ABSTRACT** This paper presents a novel forecasting method for financial time series using artificial neural networks. Financial time series are characterized by chaos and are difficult to forecast by traditional function approximation techniques. They exhibit different cyclical patterns with different time periods. The exponential moving averages of higher time are easier to predict accurately. The higher time period moving averages show smooth patterns that can be captured by nonlinear function approximation by fuzzy neural networks. Although several prediction techniques have been applied to the task of financial time series forecasting, the naive prediction of no change is difficult to beat. The goal is to predict the future values with more accuracy than the naive prediction and also provide upper and lower bounds with 99% confidence. Experimental results on five stock indices show that the proposed method exhibits higher accuracy and reliability.

**KEYWORDS :** Time series forecasting, financial time series, stock returns, artificial neural networks, exponential moving average

### 1. Introduction

Time series forecasting [1] is a mature research area that has wide application in several varied fields such as climate science, econometrics, life sciences, astronomy, control theory etc. The basic task is to predict the next value in a time series data. The fundamental assumption in forecasting is that the time series follows a fixed underlying dynamics that exhibits itself in different patterns [2]. A Trend is a long term gradual change in the time series data in a single direction. Seasonality is the Dealing with randomness is a crucial part of time series forecasting. A time series is a special case of multidimensional data set called panel data. A time series is a one-dimensional panel. Some examples of time series are earthquake occurrence, amount of rainfall, supply and demand of a commodity, sun spots, electroencephalography, stock prices etc.

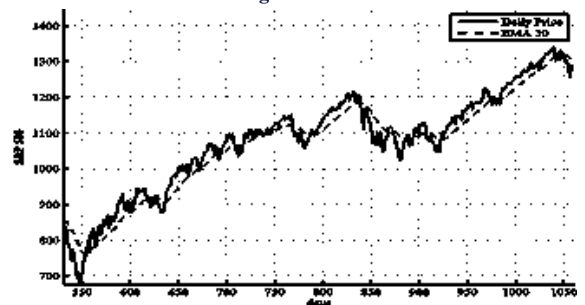
Time series analysis comprise of methods to predict future values of time series based on past values. Broadly time series analysis falls under two categories: frequency domain methods and time domain methods. In frequency domain methods spectral and wavelet transforms are employed to evaluate the correlation between time series values [3]. In time domain methods, auto-correlation and cross-correlation analysis are employed. New classes of methods based on fractal theory [4] are also widely researched. In some of the methods, the fundamental stationary mechanics that generated the time series is described using a set of parameters. These parameters are then estimated from past data and used to forecast future values. These are called parametric approach to time series analysis. The moving average and auto regressive models [5] are parametric models. Non-parametric approaches estimate the covariance or spectrum of the process without assumptions about the underlying process. Nonlinearity assumption creates more powerful models but require more data points and sensitive to randomness. Multivariate methods work on more than one time series variables at a time. They include the cross correlation among the different variables to arrive at better results. Hidden Markov Models [7] are examples of this approach.

Financial Time Series such as stock indices and prices are marked by chaotic behavior. The underlying mechanism undergoes drastic changes synchronous with political changes, disruptive technology. Technical analysis combined with inputs from fundamental analysis has been the cornerstone of success in the investing field. It has to be applied in conjunction with various risk management strategies to tide over unexpected shocks that are more frequent in the financial world than one would expect. Fuzzy time series (FTS) models have been applied to financial time series with good measure of success.

This paper presents a neuro fuzzy solution to FTS modeling of stock prices. The modeling is applied to multiple time scales based on exponential moving averages. The underlying mechanics of different time periods are collated to arrive at the final prediction. The next section briefly explains the proposed method.

### 2. Methodology

#### 2.1 Multi-scale FTS Modeling



**Figure 1. S&P 500 Daily Prices vs. 30 day EMA**

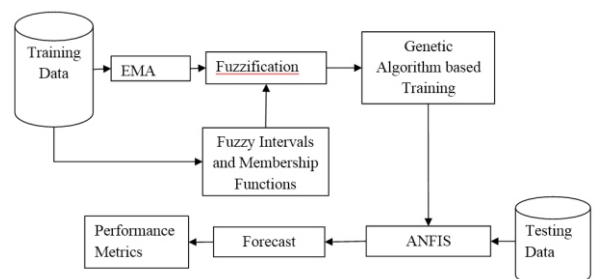
Figure 1 displays the daily price vs. 30 day exponential moving average (EMA) of Standard and Poor's 500 (S&P 500) index. The exponential moving average of a time series  $X$  is defined as

$$EMA = (Close - EMA(\text{previous day})) * \text{multiplier} + EMA(\text{previous day}) \dots (1)$$

The multiplier is a factor that depends on the time period of the EMA.

$$\text{multiplier} = \frac{2}{d+1} \dots (2)$$

Adaptive Neuro Fuzzy Inference System (ANFIS) is an artificial neural network based on Takagi- Sugeno fuzzy inference system. It combines fuzzy logic and neural networks in a single framework. The inference system is based on IF-THEN rules that approximate nonlinear functions. It is considered to be a universal estimator. The parameters are estimated using genetic algorithm similar to training of neural networks. The workflow of the proposed method is illustrated in figure 2.



**Figure 2. Architecture of the Proposed Method**

#### 2.2 Fuzzy Time Series

Fuzzy Time Series (FTS) methods divide the universe of discourse  $U = \{u_1, u_2, \dots, u_b\}$  into several fuzzy sets  $A_i$  defined as

$$A_i = \frac{f_{A_1}(u_1)}{u_1} + \frac{f_{A_2}(u_2)}{u_2} + \dots + \frac{f_{A_b}(u_b)}{u_b} \quad \dots (3)$$

Where  $f_{A_i}: U \rightarrow [0,1]$

is the membership function of the fuzzy set  $A_i$  that maps each element to a real number in the unit interval representing its degree of belongingness in the set.

A Fuzzy Time Series on real numbers  $Y(t)$  is defined as the collection  $F(t)$  of fuzzy sets  $f_i(t), (i=1,2,\dots)$  that are defined using  $Y(t)$  as the universe of discourse.

A Fuzzy Relation between  $F(t)$  and  $F(t-1)$  is denoted by  $R(t-1,t)$  and written as

$$F(t) = F(t-1) \circ R(t-1,t) \quad \dots (4)$$

If such a relation exist then  $F(t)$  is said to be caused by  $F(t-1)$ . The variable  $t$  denotes time. In short a fuzzy relation is expressed as  $F(t-1) \rightarrow F(t)$ . This enables FTS models to capture human like intelligence. The right hand side is the fuzzy forecast and the left hand side can involve more than one fuzzy sets. If there are  $N$  fuzzy sets in the left hand side, it is referred to as a  $N$  - order relation. High order relations were introduced by Chen. A group of fuzzy relations is a fuzzy relationship group (FRG).

**2.3 Genetic Algorithms**

Genetic algorithms (GA) [16] fall under the category of evolutionary optimization algorithms. GA is based on heuristics and pseudo random numbers are used in the calculations. A set of possible candidate solutions called the gene pool or population is randomly initiated. The candidates are coded as binary strings. The forecasting accuracy is used as the fitness of each candidate. Then two steps namely crossover and mutation are utilized to advance the population to the next generation. Crossover involves selecting two individual solutions and combining their features to create an offspring. Single site crossover technique is employed. Mutation is a random change in the candidate to create the next generation. GAs has better chance of converging to global optimum rather than getting stuck at local optima. The number of input nodes is three corresponding to the last three EMA data points. The number of neurons in the hidden layer is optimally fixed at 6. Different ANFIS models are trained for three different EMAs of periods 30, 60 and 90 days. The weights of the neurons are binarized to get the genes. Four bit genes are used for illustration while several weights are used in the training of ANFIS. The forecast values of the EMAs are used to arrive at a prediction for the next day closing price based on weighted average. The EMAs are weighted according to their time period. This makes sense due to the fact that higher period EMAs are predicted with better accuracy.

**2.4 Performance Metrics**

The performance metrics used to evaluate the proposed method against ANFIS applied on closing price are as follows with  $P_i$  and  $\hat{P}_i$  represent the actual and predicted values.

Mean Squared Error (MSE) is defined as  $mse = \frac{1}{n} \sum_{i=1}^n (P_i - \hat{P}_i)^2$

... (2) Root Mean Squared Error (RMSE) is defined as

$$rmse = \sqrt{mse}$$

Mean Absolute Deviation (MAD) is defined as

$$mad = \frac{1}{n} \sum_{i=1}^n |P_i - \hat{P}_i| \quad \dots (4)$$

It is the average of all absolute deviations of the predicted from the actual values.

Mean Absolute Prediction Error (MAPE) [] is also known as Mean Absolute Percentage Deviation (MAPD). It is defined as

$$mape = \frac{1}{n} \sum_{i=1}^n \left| \frac{P_i - \hat{P}_i}{P_i} \right| \quad \dots (5)$$

MAPE can be used in our current application since there are no zero values in the predicted variable and does not cause division by zero error. When MAPE is multiplied by 100, it is expressed as a

percentage. MSE, RMSE, MAD and MAPE must be low for a good prediction. RMSE, MAD and MAPE are expressed in the same units as the predicted variable i.e., the corresponding currency of the indices.

**2.5 Data**

Five stock indices namely S&P 500, NASDAQ, DJIA, NIFTY and BANKNIFTY are used in the experiments. Daily closing prices of the indices from 2007 to 2015 are collected. Half of the data is used the training set and the other half as the test set. Figure 4-8 show the data and its autocorrelation. The autocorrelation demonstrates the efficiency of naive forecasting i.e., using the current day's closing price as the prediction for the next day.

**3. Experimental Results and Analysis**

The proposed method is compared against the following methods.

- a) Naive forecasting which is the prediction of zero change in daily closing price levels
- b) Artificial Neural Networks with two input neurons corresponding to the previous two days' closing prices.
- c) Second order ANFIS prediction of closing price from the previous two days' closing prices.

Tables 1-5 present the results on the five indices in the dataset.

**Table 1. Performance Measures for S&P 500 index**

	Naive Forecasting	Second Order ANN	Second Order ANFIS	Proposed Method
MSE	226.96	230.65	227.63	204.83
RMSE	15.07	15.19	15.09	14.31
MAD	10.85	11.01	10.85	10.31
MAPE (%)	0.6091	0.6089	0.6089	0.5787

**Table 2. Performance Measures for NASDAQ**

	Naive Forecasting	Second Order ANN	Second Order ANFIS	Proposed Method
MSE	1605.90	1600.50	1602.50	1449.30
RMSE	40.07	40.01	40.03	38.07
MAD	28.98	28.86	28.89	27.53
MAPE (%)	0.7133	0.7109	0.7113	0.6777

**Table 3. Performance Measures DJIA index**

	Naive Forecasting	Second Order ANN	Second Order ANFIS	Proposed Method
MSE	1612.40	1611.50	1611.50	1.4552
RMSE	126.98	126.95	126.95	87.76
MAD	92.38	92.33	92.33	87.76
MAPE (%)	0.5855	0.5853	0.5853	0.5563

**Table 4. Performance Measures for NIFTY index**

	Naive Forecasting	Second Order ANN	Second Order ANFIS	Proposed Method
MSE	4570.1	4572.7	4560.8	4124.5
RMSE	67.60	67.62	67.53	64.22
MAD	50.14	50.10	50.07	47.64
MAPE (%)	0.7451	0.7445	0.7441	0.7078

**Table 5. Performance Measures for BANKNIFTY index**

	Naive Forecasting	Second Order ANN	Second Order ANFIS	Proposed Method
MSE	41267	41236	41253	37243
RMSE	203.14	203.07	203.11	192.98
MAD	148.97	148.87	148.94	141.53
MAPE (%)	1.1059	1.1051	1.1057	1.0506

Table 1-5 clearly show that the proposed method performs better than naive forecasting. It also shows the folly of using ANNs and ANFIS to predict stock or index prices. The correlation with the previous day's price is very high as indicated by the autocorrelation and partial correlation plots of S&P 500 index in figures 3 and 4. Other indices also

exhibit similar behavior. Naive forecasting however is not useful to make investment decisions other than simple buy and hold strategy.

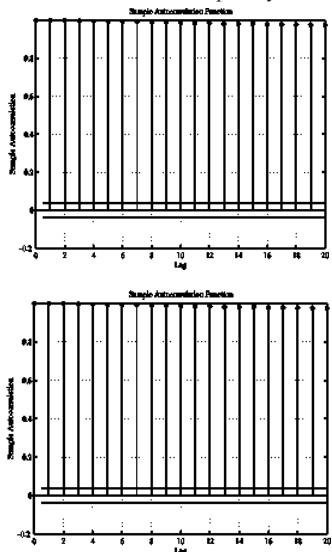


Figure 3. Autocorrelation of S&P 500 Time Series Data Figure 4. Partial Correlation of S&P 500 Time Series Data

Figures 5-9 show the prediction errors in exponential moving average EMA-30 of the five indices. Higher EMAs are smoother and easier to predict due to the slow rate of change.

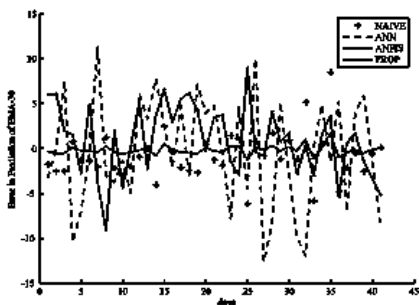


Figure 5. Prediction Error in EMA-30 for S&P 500

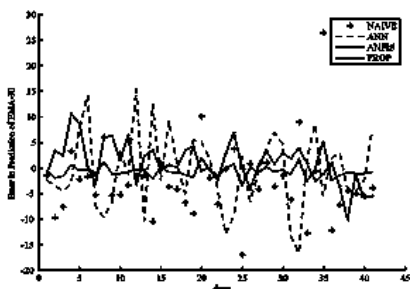


Figure 6. Prediction Error in EMA-30 for NASDAQ

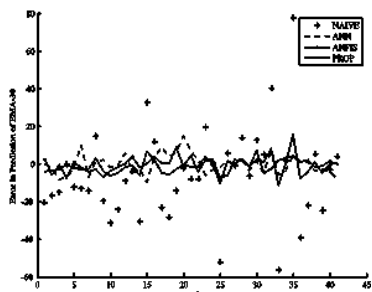


Figure 7. Prediction Error in EMA-30 for DJIA

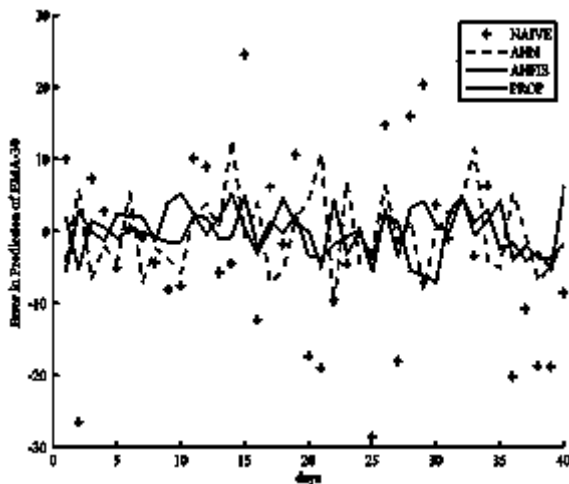


Figure 8. Prediction Error in EMA-30 for NIFTY 50

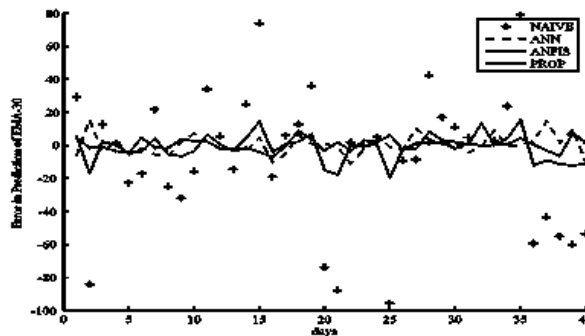


Figure 9. Prediction Error in EMA-30 for BANKNIFTY

The proposed method outperforms other methods in all the cases as evidenced in figures 5-9. The prediction of EMAs at different scales and their weighted combination has produced a more stable prediction.

**4. Conclusion**

This paper presented a novel time series forecasting method suited for financial time series. The patterns existing at multiple time scales are separately predicted and suitably combined to get a powerful predictor. The experimental results indicated the superior performance when compared against direct function approximation on the price with past prices as dependent variables. The forecasting is applied on five indices and the performance demonstrated with various measures. However successful application of any prediction to any investing strategy must involve more risk management and hedging strategies in combination with prediction. Future work may focus on the successful application of the prediction in practice and the evaluation of advanced strategy selection algorithms.

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