Original Research Paper

Volume-8 | Issue-8 | August-2018 | PRINT ISSN No 2249-555X

Science

A STUDY ON OPTIMIZATION & STRATEGY OF GAME

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ABSTRACT In this paper a discussion has been done about the various types of games under consideration and various techniques to optimize the solution of these games. We consider the games of pure and mixed Strategies Pattern. By studying the pay offs and co-operation between different player is different methods, this paper form a common procedure of techniques Comparisons for testing the effectiveness of the techniques. At the end, the advantage and demerits of Each optimization technique with respect to an algorithm point of view has been mentioned.

KEYWORDS : Gametheory, optimization,Nash equilibrium, Linear programming.

INTRODUCTION

Game theory is a formal way to analyze conflict of Interest among rational agents. The theory is its true sense deals with the ability of an entity or of individual to take a certain decision keeping in (a player) in view the effects of other entities decisions on him in a Situation of conformation. Different players maybe allowed different moves, but each players knows the moves available to the other players. Game theory concepts are applicable to any actions on processes which contains several agents. Which are inter dependent to attaining similar goals. These agents may be players, individuals, organization etc. Applying the concept of game theory a game B a formal model of an interactive situation typically Involving players. There are two main types of game; Which are static games and the Dynamic game. In the static games the players have a first Set of pure strategies and in dynamic games the strategies are infinite. Strategies of a players classified into two types; ie pure and mixed Strategies. In case of pure Strategy there a predetermined place that prescribes for a players the sequence of moves and counter moves the players would make during a complete games. A mixed Strategies B defined by a probability the distribution over the set of pure strategies. The concept of modern game theory was Introduced by John Von Neumann and Oskar Morgenstern (1944), Who describe the word 'game' for the first time By systematically, specifying the rules of the game; The Move of the theirs moves and the outcome for each players at the end of the game, of each players has chosen strategy and no players can benefit by changing his strategy unchanged, their the current set of strategy closer and the corresponding payoffs continue. Nash equilibrium in case of finite games which of this players according to mas - colell at a "Every finite games of perfect information to has a pure strategy Nash equilibrium that can be derived by backward induction.

Analyzation

2.1 Pure strategy

A pure strategy is a predetermined plan that payoff for a player the sequence of moves and counter moves be will make during complete game. An optimal solution to the game is said to be reached of neither player finds it beneficial to alter his strategy. In this case the game is said to be is a state of equilibrium.The equilibrium is called as Nash Equilibrium. The game matrix is usually expressed in terms of a payoff to a player. Thus a payoff matrix give a complete characterization of a game.

2.2.Solution Procedure of two-person zero-sum Game:

For solving two person zero-sum game we the row minimum (maximum) and column maximum (minimum) values. These values provides us with the value of the game and determine of a saddle point B in the game on not. In case the saddle point is found in a game then the game is bound to have been played by a pure strategy which would minimize the losses of each players and maximum profits correlating each other. This saddle point also represent the Nash Equilibrium that will be present in the game. The Nash Equilibrium in any game can be achieved by elimination of the unfeasible moves for each players.

Players II	Players I			
		A_1	A_2	A_3
	B_1	2,0	2,4	$4,_0$
	B_2	0,3	0,2	4,0
	B_3	0,3	3,5	6,4

Applying the elimination rows. It is clearly for player I strategy A3 is not feasible compared to A1 or A2 and so it is logical to assume that the players I would not play this strategy. This makes it safe to eliminate this strategy 'Now for player II, Strategy B1 and B3 clearly dominate Strategy B2 and hence B2 can also be eliminate.

Player II	Player I		
		A^1	A_2
	B_1	2,0	2,4
	B_3	0,3	3,5

Now considering this new matrix, for player I the optimal strategy is A2 and so column A1 can also be eliminate. Similarly row B1 can also be eliminated for players I. This shows that for player I the optimal strategy A2 while for the player II it is B3. This solution to any game is always a stable one and called as the Nash Equilibrium. Considering another3*3 payoff matrix. Now we calculate the minimum and maximum. So as to find the saddle point of the game of any.

Player y	Player x			
	5	3	1	1
	7	4	5	4
	1	2	6	1
	7	4	6	

In the above payoff matrix if player X choose his first strategy he can lose at the minimum of [5 ,3 ,1]. For player Y, playing the first strategy he can obtain a sure minimum gain of [7, 4, 5]. New if the player X plays his second strategy be can lose a maximum of [5,7,1] and a maximum of [3,4,2] If he plays his third strategy . Players Y playing his second and third strategy can obtain a minimum of [7 ,4 ,5]and [1,2 ,6]. Thus the maximum value in each column represent the maximum loss that player X will have to accommodate playing that strategy. The minimum value is each row represent the minimum gain that player Y can get playing that strategy, player X by selecting the second strategy is minimizing his loss which player Y by selecting his second strategy is maximizing his gains. In the above case the maximin (M1) = Minimax(M2).

This equality gives the saddle point of the game. The condition of optimality is reached here. Since players are not tempted to change their strategy. The saddle point also the value of the game. The value of the game satisfies the following equality.

Value of the game GV in between the max mini and mini max.
(ie) value is GV € [m1,m2]
(ie) m1 $\leq$ GV $\leq$ m2, where GV is the value of the game.

2.3. Mixed Strategies:

Mixed strategies are very useful to solve the unstable games. According to the unstable situation, there is no pure strategy for a player, will play a multiple number of strategies to obtain the optimality. This condition of optimality can be found out procedure that the random payoff is replaced by its expected value. For a game with two players X & Y, the mixed strategy for X would the vector x.

$\overline{X}$ = [x_1,x_2,…..xn]T=↑ [where xn is the probability of selecting the n^{th} strategy.]

(ie) $\sum_{i=1}^{n} x_i = 1$

Similarly few players Y the mixed strategy would be vector Y .$\overline{Y}$= $[y1,y2,\ldots yn]^T$

(ie) $\sum_{j=1}^{n} y_i = 1$

If we represent the I, j entry of the payoff ,matrix of the game as Z_{ij}, the payoff matrix can be formed as

Player X	Player Y			
	Z11	Z12	……	Zij
	Z21	Z22	……	Z2j
	…..	……….	……	…….
	Zi1	Zi2	……	zij

Now considering the minimax –criterion for a mixed strategy.

(ie)The maximum value of the minimum expected gain for player X and the minimum value of the maximum expected for players Y. For any game matrix there exit optimal strategies A and B such that E(A,B)=X=Y=GV, when GV the expected value of the game and E(A,B)= $\sum_{i=1}^{m}\sum_{i=1}^{m} g_{ij}\, a_{ij} b_{ij}$

where $\bar{x}$ and $\bar{y}$ are the corresponding vectors representing. The probability distribution of the strategies of players. According to the every pure strategy game is a special case of the mixed strategy game. Considering that almost all practical games and their application contain mixed strategies analyzing their optimal solution of games.

optimization of linear programming method:

The objective function of linear programming is either maximization of profit or minimization to the cost. In game, each player is trying to maximize his profit or the objective function of two linear problems are identical to the value of the game when both players select their optimal strategies one players highest profit is the other players highest loss Or the dual of the first players programming problems. By solving for any one of the players objective the other players objective can be cagily found. Playing As objective strategy can be represented as

Max[A= min $\{\sum_{i=1}^{m} a_{i1}\, x_{i1}, \sum_{i=1}^{m} a_{i2}\, x_{i2} \ldots \sum_{i=1}^{m} a_{ij}\, x_{ij}\}$

Subject to the constraints $x_i \geq 0$, i=1,2,,…m and $\sum_{i=1}^{m} x_{ij}$

Then for player A the problems become of maximization of Z subject to the consideration

Max Z= x1+x2+…………………….+xm

Subject to

$a_{11}x_1+a_{12}x_2+\ldots\ldots am_1x_m \geq 1$

$a_{21}x1+a_{22}x2+\ldots\ldots am_2x_m \geq 1$

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$an_1x_1+a_{n2}x_2+\ldots\ldots+a_{mn}+x_m \geq 1$

for each xi ≥ 0 i=1,2,,……m

where Z = $\frac{1}{W}$ while playersB objective strategy can be represented as

Min[B= max $\{\sum_{j=1}^{n} a_{1j}\, y_j, \sum_{j=2}^{n} a_{2j}\, y_j \ldots \sum_{j=m}^{n} a_{mj}\, y_j\}$ then for player B the problem become of

minimization of W subject to the construction,

Min W = $y_1+y_2+\ldots\ldots+y_n$

Subject to,

$a_{11}y_1+a_{12}y_2+\ldots\ldots a_{m1}y_n \leq 1$

$a_{21}y_1+a_{22}y_2+\ldots\ldots a_{m2}y_n \leq 1$

……………………………………………..

………………………………………………

$a_{n1}y_1+a_{n2}y_2+\ldots\ldots+a_{mn}y_n \leq 1$

for each $y_i \geq 0$, i=1,2,….n, where W= $\bar{v}$

Numerical Example:

Consider the following LPP.

The LPP for B is the dual of A and hence solving any one of them yields the solution of the other.

Let us take 3x3 game which can be represented by the following Lpp.

Player A	Player B		
	8	3	-1
	-2	-3	7
	-6	4	2

Max W = $y_1+y_2+y_3$

Subject to,

$8y_1+3y_2-y_3 \quad \leq 1$

$-2y_1-3y_2+7y_3 \leq 1$

$-6y_1+4y_2+2y_3 \leq 1$, where $y_1,y_2,y_3 \geq 0$

The optimal strategy for B is game by

W= $\frac{101}{180}$, y1= $\frac{13}{180}$, y2= $\frac{41}{180}$, y3= $\frac{47}{180}$

But v= $\frac{1}{W}$, y1= $\frac{y1}{W}$; Y2= $\frac{y2}{W}$; y3= $\frac{y3}{W}$

Ie v= $\frac{101}{180}$, Y1= $\frac{13}{180}$, Y2= $\frac{41}{180}$, Y3= $\frac{47}{180}$,

Here Y1+Y2+Y3=1

The optimal strategy for A is the dual of the above given by,

Z=W= $\frac{101}{180}$, x1= $\frac{49}{180}$; x2= $\frac{5}{36}$; x3= $\frac{3}{20}$

But v= $\frac{1}{Z}$; x1= $\frac{x1}{Z}$; Y2= $\frac{x2}{Z}$; x3= $\frac{x3}{N}$

Ie v= $\frac{180}{101}$; x1= $\frac{40}{101}$; x2= $\frac{25}{101}$; x3= $\frac{27}{101}$;

Conclusion:

In the paper, We study the various types of games and their methodology of optimization with numerical examples and theoretical concept.Usinglinear programming problem method to optimize two-person zero-sum games one of the conclusion that can be drawn. Every game can be represented by a LPP of maximization or minimization. The presence of the dual of a LPP facilitates the use of linear programming since using one objective function and solving for it the objective of the other player can also be found from an algorithm point of view linear programming is more feasible compared to graphical methods since it is modulur in nature and works step by step.

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