



CLASS OF RATIO AND PRODUCT- TYPE ESTIMATOR UNDER STRATIFIED RANDOM SAMPLING

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ABSTRACT In the present paper, an improved class of ratio and product- type estimators under stratified random sampling has been proposed for estimating the population mean of characteristic under study. The proposed estimator includes number of estimators that utilizes the known value of population parameters as coefficient of variation, correlation coefficient etc. of auxiliary variable. The bias and mean square error of proposed estimator have been derived up to first order approximation. The optimum mean square error of proposed estimator has also been obtained which is equal to the mean square error of classical separate linear regression that is considered to be least in literature. Numerical illustration has been carried out.

KEYWORDS : Bias, Efficiency, Separate ratio estimator, Stratified sampling.

1. Introduction

Several sampling strategies heavily depend on the use of auxiliary information. Cochran (1942) suggested method of utilizing the auxiliary information for the estimation of population mean in order to increase the precision of the estimate. Contrary to the situation of ratio estimator, the product estimator was proposed by Murthy (1964). Later on, in literature, many authors suggested modified ratio-type estimators which are generally developed either by using one or more unknown constants or by introducing a convex linear combination of sample and population means of auxiliary variable with unknown weights. In both the cases, optimum choices of unknown parameters are made by minimizing the mean square error of modified estimators so that they become superior to the conventional one. Walsh (1970), Ray et al (1979), Srivenkataramana and Tracy (1979), Vos (1980), Srivenkataramana, (1980), Singh et-al (2015) and Singh (2015) considered a ratio –type estimators with the use of weighted mean of \bar{X} and \bar{x} in place of \bar{x} in classical ratio and product estimators. Singh and Singh (2007), Singh and Agnihortie (2008) did some other remarkable works in this direction.

The works of above mentioned authors assume homogeneity of population units based on simple random sampling. If population units are heterogeneous in nature stratified random sampling scheme is available in literature. Kadilar and Cingi (2003) modified the estimator suggested by Sisodiya and Dwivedi (1981), Singh and Kakran (1993) and Upadhyaya and Singh (1999) into stratified sampling. Again Kadilar and Cingi (2005) suggested a new ratio-type estimator in stratified sampling and found it more efficient than the combined ratio estimator in all conditions.

2. Existing Estimators

2.1 Separate ratio estimator and product estimators are defined as

$$\bar{y}_{sr} = \sum_{h=1}^L w_h \frac{\bar{y}_h}{\bar{x}_h} \bar{X}_h \quad (1)$$

$$\bar{y}_{sp} = \sum_{h=1}^L w_h \frac{\bar{y}_h \bar{x}_h}{\bar{x}_h} \quad (2)$$

The MSE of \bar{y}_{sr} and \bar{y}_{sp} up to first order of approximation are given by

$$MSE(\bar{y}_{sr}) = \sum_{h=1}^L w_h^2 \bar{V}_h^2 \theta_h [C_{xh}^2 + C_{yh}^2 - \rho_h C_{xh} C_{yh}] \quad (3)$$

$$MSE(\bar{y}_{sp}) = \sum_{h=1}^L w_h^2 \bar{V}_h^2 \theta_h [C_{xh}^2 + C_{yh}^2 + \rho_h C_{xh} C_{yh}] \quad (4)$$

2.2 The combined ratio and product estimator are defined as:

$$\bar{y}_{cr} = \frac{\bar{y}_{st}}{\bar{x}_{st}} \bar{X} \quad (5)$$

$$\bar{y}_{cp} = \frac{\bar{y}_{st} \bar{x}_{st}}{\bar{X}} \quad (6)$$

The MSE of \bar{y}_{cr} and \bar{y}_{cp} up to first order of approximation are given by

$$MSE(\bar{y}_{cr}) = \sum_{h=1}^L w_h^2 \theta_h [S_{Yh}^2 + R^2 S_{Xh}^2 - R S_{XYh}] \quad (7)$$

$$MSE(\bar{y}_{cp}) = \sum_{h=1}^L w_h^2 \theta_h [S_{Yh}^2 + R^2 S_{Xh}^2 + R S_{XYh}] \quad (8)$$

Kadilar and Cingi (2003) Estimator

2.3 Cem Kadilar and Hulya Cingi (2003) suggested ratio estimator under stratified random sampling which is as follows:

Sisodia and Dwivedi (1981) proposed a ratio estimator under simple random sampling using coefficient of variation. Later on Kadilar and Cingi (2003) modified it under stratified random sampling as

$$\bar{y}_{stSD} = \bar{y}_{st} \frac{\sum_{h=1}^L W_h (\bar{X}_h + C_{Xh})}{\sum_{h=1}^L W_h \bar{X}_h} \tag{9}$$

The MSE is obtained as;

$$MSE(\bar{y}_{stSD}) = \sum_{h=1}^L W_h^2 \theta_h \left[S_{yh}^2 - 2 \left(\frac{\sum_{h=1}^L W_h \bar{Y}_h}{\sum_{h=1}^L W_h (\bar{X}_h + C_{Xh})} \right) S_{yxh} + \left(\frac{\sum_{h=1}^L W_h \bar{Y}_h}{\sum_{h=1}^L W_h (\bar{X}_h + C_{Xh})} \right)^2 S_{Xh}^2 \right] \tag{10}$$

2.3.2 Singh and Kakran (1993) developed a ratio estimator by using coefficient of kurtosis of auxiliary variable X. Later on Kadilar and Cingi (2003) modified it under stratified sampling as

$$\bar{y}_{stSK} = \bar{y}_{st} \frac{\sum_{h=1}^L W_h (\bar{X}_h + \beta_{2h}(x))}{\sum_{h=1}^L W_h (\bar{X}_h + \beta_{2h}(x))} \tag{11}$$

Now the MSE is obtained as

$$MSE(\bar{y}_{stSK}) = \sum_{h=1}^L W_h^2 \theta_h \left[S_{yh}^2 - 2 \left(\frac{\bar{Y}_{st}}{\sum_{h=1}^L W_h (\bar{X}_h + \beta_{2h}(x))} \right) S_{yxh} + \left(\frac{\bar{Y}_{st}}{\sum_{h=1}^L W_h (\bar{X}_h + \beta_{2h}(x))} \right)^2 S_{Xh}^2 \right] \tag{12}$$

2.3.3 Upadhyaya and Singh (1999) suggested two ratio estimator using coefficient of variation and Kurtosis. Kadilar and Cingi (2003) modified both the estimator under stratified random Sampling as

$$\bar{y}_{stUS1} = \bar{y}_{st} \frac{\sum_{h=1}^L W_h (\bar{X}_h \beta_{2h}(x) + C_{Xh})}{\sum_{h=1}^L W_h (\bar{X}_h \beta_{2h}(x) + C_{Xh})} \tag{13}$$

$$\bar{y}_{stUS2} = \bar{y}_{st} \frac{\sum_{h=1}^L W_h (\bar{X}_h C_{Xh} + \beta_{2h}(x))}{\sum_{h=1}^L W_h (\bar{X}_h C_{Xh} + \beta_{2h}(x))} \tag{14}$$

The MSE's are given as

$$MSE(\bar{y}_{stUS1}) = \sum_{h=1}^L W_h^2 \theta_h \left[S_{yh}^2 - 2 \left(\frac{\bar{Y}_{st}}{\sum_{h=1}^L W_h (\bar{X}_h \beta_{2h}(x) + C_{Xh})} \right) S_{yxh} + \left(\frac{\bar{Y}_{st}}{\sum_{h=1}^L W_h (\bar{X}_h \beta_{2h}(x) + C_{Xh})} \right)^2 S_{Xh}^2 \right] \tag{15}$$

$$SE(\bar{y}_{stUS2}) = \sum_{h=1}^L W_h^2 \theta_h \left[S_{yh}^2 - 2 \left(\frac{\bar{Y}_{st}}{\sum_{h=1}^L W_h (\bar{X}_h C_{Xh} + \beta_{2h}(x))} \right) S_{yxh} + \left(\frac{\bar{Y}_{st}}{\sum_{h=1}^L W_h (\bar{X}_h C_{Xh} + \beta_{2h}(x))} \right)^2 S_{Xh}^2 \right] \tag{16}$$

Kadilar and Cingi (2005) Estimator

2.4 Cem Kadilar and Hulya Cingi (2005) suggested improved combined ratio estimator which is given as

$$\bar{y}_{stP} = k^* \bar{y}_{RC} = k^* \frac{\bar{Y}_{st} \bar{X}}{\bar{X}_{st}} \tag{17}$$

where k^* is a constant

The MSE of \bar{y}_{stP} is:

$$MSE(\bar{y}_{stP}) = k^{*2} \sum_{h=1}^L W_h^2 \theta_h (S_{yh}^2 - R S_{YXh} + R^2 S_{Xh}^2) + (k^* - 1)^2 \bar{Y}^2 \tag{18}$$

2.5 H.P. Singh and N. Agnihotrie (2008) proposed ratio-product estimators under simple random sampling to estimate the population mean as:

$$T = \bar{y} \left[\alpha \frac{(a\bar{X}+b)}{(a\bar{X}+b)} + (1 - \alpha) \frac{(a\bar{X}+b)}{(a\bar{X}+b)} \right] \tag{19}$$

Where $0 < \alpha < 1$ is an unknown weight.

$a (\neq 0)$ and b are known auxiliary information.

The MSE of estimator T up to first order of approximation is given by:

$$MSE(T) = \bar{Y}^2 \theta [C_y^2 + P^2(1 - 2\alpha)^2 C_x^2 + 2P(1 - 2\alpha)\rho C_x C_y] \tag{20}$$

Where $P = \frac{a\bar{x}}{a\bar{x}+b}$ (21)

Estimator T exhibit a number of estimators as its particular cases for different value of α and P . All these estimators are useful to estimate the population mean under simple random sampling. We know that separate ratio estimator is better than the combined ratio estimator unless α is same from stratum to stratum which is not often possible. In present paper, Singh and Agnihotrie (2008) estimator has been modified for stratified sampling scheme.

3. The proposed Estimator

The proposed estimator is given by

$$t_P = \sum_{h=1}^L W_h \bar{y}_h \left[\alpha \frac{(a_h \bar{X}_h + b_h)}{(a_h \bar{X}_h + b_h)} + (1 - \alpha) \frac{(a_h \bar{X}_h + b_h)}{(a_h \bar{X}_h + b_h)} \right] \tag{22}$$

Where $0 \leq \alpha \leq 1$ is known weight within strata,

$a_h \neq 0$ and b_h are known auxiliary information within strata like kurtosis, skewness, etc.,

\bar{X}_h is Population mean of auxiliary variable of h^{th} stratum and assumed to be known,

3.1 Bias and Mean square error of proposed Estimator t_p

To obtain the bias and mean square error of the estimator t_p to first order of approximation, let us define

$$\text{Let } \bar{y}_h = \bar{Y}_h(1 + e_{0h}), \bar{x}_h = \bar{X}_h(1 + e_{1h}),$$

$$E(e_{0h}) = E(e_{1h}) = 0, E(e_{0h}^2) = \theta_h C_{y_h}^2, E(e_{1h}^2) = \theta_h C_{x_h}^2, E(e_0 e_1) = \theta_h \rho_h C_{x_h} C_{y_h} \quad \text{where } \theta_h = \left(\frac{1}{n_h} - \frac{1}{N_h}\right) \text{ and } \rho_h = \frac{S_{x_h y_h}}{S_{x_h} S_{y_h}}$$

The bias and Mean square error of proposed Estimator have been derived up to first order of approximation and is given as

$$\text{Bias}(t_p) = \sum_{h=1}^L w_h \bar{Y}_h \theta_h [\alpha P_h^2 C_{x_h}^2 + (1 - 2\alpha) P_h \rho_h C_{x_h} C_{y_h}] \tag{23}$$

$$\text{and, MSE}(t_p) = \sum_{h=1}^L w_h^2 \bar{Y}_h^2 \theta_h [C_{y_h}^2 + P_h^2 (1 - 2\alpha)^2 C_{x_h}^2 + 2 P_h (1 - 2\alpha) \rho_h C_{x_h} C_{y_h}] \tag{24}$$

Where $P_h = \frac{a_h \bar{x}_h}{a_h \bar{x}_h + b_h}$ (25)

3.2 Optimum Mean square Error of the proposed estimator t_p

The optimum MSE of the proposed estimator can be obtained by getting the first order partial derivatives of MSE (t_p) with respect to α gives,

$$\alpha = \frac{1}{2} + \frac{\rho_h C_{y_h}}{2 P_h C_{x_h}} \tag{26}$$

Thus, $\text{MSE}(t_p)_{\min} = \sum_{h=1}^L w_h^2 \bar{Y}_h^2 \theta_h (1 - \rho_h^2) C_{y_h}^2$ (26)

That is the MSE of linear regression estimator under stratified random sampling.

3.3 Bias and Mean Square Error of particular cases of t_p

Ratio Type Estimators

3.3.1 Bias and mean square error of separate ratio type estimators which are particular cases of t_p given in Table 1, are :

$$\text{Bias}(t_1) = \sum_{h=1}^L w_h \bar{Y}_h \theta_h [C_{x_h}^2 - \rho_h C_{x_h} C_{y_h}] \quad i=1 \tag{27}$$

$$\text{MSE}(t_1) = \sum_{h=1}^L w_h^2 \bar{Y}_h^2 \theta_h [C_{y_h}^2 + C_{x_h}^2 - 2 \rho_h C_{x_h} C_{y_h}] \quad i=1 \tag{28}$$

$$\text{Bias}(t_i) = \sum_{h=1}^L w_h \bar{Y}_h \theta_h \left[\frac{P_{(i-1)}^2}{2} C_{x_h}^2 - \frac{P_{(i-1)}}{2} \rho_h C_{x_h} C_{y_h} \right] \quad i=3,5,\dots,13 \tag{29}$$

$$\text{MSE}(t_i) = \sum_{h=1}^L w_h^2 \bar{Y}_h^2 \theta_h \left[C_{y_h}^2 + \frac{P_{(i-1)}^2}{2} C_{x_h}^2 - 2 \frac{P_{(i-1)}}{2} \rho_h C_{x_h} C_{y_h} \right] \quad i=3,5,\dots,13 \tag{30}$$

Product Type Estimators

3.3.2 Bias and mean square Error of separate product type estimators which are particular cases of t_p given in Table 11, are:

$$\text{Bias}(t_2) = \sum_{h=1}^L w_h \bar{Y}_h \theta_h \rho_h C_{x_h} C_{y_h} \quad j=2 \tag{31}$$

$$\text{MSE}(t_2) = \sum_{h=1}^L w_h^2 \bar{Y}_h^2 \theta_h [C_{y_h}^2 + C_{x_h}^2 + 2 \rho_h C_{x_h} C_{y_h}] \quad j=2 \tag{32}$$

$$\text{Bias}(t_j) = \sum_{h=1}^L w_h \bar{Y}_h \theta_h \left[\frac{P_{j-1}^2}{2} C_{x_h}^2 + \frac{P_{j-1}}{2} \rho_h C_{x_h} C_{y_h} \right] \quad j=4,6, \dots, 14 \tag{33}$$

$$\text{MSE}(t_j) = \sum_{h=1}^L w_h^2 \bar{Y}_h^2 \theta_h \left[C_{y_h}^2 + \frac{P_{j-1}^2}{2} C_{x_h}^2 + 2 \frac{P_{j-1}}{2} \rho_h C_{x_h} C_{y_h} \right] \quad j=4,6, \dots, 14 \tag{34}$$

Where,

$$P_{1h} = \frac{\bar{x}_h}{\bar{x}_h + C_{x_h}}; \quad P_{2h} = \frac{\bar{x}_h}{\bar{x}_h + \rho_{xyh}}; \quad P_{3h} = \frac{C_{x_h} \bar{x}_h}{C_{x_h} \bar{x}_h + \rho_{xyh}}; \quad P_{4h} = \frac{\rho_{xyh} \bar{x}_h}{\rho_{xyh} \bar{x}_h + C_{x_h}}; \quad P_{5h} = \frac{S_{x_h} \bar{x}_h}{S_{x_h} \bar{x}_h + C_{x_h}}; \quad P_{6h} = \frac{S_{x_h} \bar{x}_h}{S_{x_h} \bar{x}_h + \rho_{xyh}}$$

S/No.	ESTIMATORS	α	a_h	b_h
SEPARATE RATIO -TYPE ESTIMATORS				
1	$t_1 = \bar{y}_{sr} = \sum_{h=1}^L w_h \frac{\bar{y}_h}{\bar{x}_h} \bar{x}_h$	1	1	0
2	$t_3 = \sum_{h=1}^L w_h \bar{y}_h \frac{\bar{x}_h + C_{x_h}}{\bar{x}_h + C_{x_h}}$	1	1	C_{x_h}
3	$t_5 = \sum_{h=1}^L w_h \bar{y}_h \frac{\bar{x}_h + \rho_{xyh}}{\bar{x}_h + \rho_{xyh}}$	1	1	ρ_{xyh}
4	$t_7 = \sum_{h=1}^L w_h \bar{y}_h \frac{C_{x_h} \bar{x}_h + \rho_{xyh}}{C_{x_h} \bar{x}_h + \rho_{xyh}}$	1	C_{x_h}	ρ_{xyh}
5	$t_9 = \sum_{h=1}^L w_h \bar{y}_h \frac{\rho_{xyh} \bar{x}_h + C_{x_h}}{\rho_{xyh} \bar{x}_h + C_{x_h}}$	1	ρ_{xyh}	C_{x_h}
6	$t_{11} = \sum_{h=1}^L w_h \bar{y}_h \frac{S_{x_h} \bar{x}_h + C_{x_h}}{S_{x_h} \bar{x}_h + C_{x_h}}$	1	S_{x_h}	C_{x_h}
7	$t_{13} = \sum_{h=1}^L w_h \bar{y}_h \frac{S_{x_h} \bar{x}_h + \rho_{xyh}}{S_{x_h} \bar{x}_h + \rho_{xyh}}$	1	S_{x_h}	ρ_{xyh}

SEPARATE PRODUST-TYPE ESTIMATORS				
1	$t_2 (\bar{y}_{sp}) = \sum_{h=1}^L w_h \bar{y}_h \frac{\bar{x}_h}{\bar{X}_h}$	0	1	0
2	$t_4 = \sum_{h=1}^L w_h \bar{y}_h \frac{\bar{x}_h + C_{xh}}{\bar{X}_h + C_{xh}}$	0	1	C_{xh}
3	$t_6 = \sum_{h=1}^L w_h \bar{y}_h \frac{\bar{x}_h + \rho_{xyh}}{\bar{X}_h + \rho_{xyh}}$	0	1	ρ_{xyh}
4	$t_8 = \sum_{h=1}^L w_h \bar{y}_h \frac{C_{xh} \bar{x}_h + \rho_{xyh}}{C_{xh} \bar{X}_h + \rho_{xyh}}$	0	C_{xh}	ρ_{xyh}
5	$t_{10} = \sum_{h=1}^L w_h \bar{y}_h \frac{\rho_{xyh} \bar{x}_h + C_{xh}}{\rho_{xyh} \bar{X}_h + C_{xh}}$	0	ρ_{xyh}	C_{xh}
6	$t_{12} = \sum_{h=1}^L w_h \bar{y}_h \frac{S_{xh} \bar{x}_h + C_{xh}}{S_{xh} \bar{X}_h + C_{xh}}$	0	S_{xh}	C_{xh}
7	$t_{14} = \sum_{h=1}^L w_h \bar{y}_h \frac{S_{xh} \bar{x}_h + \rho_{xyh}}{S_{xh} \bar{X}_h + \rho_{xyh}}$	0	S_{xh}	ρ_{xyh}

4. NUMERICAL ILLUSTRATION

In this section, Theoretical findings have been supported by numerical illustration. For this purpose the data of Kadilar and Cingi (2003) have been used. Apple production amount as a variable of interest and number of apple trees as auxiliary variable in 854 villages of Turkey in 1999. Samples from each stratum have been randomly selected by using the Neyman allocation

$$n_h = n \frac{N_h S_h}{\sum_{h=1}^L N_h S_h}$$

Table 2 Data Statistics

N=854	N ₁ =106	N ₂ =106	N ₃ =94	N ₄ =171	N ₅ =204	N ₆ =173
N=140	n ₁ =9	n ₂ =17	n ₃ =38	n ₄ =67	n ₅ =7	n ₆ =2
$\bar{X}=37600$	$\bar{X}_1=24375$	$\bar{X}_2=27421$	$\bar{X}_3=72409$	$\bar{X}_4=74365$	$\bar{X}_5=26441$	$\bar{X}_6=9844$
$\bar{Y}=2930$	$\bar{Y}_1=1536$	$\bar{Y}_2=2212$	$\bar{Y}_3=9384$	$\bar{Y}_4=5588$	$\bar{Y}_5=967$	$\bar{Y}_6=404$
$S_y=144794$	$S_{y1}=49189$	$S_{y2}=57461$	$S_{y3}=160757$	$S_{y4}=285603$	$S_{y5}=45403$	$S_{y6}=18794$
$S_y=17106$	$S_{y1}=6425$	$S_{y2}=11552$	$S_{y3}=29907$	$S_{y4}=28643$	$S_{y5}=2390$	$S_{y6}=946$
$C_y=3.8509$	$C_{y1}=2.0810$	$C_{y2}=2.0955$	$C_{y3}=2.2201$	$C_{y4}=3.8405$	$C_{y5}=1.7171$	$C_{y6}=1.9091$
$C_y=5.8382$	$C_{y1}=4.1829$	$C_{y2}=5.2224$	$C_{y3}=3.1870$	$C_{y4}=5.1258$	$C_{y5}=2.4716$	$C_{y6}=2.3416$
ρ_h	$\rho_1=0.82$	$\rho_2=0.86$	$\rho_3=0.90$	$\rho_4=0.99$	$\rho_5=0.71$	$\rho_6=0.89$
θ_h	$\theta_1=0.102$	$\theta_2=0.049$	$\theta_3=0.016$	$\theta_4=0.009$	$\theta_5=0.138$	$\theta_6=0.006$
W_h^2	$W_1^2=0.015$	$W_2^2=0.015$	$W_3^2=0.012$	$W_4^2=0.04$	$W_5^2=0.057$	$W_6^2=0.041$

Table 3: MSE of particular cases of t_p and $\bar{y}_{stSD}, \bar{y}_{stSK}, \bar{y}_{stUS1}, \bar{y}_{stUS2}, \bar{y}_{stP}$

Estimators	$t_1 (\bar{y}_{st})$	t_3	t_5	t_7	t_9	t_{11}	t_{13}
MSE	164244.6	164256.8	164248.7	164246	164257.9	164244.6	164244.6
Estimators	$t_2 (\bar{y}_{sp})$	t_4	t_6	t_8	t_{10}	t_{12}	t_{14}
MSE	1860501	1860419	1860473	1860490	1860411	1860501	1860501
Estimators	\bar{y}_{cr}	\bar{y}_{stSD}	\bar{y}_{stSK}	\bar{y}_{stUS1}	\bar{y}_{stUS2}	\bar{y}_{stP}	
MSE	215710.4	215717.6	215841.6	665347.7	429527.2	210423.6	

From table 3, it is seen that all the ratio and product-type estimators that are particular cases of proposed estimator are almost equally efficient.including separate ratio and product estimator.

Again table 3 revealed that all particular cases of proposed estimator whether ratio or product-type are much more efficient than the estimators suggested by Kadilar and Cingi (2003) $\bar{y}_{stSD}, \bar{y}_{stSK}, \bar{y}_{stUS1},$ and \bar{y}_{stUS2} Kadilar and Cingi (2005) \bar{y}_{stP} including combined ratio estimator \bar{y}_{cr} . Hence, we may conclude that to get the precise estimate of population mean of variable of interest under stratified random sampling, it is advisable to use proposed ratio and product-type estimators including separate ratio and product estimators.

5 CONCLUSION

In the present study, proposed estimators performed better than the classical combined ratio estimator and the estimators proposed by Kadilar and Cingi (2003) as well as Kadilar and Cingi (2005). Also all particular cases of proposed estimator including separate ratio and product estimator are equally efficient.

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