



POTENTIAL ENERGY DENSITY USING ISOTROPIC CO-ORDINATE SYSTEM

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ABSTRACT The Potential Energy in Isotropic coordinate system has also been calculated in the case of quasi-static sphere of perfect fluid of uniform density in the present investigation. The order of potential energy in this case agrees well with Newton's theory. The time dependent solution has been studied by Narlikar et.al. [1935] by Einstein's theory. The motion in the case of Narlikar's solution is not a unique motion, like Newtonian motion with acceleration, given by Kelkar et.al. The time dependent solutions of Narlikar [1935] contains arbitrary function of time, so that the solutions can give any arbitrary motion having no correlation with Newton's theory. The Potential Energy is not negligible as compared with Newton's Theory in the isotropic Coordinate system. But the radial motion in the isotropic coordinate system has not been calculated so far. Narlikar has obtained the solution of time dependent equation in the isotropic coordinate system using Einstein's theory but has not interpreted the solution in the terms of motion of the fluid. Potential energy density of quasi-static sphere of perfect fluid of uniform density is calculated using isotropic coordinate system in the present investigation only.

KEYWORDS :

2. Introduction

The actual expression [7] (Tolman art.87.14, pp224) in conservative theorem is

$$\frac{\partial}{\partial x^i} [\bar{F}_i^{\nu} + t_i^{\nu}] = 0 \quad \text{This is not a tensor equation but a covariant expression.}$$

This has been used to prove conservation of momentum and energy. One uses the theorem

$$\int_V \text{div} V d\tau = \int_V V ds$$

to prove conservation of energy and momentum. Now the fluid is considered as an isolated material in empty space. Naturally the surface of the fluid divides the entire space into two distinct regions namely exterior and interior. In the interior region fluid exists therefore ρ_0 exists and e^{α} and e^{ν} are of the type $1+\alpha+\beta r^2$. And for exterior region ρ_0 does not exist but e^{α} and e^{ν} are of the type $\frac{a}{r} + \frac{b}{r^2}$ etc. But the expression for $T_{\alpha}^{\nu} + t_{\alpha}^{\nu}$ covers both exterior and interior regions.

Now we are interested in the Potential Energy Density (P.E.D.) of the fluid so the terms in which 'r' occurs in the denominator will not count. Also any constant added to P. E.D. does not matter because it is the potential difference which matters. However the order of the term in each Christoffel symbol in the Cartesian system is 10^{-18} , for earth. So the product of Christoffel symbols is of the order of 10^{-36} in that system, for earth. This is much less than ρ because ρ is of the order of 10^{27} for earth.

Now the expression of total energy density is covariant. Therefore for spherical coordinate system also the order of total energy density is the same as in Cartesian system. Since the field is weak the space is almost flat. Now at a point, the neighborhood can be regarded as a tangentially flat space. Hence it is possible to find out coordinates such that $g_{\alpha\nu} = \text{constant}$, i.e. $g_{\alpha\nu} \approx \pm 1$ for $u = v$ and $g_{uv} = 0$ for $u \neq v$. (In tangentially flat space Christoffel symbols vanish). Now if we calculate the total energy density in the tangentially flat space it will be $\rho\sqrt{-g}$. $\sqrt{-g}$ will be different from one because space is tangentially flat but coordinates are curvilinear and observer's coordinates are used in $\sqrt{-g}$.

3. Expression for t_{α}^{ν} and L

Tolman [7] has given (in eq.87.12, on pp.224)

$$t_{\beta}^{\alpha} = \frac{1}{16\pi} \left[-g_{\beta}^{\nu} \frac{\partial L}{\partial g_{\alpha\nu}} + g_{\beta}^{\alpha} L + 2g_{\beta}^{\alpha} \Lambda \sqrt{-g} \right]$$

$$\alpha = 4, \beta = 4 \text{ for } t_4^4$$

$$g_{\beta}^{\nu} = \frac{\partial(g^{\nu\alpha} \sqrt{-g})}{\partial \beta}$$

$$g_4^4 = \frac{\partial(g^{\nu\alpha} \sqrt{-g})}{\partial t}$$

$$\therefore g_4^4 = 0$$

Since for time independent case $\frac{\partial g^{\nu\alpha}}{\partial t} = 0$

$$\therefore t_{\beta}^{\alpha} = \frac{1}{16\pi} [g_{\beta}^{\alpha} \rho] \quad \text{for } \beta = \alpha = 4$$

as $\Lambda = 0$ for earth whose radius is much less than the solar system

$$\therefore t_4^4 = \frac{1}{16\pi} [g_4^4 \rho] = \frac{1}{16\pi} [\rho]$$

$$g_{\mu}^{\mu} = 1; \quad \text{if } \mu = 4$$

$$g_{\mu}^{\mu} = 0; \quad \text{if } \mu \neq 4$$

Tolman [7] has given (on page no.222 art.87).

$$L = \sqrt{-g} g^{\alpha\nu} [\frac{1}{2} \rho_{\alpha\beta} \rho_{\nu\alpha} - \frac{1}{2} \rho_{\alpha\beta} \rho_{\alpha\beta}]$$

$$\begin{aligned} L = \sqrt{-g} [& g^{11} [\{11,1\}\{11,1\} + \{11,2\}\{12,1\} + \{11,3\}\{13,1\} + \{11,4\}\{14,1\} \\ & + \{12,1\}\{11,2\} + \{12,2\}\{12,2\} + \{12,3\}\{13,2\} + \{12,4\}\{14,2\} \\ & + \{13,1\}\{11,3\} + \{13,2\}\{12,3\} + \{13,3\}\{13,3\} + \{13,4\}\{14,3\} \\ & + \{14,1\}\{11,4\} + \{14,2\}\{12,4\} + \{14,3\}\{13,4\} + \{14,4\}\{14,4\}] \\ & g^{11} [\{11,1\}\{11,1\} + \{11,1\}\{12,2\} + \{11,1\}\{13,3\} + \{11,1\}\{14,4\} \\ & + \{11,2\}\{21,1\} + \{11,2\}\{22,2\} + \{11,2\}\{23,3\} + \{11,2\}\{24,4\} \\ & + \{11,3\}\{31,1\} + \{11,3\}\{32,2\} + \{11,3\}\{33,3\} + \{11,3\}\{34,4\} \\ & + \{14,4\}\{41,1\} + \{11,4\}\{42,2\} + \{11,4\}\{43,3\} + \{11,4\}\{44,4\}] \\ & + g^{22} [\{21,1\}\{21,1\} + \{21,2\}\{22,1\} + \{21,3\}\{23,1\} + \{21,4\}\{24,1\} \\ & + \{22,1\}\{21,2\} + \{22,2\}\{22,2\} + \{22,3\}\{23,2\} + \{22,4\}\{24,2\} \\ & + \{23,1\}\{21,3\} + \{23,2\}\{22,3\} + \{23,3\}\{23,3\} + \{23,4\}\{24,3\} \\ & + \{24,1\}\{21,4\} + \{24,2\}\{22,4\} + \{24,3\}\{23,4\} + \{24,4\}\{24,4\}] \\ & - g^{22} [\{22,1\}\{11,1\} + \{22,1\}\{12,2\} + \{22,1\}\{13,3\} + \{22,1\}\{14,4\} \\ & + \{22,2\}\{21,1\} + \{22,2\}\{22,2\} + \{22,2\}\{23,3\} + \{22,2\}\{24,4\} \\ & + \{22,3\}\{31,1\} + \{22,3\}\{32,2\} + \{22,3\}\{33,3\} + \{22,3\}\{34,4\} \\ & + \{22,4\}\{41,1\} + \{22,4\}\{42,2\} + \{22,4\}\{43,3\} + \{22,4\}\{44,4\}] \\ & + g^{33} [\{31,1\}\{31,1\} + \{31,2\}\{32,1\} + \{31,3\}\{33,1\} + \{31,4\}\{34,1\} \\ & + \{32,1\}\{31,2\} + \{32,2\}\{32,2\} + \{32,3\}\{33,2\} + \{32,4\}\{34,2\} \\ & + \{33,1\}\{31,3\} + \{33,2\}\{32,3\} + \{33,3\}\{33,3\} + \{33,4\}\{34,3\} \\ & + \{34,1\}\{31,4\} + \{34,2\}\{32,4\} + \{34,3\}\{33,4\} + \{34,4\}\{34,4\}] \\ & - g^{33} [\{33,1\}\{11,1\} + \{33,1\}\{12,2\} + \{33,1\}\{13,3\} + \{33,1\}\{14,4\} \\ & + \{33,2\}\{21,1\} + \{33,2\}\{22,2\} + \{33,2\}\{23,3\} + \{33,2\}\{24,4\} \\ & + \{33,3\}\{31,1\} + \{33,3\}\{32,2\} + \{33,3\}\{33,3\} + \{33,3\}\{34,4\} \\ & + \{33,4\}\{41,1\} + \{33,4\}\{42,2\} + \{33,4\}\{43,3\} + \{33,4\}\{44,4\}] \\ & + g^{44} [\{41,1\}\{41,1\} + \{41,2\}\{42,1\} + \{41,3\}\{43,1\} + \{41,4\}\{44,1\} \\ & + \{42,1\}\{41,2\} + \{42,2\}\{42,2\} + \{42,3\}\{43,2\} + \{42,4\}\{44,2\} \\ & + \{43,1\}\{41,3\} + \{43,2\}\{42,3\} + \{43,3\}\{43,3\} + \{43,4\}\{44,3\} \\ & + \{44,1\}\{41,4\} + \{44,2\}\{42,4\} + \{44,3\}\{43,4\} + \{44,4\}\{44,4\}] \\ & - g^{44} [\{44,1\}\{11,1\} + \{44,1\}\{12,2\} + \{44,1\}\{13,3\} + \{44,1\}\{14,4\} \\ & + \{44,2\}\{21,1\} + \{44,2\}\{22,2\} + \{44,2\}\{23,3\} + \{44,2\}\{24,4\} \\ & + \{44,3\}\{31,1\} + \{44,3\}\{32,2\} + \{44,3\}\{33,3\} + \{44,3\}\{34,4\} \\ & + \{44,4\}\{41,1\} + \{44,4\}\{42,2\} + \{44,4\}\{43,3\} + \{44,4\}\{44,4\}] \end{aligned}$$

The isotropic line element is

$$ds^2 = -e^u (dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2) + e^v dt^2$$

$$ds^2 = g_{\alpha\nu} dx^{\alpha} dx^{\nu}$$

$$g_{11} = -e^u; \quad g_{22} = -e^u r^2$$

$$g_{33} = -e^u r^2 \sin^2 \theta; \quad g_{44} = e^v$$

$$g^{11} = -e^{-\mu}; \quad g^{22} = -\frac{e^{-\mu}}{r^2}; \quad g^{33} = -\frac{e^{-\mu}}{r^2 \sin^2 \theta}; \quad g^{44} = e^{-\nu}$$

Christoffel symbols corresponding to isotropic line element in the static cases are given by Tolman [7] (Page 242).

$$\begin{aligned} \{11, 1\} &= \frac{1}{2} \mu' = A & \{21, 2\} &= \frac{1}{r} + \frac{1}{2} \mu' = B \\ \{12, 2\} &= \frac{1}{r} + \frac{1}{2} \mu' = B & \{22, 1\} &= -\left(r + \frac{1}{2} r^2 \mu'\right) = D \\ \{13, 3\} &= \frac{1}{r} + \frac{1}{2} \mu' = B & \{23, 3\} &= \text{Cot} \theta = E \\ \{14, 4\} &= \frac{1}{2} \nu' = C & & \\ \{31, 3\} &= \frac{1}{r} + \frac{1}{2} \mu' = B & \{41, 4\} &= \frac{1}{2} \nu' = C \\ \{32, 3\} &= \text{Cot} \theta = E & \{44, 1\} &= \frac{1}{2} e^{\nu-\mu} \nu' = H \\ \{33, 1\} &= -\left(r + \frac{1}{2} r^2 \mu'\right) \sin^2 \theta = F \\ \{33, 2\} &= -\text{Sin} \theta \text{Cos} \theta = G \end{aligned}$$

Keeping those terms for which Christoffel symbols exist in the Lagrangian we get

$$\begin{aligned} \mathcal{L} = \sqrt{-g} & - (g^{11} \{12, 2\} \{12, 2\} + \{13, 3\} \{13, 3\} + \{14, 4\} \{14, 4\} - \\ & \{11, 1\} \{12, 2\} - \{11, 1\} \{13, 3\} - \{11, 1\} \{14, 4\}) \\ & + g^{22} \{21, 2\} \{22, 1\} + \{22, 1\} \{21, 2\} + \{23, 3\} \{23, 3\} - \{22, 1\} \{11, 1\} \\ & - \{22, 1\} \{12, 2\} - \{22, 1\} \{13, 3\} - \{22, 1\} \{14, 4\}) \\ & + g^{33} \{31, 3\} \{33, 1\} + \{32, 3\} \{33, 2\} + \{33, 1\} \{31, 3\} + \{33, 2\} \{32, 3\} \\ & - \{33, 1\} \{11, 1\} - \{33, 1\} \{12, 2\} - \{33, 1\} \{13, 3\} - \{33, 1\} \{14, 4\} \\ & - \{33, 2\} \{23, 3\}) \\ & + g^{44} \{41, 4\} \{44, 1\} + \{44, 1\} \{41, 4\} - \{44, 1\} \{11, 1\} - \{44, 1\} \{12, 2\} \\ & - \{44, 1\} \{13, 3\} - \{44, 1\} \{14, 4\}) \end{aligned}$$

$$\mathcal{L} = \sqrt{-g} - (g^{11} [B^2 + B^2 + C^2 - AB - AB - AC] + g^{22} [BD + BD + E^2 - AD - BD - CD] + g^{33} [BF + EG + BF + EG - AF - BF - BF - CF - EG] + g^{44} [CH + CH - AH - BH - CH])$$

$$\mathcal{L} = \sqrt{-g} (g^{11} [2B^2 + C^2 - 2AB - AC] + g^{22} [E^2 - AD - CD] + g^{33} [EG - AF - CF] + g^{44} [CH - AH - 2BH]) \quad \dots (11.1)$$

$$\begin{aligned} g^{11} [2B^2 + C^2 - 2AB - AC] &= \\ -e^{-\mu} \left[2 \left(\frac{1}{r} + \frac{1}{2} \mu' \right)^2 + \left(\frac{1}{2} \nu' \right)^2 - 2 \frac{1}{2} \mu' \left(\frac{1}{r} + \frac{1}{2} \mu' \right) - \frac{1}{2} \mu' \frac{1}{2} \nu' \right] \\ &= -e^{-\mu} \left[2 \left(\frac{1}{r^2} + \frac{\mu'^2}{4} + \frac{\mu'}{r} \right) + \frac{\nu'^2}{4} - \frac{\mu'}{r} + \frac{\mu'}{2} - \frac{\mu \nu'}{4} \right] \\ &= -e^{-\mu} \left[\frac{2}{r^2} + \frac{\mu'^2}{2} + \frac{2\mu'}{r} + \frac{\nu'^2}{4} - \frac{\mu'}{r} + \frac{\mu'}{2} - \frac{\mu \nu'}{4} \right] \\ &= -e^{-\mu} \left[\frac{2}{r^2} + \frac{\mu'}{r} + \frac{\nu'^2}{4} - \frac{\mu \nu'}{4} \right] \quad \dots (a) \end{aligned}$$

$$\begin{aligned} g^{22} [E^2 - AD - CD] &= -\frac{e^{-\mu}}{r^2} \left[\text{cot}^2 \theta + \frac{\mu'}{2} \left(r + \frac{r^2 \mu'}{2} \right) + \frac{\nu'}{2} \left(r + \frac{r^2 \mu'}{2} \right) \right] \\ &= -\frac{e^{-\mu}}{r^2} \left[\text{cot}^2 \theta + \frac{\mu' r}{2} + \frac{r^2 \mu'^2}{4} + \frac{\nu' r}{2} + \frac{r^2 \mu' \nu'}{4} \right] \quad \dots (b) \end{aligned}$$

$$\begin{aligned} g^{33} [EG - AF - CF] &= \\ g^{33} \left[-\text{cot} \theta \sin \theta \cos \theta + \frac{\mu'}{2} \left(r + \frac{1}{2} r^2 \mu' \right) \sin^2 \theta + \frac{\nu'}{2} \left(r + \frac{1}{2} r^2 \mu' \right) \sin^2 \theta \right] \\ &= -\frac{e^{-\mu}}{r^2 \sin^2 \theta} \left[-\cos^2 \theta + \frac{\mu'}{2} r \sin^2 \theta + \frac{\mu'^2}{4} r^2 \sin^2 \theta + \frac{\nu' r}{2} \sin^2 \theta + \frac{1}{4} r^2 \mu' \nu' \sin^2 \theta \right] \\ &= -\frac{e^{-\mu}}{r^2} \left[-\text{cot}^2 \theta + \frac{\mu'}{2} r + \frac{\mu'^2}{4} r^2 + \frac{\nu' r}{2} + \frac{1}{4} r^2 \mu' \nu' \right] \quad \dots (c) \end{aligned}$$

$$\begin{aligned} g^{44} [CH - AH - 2BH] &= \\ g^{44} \left[\frac{\nu'}{2} \frac{\nu'}{2} e^{-\nu-\mu} - \frac{\mu'}{2} \frac{\nu'}{2} e^{-\nu-\mu} - 2 \left(\frac{1}{r} + \frac{1}{2} \mu' \right) \frac{\nu'}{2} e^{-\nu-\mu} \right] \\ &= e^{-\nu} \left[\frac{\nu'^2}{4} e^{-\nu-\mu} - \frac{\mu' \nu'}{4} e^{-\nu-\mu} - \frac{\nu'}{r} e^{-\nu-\mu} - \frac{\mu' \nu'}{2} e^{-\nu-\mu} \right] \\ &= e^{-\nu} \left[\frac{\nu'^2}{4} - \frac{3\mu' \nu'}{4} - \frac{\nu'}{r} \right] \quad \dots (d) \end{aligned}$$

Substitute equation (a), (b), (c) and (d) in equation (11.1)

$$\begin{aligned} \mathcal{L} = \sqrt{-g} & \left(-e^{-\mu} \left[\frac{2}{r^2} + \frac{\mu'}{r} + \frac{\nu'^2}{4} - \frac{\mu \nu'}{4} \right] - \frac{e^{-\mu}}{r^2} \left[\text{Cot}^2 \theta + \frac{\mu' r}{2} + \frac{r^2 \mu'^2}{4} + \frac{\nu' r}{2} + \frac{r^2 \mu' \nu'}{4} \right] \right. \\ & \left. - \frac{e^{-\mu}}{r^2} \left[-\text{cot}^2 \theta + \frac{\mu'}{2} r + \frac{\mu'^2}{4} r^2 + \frac{\nu' r}{2} + \frac{1}{4} r^2 \mu' \nu' \right] + e^{-\mu} \left[\frac{\nu'^2}{4} - \frac{3\mu' \nu'}{4} - \frac{\nu'}{r} \right] \right) \\ \mathcal{L} = \sqrt{-g} & \left(e^{-\mu} \left[\frac{\nu'^2}{4} - \frac{3\mu' \nu'}{4} - \frac{\nu'}{r} - \frac{2}{r} - \frac{\mu'}{r} - \frac{\nu'^2}{4} + \frac{\mu \nu'}{4} \right] \right. \\ & \left. - \frac{e^{-\mu}}{r^2} \left[\text{cot}^2 \theta + \frac{\mu' r}{2} + \frac{r^2 \mu'^2}{4} + \frac{\nu' r}{2} + \frac{r^2 \mu' \nu'}{4} - \text{cot}^2 \theta + \frac{\mu' r}{2} + \frac{r^2 \mu'^2}{4} + \frac{\nu' r}{2} + \frac{r^2 \mu' \nu'}{4} \right] \right) \\ & = \sqrt{-g} \left(e^{-\mu} \left[-\frac{\mu \nu'}{2} - \frac{\nu'}{r} - \frac{2}{r} - \frac{\mu'}{r} - \frac{\nu'^2}{4} + \frac{\mu \nu'}{4} \right] \right) \\ & = \sqrt{-g} \left(e^{-\mu} \left[-\frac{\mu \nu'}{2} - \frac{\nu'}{r} - \frac{2}{r} - \frac{\mu'}{r} - \frac{\nu'^2}{4} + \frac{\mu \nu'}{4} \right] \right) \\ \mathcal{L} = e^{\left(\frac{2\mu-\nu}{2} \right)} & r^2 \sin \theta \left[-\mu' \nu' - 2 \frac{\nu'}{r} - \frac{2}{r} - 2 \frac{\mu'}{r} - \frac{\mu'^2}{2} \right] \quad \dots (11.2) \end{aligned}$$

The terms $-2 \frac{\nu'}{r}$ & $-2 \frac{\mu'}{r}$ are cancelled due to assumed values $\mu = 2\Omega$ and $\nu = 2\Omega$. Hence the order of \mathcal{L} is 10^{-36} the same as $\mu' \nu'$ and $\frac{\mu'^2}{2}$. Now $-\frac{2}{r}$ is multiplied r^2 giving -2 which is a constant and does not affect the potential difference.

As shown by Eddington¹ [8] (equation 46.2, art46, page 101), Newtonian approximation can be achieved only by the line element $ds^2 = -(1 + 2\Omega)(dx^2 + dy^2 + dz^2) + (1 - 2\Omega)dt^2$

$$\begin{aligned} \text{Hence} \\ \rho \sqrt{-g} & \cong \frac{\rho_0}{1 - \beta^2} (1 + 2\Omega) \quad \text{since } \sqrt{-g} \cong (1 + 2\Omega) \\ & \cong \rho_0 (1 + \beta^2) (1 + 2\Omega) \end{aligned}$$

$$\cong \rho_0 + \rho_0 \beta^2 + \rho_0 2\Omega \quad \text{since } 2\beta^2 \Omega \rho_0 \text{ is negligible,}$$

where,

ρ_0 - is rest mass Energy Density,

$\rho_0 \beta^2$ - is double Kinetic energy Density given by Special Relativity [or Newton if β is small].

$\rho_0 2\Omega$ - is double Potential Energy Density given by Newton. The factor 2 comes is because of fitzguard contraction as shown by Eddington.

Potential Energy Density and Kinetic Energy Density are individually double of that given by special relativity. But this does not alter the total energy because both P.E.D. and K.E.D. are approximately double as given by Newton. The P. E. is negative in gravitation and K.E. is always positive. Hence the total energy is conserved like Newtonian mechanics. We were misled by Tolman's remark [7] (page 224) that t_4^4 is density of gravitational field so we had to test the remark and found that t_4^4 has nothing to do with total energy. It however gives a very powerful mathematical method of proving conservation of energy and momentum. The great advantage of t_4^4 is that it vanishes in tangentially flat space and hence $T_4^4 \sqrt{-g}$ is the only expression of total energy density in tangentially flat space. But total energy density must be covariant since it does not depend on coordinate system. Hence is the total energy of the fluid. In the observer's coordinates $\sqrt{-g}$ is not equal to 1 in the Cartesian system. But one can find a coordinate system in which $\sqrt{-g} = 1$. This is easily achieved by transformation of coordinates such as $x' = \alpha x$ etc. so that $dx' = \alpha dx$ etc.

In the x,y,z,t system of observer $\sqrt{-g} \neq 1$ but after transformation given above $\sqrt{-g}$ can be made to be equal one in primed system.

After integration over an infinite sphere the energy momentum has been given by Tolman [7](art.91, on page 233) as covariant expression $J_p = (0,0,0,m)$.

where 'm' is the total energy of the fluid. 'm' occurs in the exterior solution as integration constant. This proves the conservation of energy and momentum.

Supposing the fluid has both kinetic energy and potential energy zero at $t=0$. Total energy will be conserved. Supposing the kinetic energy is not zero at $t=0$ then the fluid must have started contracting at some earlier time when Kinetic energy is zero and we can adjust the Potential

energy to be zero at $t=0$. If the fluid is expanding with kinetic energy at $t=0$ we can adjust the potential energy so that the sum of kinetic energy and potential energy is zero.

4. Conclusion

Total energy density, $\rho\sqrt{-g} = \rho\sqrt{-g} ? (1+2\Omega)\rho$ in Cartesian coordinates. so that in a tangentially flat space the Christoffel symbols vanish completely and total energy is given by $\rho\sqrt{-g}$. Energy of gravitation (potential) does not vanish like that of the anisotropic case and $T^i_j\sqrt{-g}$ gives the total energy of the fluid in the isotropic coordinate system. It was concluded that the potential energy in the anisotropic coordinate system is negligible as compared with Newtonian potential energy. On the other hand in the isotropic coordinate system both Potential Energy and Kinetic Energy are double the values of Newtonian Energy. But the sum of Potential Energy and Kinetic Energy is conserved and has approximately the Newtonian value just like Newton's theory. Potential energy can be adjusted by adding a suitable constant so that at some instant both potential energy and kinetic energy becomes zero. Actually the case of interest is contraction of a fluid at rest at time $t=0$.

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