



ON THE MULTIPLICATIVE TOPOLOGICAL INDICES OF CARBON NANOCONES

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ABSTRACT Let G be a simple and connected graph with the vertex set $V(G)$ and edge set $E(G)$. The nanocones can be considered as the nanoscale conical carbon-based material. In carbon nanocones $CNC_k[n]$, the parameter k defines the length of inner cycle and n defines the number of layers of the graph. In this paper the multiplicative version of $SK(G)$, $SK_1(G)$, $SK_2(G)$, Reciprocal Randic index, Reduced Reciprocal Randic index, and Root mean square index for nanocones $CNC_5[n]$ where $n=1-5, 10, 12$ and 20 are investigated.

KEYWORDS : Topological indices, multiplicative topological indices, carbon nanocones, degree, pentagonal nanocone.

1. Introduction

Let G be a molecular graph. The vertex-set and edge-set of G denoted by $V(G)$ and $E(G)$ respectively. The number of vertices of G , adjacent to a given vertex v , is the degree of this vertex and will be denoted by d_v . A topological index is a number related to a molecular graph invariant automorphism of Graph. The ability of elemental carbon to form extended two dimensional sheet structures with extremely strong bond makes it a stable material to produce isolated objects. The sheets can be resealed notionally to form a cone or horn [1]. Nanocones are discovered in 1994 [2]. Nanocones are carbon based structures formed by introducing 60° inclination defects in two-dimensional graphene sheets. The graph of this molecule consists of one pentagon surrounded by layers of hexagons [3]. The nanocone structure of $CNC_5[1]$ is shown in figure (1). The vertex-degree based multiplicative topological indices for molecular graphs are studied by [4-12]. Our notations are standard and mainly taken from books of graph theory [13-14]. The partitions of edges $(2,2)$, $(2,3)$ and $(3,3)$ for nanocones $CNC_5[n]$ where $n=1-5, 10, 12$ and 20 are taken from [4,15].

In carbon nanocones $CNC_k[n]$ the parameter k defines the length of the inner cycle and n defines the number of layers of the graph. The carbon nanocones $CNC_5[2]$, $CNC_5[4]$ graph consists of pentagon as its core surrounded by two and four layers of hexagons respectively fig (2). The definitions for degree-based topological indices are taken from [16-20]. In this paper the multiplicative version of $SK(G)$, $SK_1(G)$, $SK_2(G)$, Reciprocal Randic index, Reduced Reciprocal Randic index, and Root mean square index are defined and computed for nanocones $CNC_5[n]$ where $n=1-5, 10, 12$ and 20 .

2. Materials and Method

A molecular graph is constructed by representing each atom of a molecule by a vertex and bonds between atoms by edges. The number of vertices of G adjacent to a given vertex v , is the degree of this vertex and will be denoted by d_v . The 2-dimensional graph of $CNC_5[1]$ is shown in figure (1).

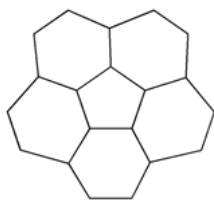


Fig.1. 2-dimensional graph of $CNC_5[1]$.

The vertices of this graph have degree either 2 or 3. The pentagon is surrounded by 5 hexagons. The molecular graph of $CNC_5[1]$ has 25 edges.

If u and v be vertices of any edge then there are 10 edges of degree $d_u = d_v = 3$, 5 edges of type $d_u = d_v = 2$, 10 edges of type $d_u = 3, d_v = 2$.

The $SK(G)$, $SK_1(G)$, $SK_2(G)$ are defined in V. Shigehalli [21], Randic index, reciprocal Randic index, reduced reciprocal Randic index are defined in [4-12, 16-20] and the root mean square index is defined in

[22]. The partitions of edges $(2,2)$, $(2,3)$ and $(3,3)$ for nanocones for $CNC_5[n]$ where $n=1-5, 10, 12$ and 20 are taken from [4,15]. The multiplicative $SK(G)$ index is computed for $CNC_k[n]$ from molecular graph of $CNC_5[1]$ figure (1). In the molecular graph of pentagonal carbon nanocones edges with endpoints having degrees 2, 2 (E_1) are 5. This may due to pentagon is a shape with five sides and five angles. The edges with endpoints 3, 3 (E_2) and 3, 2 (E_3) are counted on molecular graphs of carbon nanocones. The graphs for $CNC_5[2]$ and $CNC_5[4]$ are shown in figure (2). The number of edges with $d_u = d_v = 2$ are 5 in nanocones $CNC_5[n]$ with $n=1-5, 10, 12$ and 20 .

3. Results and discussion

In carbon nanocones $CNC_k[n]$ the parameter k defines the length of inner cycle and n defines the number of layers of the graph. The graphs for $CNC_5[2]$ and $CNC_5[4]$ are shown in figure (2).



Fig.2. Molecular graphs of $CNC_5[2]$ and $CNC_5[4]$ nanocones.

From 2-dimensional graphs of these carbon nanocones the edges of degree $(3, 3)$, $(2, 2)$ and $(3, 2)$ are counted [4, 15] and represented in table number (1).

Table 1. The number of edges ($d_u = 3, d_v = 3$ E_1), ($d_u = 2, d_v = 2$ E_2) and ($d_u = 3, d_v = 2$ E_3) for nanocones $CNC_k[n]$ where $n=1-5, 10, 12$ and 20 .

Carbon Nanocones	The number of edges of type $d_u = 3, d_v = 3$ (E_1)	The number of edges of type $d_u = 2, d_v = 2$ (E_2)	The number of edges of type $d_u = 3, d_v = 2$ (E_3)
$CNC_5[1]$	10	5	10
$CNC_5[2]$	35	5	20
$CNC_5[3]$	75	5	30
$CNC_5[4]$	130	5	40
$CNC_5[5]$	200	5	50
$CNC_5[10]$	775	5	100
$CNC_5[12]$	1110	5	120
$CNC_5[20]$	3050	5	200

The graph of pentagonal carbon nanocone (fig. 1)

$CNC_5[1]$ has 25 edges.

If u and v are vertices of any edge then, there are 10 edges of degree $d_u = d_v = 3$ (E_1),

5 edges of type $d_u = d_v = 2$ (E_2), 10 edges of type $d_u = 3, d_v = 2$ (E_3).

The SK index of a graph $G = (u, v)$ is defined as [21], $SK(G) =$

$$\sum_{u, v \in E(G)} \sum_{u, v \in E(G)} \frac{d_u + d_v}{2}$$

Where d_u and d_v are the degrees of the vertices u and v .

The multiplicative PSK (G) is defined as,

$$PSK(G) = \prod_{u,v \in E(G)} \frac{d_u + d_v}{2}$$

multiplicative SK₁ (G) index,

$$PSK_1(G) = \prod_{u,v \in E(G)} \frac{d_u \times d_v}{2}$$

multiplicative SK₂ (G) index,

$$PSK_2(G) = \prod_{u,v \in E(G)} \left(\frac{d_u + d_v}{2}\right)^2$$

multiplicative reciprocal Randic index,

$$PRRI(G) = \prod_{u,v \in E(G)} \sqrt{d_u d_v}$$

multiplicative reduced reciprocal Randic index,

$$PRRRI(G) = \prod_{u,v \in E(G)} \sqrt{(d_u - 1)(d_v - 1)}$$

multiplicative root mean square index,

$$PRMSI(G) = \prod_{u,v \in E(G)} \sqrt{(d_u^2 + d_v^2)/2}$$

Almost every topological index can be expressed in the multiplicative version. The multiplicative SK (G) for CNC₅[1] computed from figure (1) as,

$$\begin{aligned} PSK(G) &= \prod_{u,v \in E(G)} \frac{d_u + d_v}{2} \\ &= \prod_{u,v \in E_1(G)} \frac{d_u + d_v}{2} \times \prod_{u,v \in E_2(G)} \frac{d_u + d_v}{2} \times \prod_{u,v \in E_3(G)} \frac{d_u + d_v}{2} \\ &= \left(\frac{3+3}{2}\right)^{E_1} \times \left(\frac{2+2}{2}\right)^{E_2} \times \left(\frac{3+2}{2}\right)^{E_3} \\ &= \left(\frac{3+3}{2}\right)^{10} \left(\frac{2+2}{2}\right)^5 \left(\frac{3+2}{2}\right)^{10} \\ &= 7500 \end{aligned}$$

These definitions of degree-based topological indices [4-12, 16-22] are used for computing the TI's of CNC_k[n] where n=1- 5, 10, 12 and 20 and the values of TI's tabled in table 2.

The TI's follows the order PRRRI (G) < PRMSI (G) < PRRI (G) < PSK (G) < PSK₁ (G) < PSK₂ (G) for CNC₅[1].

and the order PRRRI(G) < PRMSI(G) < PRRI(G) < PSK(G) < PSK₁(G) < PSK₂(G) in CNC₅[20].

Table 2. The multiplicative topological indices of CNC_k[n] where n=1-5, 10, 12 and 20 carbon nanocones.

Topologica l indices Carbon Nanocones	PSK(G)	PSK ₁ (G)	PSK ₂ (G)	PRRI(G)	PRRRI (G)	PRMSI (G)
CNC ₅ [1]	7500	13500	112500	7348.45	1414.2	3354
CNC ₅ [2]	52500	94500	787500	51439.15	9899.4	23478
CNC ₅ [3]	168750	303750	2531250	165340.12 5	31819.5	75465
CNC ₅ [4]	390000	702000	5850000	382119.4	73538.4	174408
CNC ₅ [5]	750000	1350000	11250000	734845	141420	335400
CNC ₅ [10]	5812a5 00	1046250 0	8718750 0	5695048.7 5	109600 5	259935 0
CNC ₅ [12]	999000 0	1798200 0	1498500 00	9788135.4	188371 4.4	446752 8
CNC ₅ [20]	457500 00	8235000 0	6862500 00	44825545	862662 0	204594 00

4. Conclusion

The multiplicative version of SK(G),SK₁(G),SK₂(G), Reciprocal Randic index, Reduced Reciprocal Randic index and Root mean square index are defined and computed for molecular graphs of

CNC₅[n] where n=1-5,10,12 and 20 nanocones. The values of multiplicative version of SK(G),SK₁(G),SK₂(G), Reciprocal Randic index, Reduced Reciprocal Randic index, and Root mean square index increase with n as the layer of hexagons increase with n surrounding the pentagon core in nanocones CNC_k[n].The PSK₂(G) has highest values for CNC₅[1] and CNC₅[20] among the CNC_k[n] studied.

REFERENCES

- [1] S.Iijima,M.Yudusaka,R.Yamada et al, Nanoaggregates of single-walled graphic carbon nano-horns,Chem.Phys.Lett. 1999,309(3-4):165-170.
- [2] Krishnan,Ebbesen et al , Nature,388,(2001)241.
- [3] T.Doslic, The hyper-Wiener index of one-pentagonal carbon nanocone, Current Nanoscience,2013, 9, 557-560.
- [4] O.K.Kurucu, E.Asian, Atom bond connectivity index of Carbon nanocones and An Algorithm, Applied Mathematics and Physics, 2015, Vol.3, No.1, 6-9.
- [5] M.R.Farahani, M.K.Jamil,M.R.Rajesh Kanna,R.Pradeep Kumar, The multiplicative Zagreb Eccentricity index of Polycyclic Aromatic Hydrocarbon(PAHk),International Journal of Scientific and Engineering Research ,Vol.7,Issue 2,February 2016.
- [6] W.Gao, M.R.Farahani, M.R.R.Kanna, The multiplicative Zagreb indices of Nanostructures and Chains, Open Journal of Discrete Mathematics, 2016, 82-88.
- [7] N.K.Raut, The Zagreb group indices and polynomials, International Journal of Modern Engineering Research, Vol.6, Issue 10, October-2016, 84-87.
- [8] M.Bhanumathi, K. Easu Julia-Rani, On multiplicative Harmonic Index, Multiplicative ISI index and Multiplicative F index of TUC4C8[m, n] and TUC4[m, n] nanotubes, International Journal on Recent Trends in Life Sciences and Mathematics , Vol.4,Issue 9,Sept 2017, pp.01-08.
- [9] V.R.Kulli, A new multiplicative Arithmetic-Geometric index, International Journal of Fuzzy Mathematical Archive, Vol.12, No.2, 2017, 49-53.
- [10] V.R.Kulli, Multiplicative connectivity indices of TUC4C8[m, n] and TUC4[m, n] nanotubes, Journal of Computer and Mathematical Sciences, Vol.7(11),599-605,November-2016..
- [11] N.K.Raut, G.K.Sanap, V.P.Sangle, On the Topological indices of nanocones, Recent advances in Mathematics,National Conference,21-23 January-2016.
- [12] I.Gutman, Degree-based topological indices, Croat. Chem. Acta 86(4) (2013) 351-361.
- [13] R.Diestel, Graph theory, Electronic edition, Springer-Verlog, New York, 1997-2000.
- [14] D.B.West, Introduction to graph theory, second edition, PHI, Learning Private Ltd. New Delhi, 2009, 67-80.
- [15] <https://www.google.co.in/search?q=carbon+nanocones>.
- [16] A.Ali,A.A.Bhatti and Z.Raza,Further inequalities between vertex degree-based, topological indices, arXiv,1401-7511v1(math.CO),29 Jan 2014.
- [17] M.J.Nikmeher, N.Soleimani and M.Veylaki, Proceeding of IAM V.3, N.1, 2014, pp.89-97.
- [18] I.Gutman and J.Tosovic, Testing the quality of molecular structure descriptors, vertex-degree based topological indices J.Serb.Chem.Soc.78(6)805-810(2013).
- [19] T.Doslic, Vertex-weighted, Wiener polynomials for composite graphs, ASR Mathematica Contemporanea 1(2008)66-80.
- [20] M.R.Farahani, International Journal of Engineering and Technology Research Vol.3, No 1, Feb-2015,pp.01-06.
- [21] V.Shigehalli, R.Kanabur, Computing degree-based topological indices of Polyhex nanotubes, Journal of Mathematical Nanosciences 6(1-2)2016, 47-55.
- [22] A.Anuradha, V.Kaladevi, A.Abinayaa, International Journal in Physical and Applied Sciences, Vol.4,issue8,August-2017,pp.17-25