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Mathematics

ON NANO GB-CONTINUOUS AND SLIGHTLY NANO GB-CONTINUOUS MAPS

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ABSTRACT The aim of this paper is to establish different notions of nano gb-continuous functions using nano gb-closed sets and to derive some of their properties.

KEYWORDS : nano gb-closed set, nano gb-open set, nano gb-continuity, slightly nano gb-continuity.

Introduction

Levine [9] introduced the concept of generalized closed sets as a generalization of closed sets in topological spaces. This concept was found to be useful to develop many results in general topology. In 2004, Ekici and Caldas [7] introduced the notion of slightly γ – continuity (slightly b-continuity) which is a weaker form of b-continuity. The notion of nano topology was introduced by Lellis Thivagar [11] which was defined in terms of approximations and boundary region of a subset of an universe using an equivalence relation on it. He also established the weak forms of nano open sets namely nano α -open sets, nano semi open sets and nano pre open sets [11]. Extensive research on generalizing closedness in nano topological spaces was done in recent years by many mathematicians [5, 6, 16].

The purpose of the present paper is to introduce and investigate some of the fundamental properties of slightly nano gb-continuous functions. Also we obtain the relationship of slightly nano gb-continuity with other forms of nano continuities.

Preliminaries

Definition 2.1[15]: Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let X $\subseteq$ U

1. The lower approximation of X with respect to R is the set of all objects, which can be for certainly classified as X with respect to R and it is denoted by $L_R(X)$. That is $L_R(X) = \bigcup_{x\in U}\{R(x) : R(x) \subseteq X\}$, where R(x) denotes the equivalence class determined by x $\in$ U.

2. The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is

$$U_R(X) = \bigcup_{x\in U}\{R(x) : R(x) \cap X \neq \phi\}$$

3. The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not-X with respect to R and it is denoted by $B_R(X)$. That is $B_R(X) = U_R(X) – L_R(X)$.

Definition 2.2[11]: Let U be non-empty, finite universe of objects and R be an equivalence relation on U. Let X $\subseteq$ U. Let $\tau_R(X)$ = {U, ϕ, $L_R(X)$, $U_R(X)$, $B_R(X)$}. Then $\tau_R(X)$ is a topology on U, called as the nano topology with respect to X. Elements of the nano topology are known as the nano-open sets in U and $(U, \tau_R(X))$ is called the nano topological space. $[\tau_R(X)]^c$ is called as the dual nano topology of $\tau_R(X)$. Elements of $[\tau_R(X)]^c$ are called as nano closed sets.

Definition 2.3[12]: If $(U, \tau_R(X))$ is a nano topological space with respect to X where X $\subseteq$ U and if A $\subseteq$ U, then the nano interior of A is defined as the union of all nano-open subsets of A and it is denoted by Nint(A). That is, Nint(A) is the largest nano open subset of A. The nano closure of A is defined as the intersection of all nano closed sets containing A and is denoted by Ncl(A). That is, Ncl(A) is the smallest nano closed set containing A.

Definition 2.4: A subset A of a nano topological space $(U, \tau_R(X))$ is called nano generalized b-closed (briefly, nano gb-closed), if Nbcl(A) $\subseteq$ G whenever A $\subseteq$ G and G is nano open in U.

The complement of a nano generalized b-closed set is called nano generalized b-open (simply nano gb-open).

Definition 2.5[12]: Let $(U, \tau_R(X))$ and $(V, \tau_{R'}(Y))$ be nano topological spaces. Then a mapping $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is said to be

(i) nano continuous if $f^{-1}(B)$ is nano open in U for every nano-open set B in V.
(ii) nano α- continuous if $f^{-1}(B)$ is nano α-open in U for every nano-open set B in V.
(iii) nano semi-continuous if $f^{-1}(B)$ is nano semi-open in U for every nano-open set B in V.
(iv) nano pre-continuous if $f^{-1}(B)$ is nano pre-open in U for every nano-open set B in V.

3. Nano gb-continuous function

In this section we define nano gb-continuous function and study some of their characterizations.

Definition 3.1: Let $(U,\tau_R(X))$ and $(V,\tau_{R'}(X))$ be nano topological spaces. Then a mapping $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is said to be nano generalized b-continuous (nano gb-continuous) if the inverse image of every nano closed set in V is nano gb-closed in U.

Theorem 3.2: A function $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is nano gb-continuous if and only if $f^{-1}(G)$ is nano gb-open in U for every nano open set G in $(V,\tau_{R'}(Y))$.

Proof: Let $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ be nano gb-continuous and G be nano open set in $(V,\tau_{R'}(Y))$, then $f^{-1}(G^c)$ is nano gb-closed in $(U,\tau_R(X))$. Also $f^{-1}(G^c)=(f^{-1}(G))^c$ and so $f^{-1}(G)$ is nano gb-open in $(U,\tau_R(X))$. Conversely, let G be nano closed set in V then G^c is nano open in $(V,\tau_{R'}(Y))$. By assumption, $f^{-1}(G^c)$ is nano gb-open in $(U,\tau_R(X))$. Again $f^{-1}(G^c)=(f^{-1}(G))^c$. Thus, $f^{-1}(G)$ is nano gb-closed in $(U,\tau_R(X))$. Therefore f is nano gb-continuous.

Theorem 3.3: Let $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$. Then, every nano continuous function is nano gb-continuous.

Proof: Let G be any nano closed set in $(V,\tau_{R'}(Y))$. Then $f^{-1}(G)$ is nano closed in $(U,\tau_R(X))$. Since every nano closed set is nano gb-closed, f is nano gb-continuous.

Theorem 3.4: Let $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$. Then, every nano b-continuous, nano α-continuous, nano g-continuous, nano pre-continuous, nano semi-continuous, nano sg-continuous, nano gs-continuous, nano αg-continuous, nano gα-continuous functions are nano gb-continuous.

Proof is similar to the Theorem 3.3.

Remark 3.5: The converse of the Theorems 3.3 and 3.4 need not be true which can be seen from the following examples.

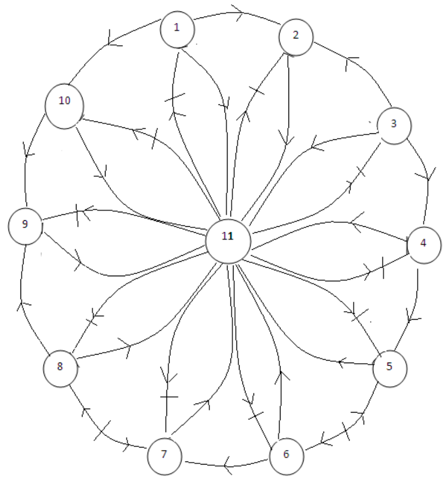
Example 3.6: Let U = {a, b, c, d} with U/R = {{a, c}, {b}, {d}} and X = {a, d}. Then the nano topology is defined as $\tau_R(X)$ = {U, ϕ, {d}, {a, c, d}, {a, c}}. Let V = {x, y, z, w} with V/R' = {{x}, {y, z}, {w}} and Y = {x, z}. Then $\tau_{R'}(Y)$ = {V, ϕ, {x}, {x, y, z}, {y, z}}. Define $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ to be f(a) = x, f(b)= y, f(c) = w, f(d) = z. Then f is nano gb-continuous but not nano continuous and nano g-continuous because $(f)^{-1}(\{x\})=\{a\}$ is not nano closed and nano g-closed in $(U,\tau_R(X))$.

Example 3.7: Let U = {a, b, c, d} with U/R = {{a}, {b, d}, {c}} and X = {b, d}. Then the nano topology is defined as $\tau_R(X)$ = {U, ϕ, {b, d}} . Let V = {x, y, z} with V/R' = {{x}, {y, z}} and Y = {x, z}. Then $\tau_{R'}(Y)$ = {V, ϕ, {x}, {y, z}}. Define $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ to be f(a) = x, f(b)= y, f(c) = y, f(d) = z. Then f is nano gb-continuous but not nano b-continuous since $(f)^{-1}(\{y,z\})=\{b,c,d\}$ is not nano b-closed in U. And f is not nano pre continuous because $(f)^{-1}(\{y,z\})=\{b,c,d\}$ is not nano pre-closed in $(U,\tau_R(X))$.

Example 3.8: Let U = {x, y, z, w} with U/R = {x}, {z}, {y, w}} and X = {x, y}. Then the nano topology is defined as $\tau_R(X)$ = {U, ϕ, {x}, {x, y, w}, {y, w}} . Let V = {a, b, c, d} with V/R' = {{a}, {b, d}, {c}} and Y = {b, d}. Then $\tau_{R'}(Y)$ = {V, ϕ, {b, d}}. Define $f:U\to V$ to be f(x) = c, f(y) = c, f(z) = b, f(w) = d. Then f is nano gb-continuous but not nano α-continuous, nano αg-continuous and gα-continuous since $(f)^{-1}(\{a,c\})=\{y\}$ is not nano α-closed nano αg-closed and nano gα-closed in U.

Example 3.9: Let U = {x, y, z, w} with U/R = {x}, {z}, {w}, {z}} and X = {y, w}. Then the nano topology is defined as $\tau_R(X)$ = {U, ϕ, { y, w}} . Let V = {a, b, c, d} with V/R' = {{a}, {b, d}, {c}} and Y = {a, b}. Then $\tau_{R'}(Y)$ = {V, ϕ, {a}, {a, b, d}, {b, d}}. Define $f:U\to V$ to be f(x) = d, f(y)= b, f(z) = a, f(w) = c. Then f is nano gb-continuous but not nano semi-continuous because $(f)^{-1}(\{a,c\})=\{z,w\}$ is not nano semi closed in U. And f is not nano sg and nano gs-continuous since $(f)^{-1}(\{b\})=\{y\}$ is not nano sg-closed and nano gs-closed in U.

1.nano continuous 2. nano pre-continuous 3. nano α-continuous 4. nano semi-continuous 5. nano b-continuous 6. nano g-continuous 7.nano g-continuous 8. nano sg-continuous 9. nano gs-continuous 10. nano g-continuous 11. nano gb-continuous



Remark 3.10: The composition of two nano gb-continuous functions need not be nano gb-continuous as seen from the following example.

Example 3.11: Let U = W = {a, b, c, d} with U/R = W/R'' ={{a}, {c}, {b, d}} and X = Z = {a, b}. Then the nano topology is defined as $\tau_R(X)$ = {U, ϕ, {a}, {a, b, d}, {b, d}} = $\tau_{R''}(Z)$. Let V = {x, y, z, w} with V/R' = {{x}, {y, w}, {z}} and Y = {y, w}. Then $\tau_{R'}(Y)$ = {V, ϕ , {y, w}}. Define f and g as f(a) = w, f(b)= z, f(c) = y, f(d) = w and g(x) = a, g(y) = a, g(z) = c, g(w) = c.. Then f and g are nano gb-continuous but $g \circ f$ is not nano gb-continuous, since for the closed set {c} in W, $(g \circ f)^{-1}(\{c\}) = \{a, b, d\}$ which is not nano gb-closed in $(U, \tau_R(X))$.

Theorem 3.12: If $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is nano gb-continuous and $g : (V, \tau_{R'}(X)) \to (W, \tau_{R''}(Z))$ is nano continuous then their composition $g \circ f : (U, \tau_R(X)) \to (W, \tau_{R''}(Z))$ is nano gb-continuous.

Proof: Let G be any nano closed set in $(W, \tau_{R''}(Z))$. Then $g^{-1}(G)$ is nano closed in $(V, \tau_{R'}(Y))$ since g is nano continuous. Since f is nano gb-continuous $(g \circ f)^{-1}(G) = f^{-1}g^{-1}(G)$ is nano gb-closed in $(U, \tau_R(X))$. Thus $g \circ f$ is nano gb-continuous.

Theorem 3.13: Let $(U, \tau_R(X))$, $(V, \tau_{R'}(Y))$ and $(W, \tau_{R''}(Z))$ be any three nano topological spaces. If $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is nano b-irresolute and $g : (V, \tau_{R'}(X)) \to (W, \tau_{R''}(Z))$ is nano b-continuous then their composition $g \circ f : (U, \tau_R(X)) \to (W, \tau_{R''}(Z))$ is nano gb-continuous.

Proof: Let G be any nano closed set in $(W, \tau_{R''}(Z))$. Since $g : (V, \tau_{R'}(X)) \to (W, \tau_{R''}(Z))$ is nano b-continuous $g^{-1}(G)$ is nano b-closed in $(V, \tau_{R'}(Y))$. Since $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is nano b-irresolute $(g \circ f)^{-1}(G) = f^{-1}g^{-1}(G)$ is nano b-closed, which is nano gb-closed in $(U, \tau_R(X))$ and so $g \circ f$ is nano gb-continuous.

4. Slightly nano gb-continuous maps

In this section the weaker form of nano gb-continuous function namely slightly nano gb-continuous function is defined. Also the relationship of slightly nano gb-continuous functions with other existing functions and its properties are discussed.

Definition 4.1: A function $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is called slightly nano gb-continuous if the inverse image of every nano clopen set in $(V, \tau_{R'}(X))$ is nano gb-open in $(U, \tau_R(X))$.

Example 4.2: Let U = V = {a, b, c, d} with U/R = {{a}, {b}, {c, d}} = V/R' and X = Y = {a, b, c}. Then $\tau_R(X) = \tau_{R'}(Y)$ = {V, ϕ , {a, b}, {c, d}}. Define $f : U \to V$ as f(a) = a, f(b)= a, f(c) = b and f(d) = b. Then the function f is slightly nano gb-continuous.

Theorem 4.3: Let $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ be a function then the following are equivalent.

(i) f is slightly nano gb-continuous.

(ii) the inverse image of every clopen set G of $(V,\tau_{R'}(X))$ is nano gb-closed in $(U,\tau_R(X))$.

(iii) the inverse image of every clopen set G of $(V,\tau_{R'}(X))$ is nano gb-clopen in $(U,\tau_R(X))$.

Proof: (i) $\Rightarrow$ (ii) Let G be nano clopen in Y. Then G^c is nano clopen in V. By (i) $f^{-1}(G^c)$ is nano gb-open in U. Since $f^{-1}(G^c)=(f^{-1}(G))^c$, $f^{-1}(G)$ is nano gb-closed.

(ii) $\Rightarrow$ (iii) By (i) and (ii) $f^{-1}(G)$ is nano gb-clopen in $(U,\tau_R(X))$.

(iii) $\Rightarrow$ (i) Let G be a nano clopen subset of $(V,\tau_{R'}(X))$. By (iii), $f^{-1}(G)$ is nano gb-clopen in $(U,\tau_R(X))$. Hence f is slightly nano gb-continuous.

Definition 4.4: A function $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is called slightly nano continuous if $f^{-1}(G)$ is nano closed in $(U,\tau_R(X))$ for every nano clopen set G in $(V,\tau_{R'}(X))$.

Theorem 4.5: Every slightly nano continuous function is slightly nano gb-continuous.

Proof: Let G be a nano clopen set in $(V,\tau_{R'}(X))$, then $f^{-1}(G)$ is nano open in $(U,\tau_R(X))$. Since every nano open set is nano gb-open, $f^{-1}(G)$ is nano gb-open. Hence f is slightly nano gb-continuous.

Remark 4.6: The converse of the above theorem need not be true which can be seen from the following example.

Example 4.7: Let U = V = {a, b, c, d} with U/R = {{a}, {b}, {c, d}} = V/R' and X = Y = {a, b, c}. Then $\tau_R(X)=\tau_{R'}(Y)$ = {V, ϕ, {a, b}, {c, d}}. Define $f:U\to V$ as f(a) = a, f(b)= a, f(c) = b and f(d) = c. Then the function f is slightly nano gb-continuous but not slightly nano continuous because $f^{-1}(\{a,b\})$ = {a, b, c} is not nano closed in $(U,\tau_R(X))$.

Theorem 4.8: Every nano gb-continuous function is slightly nano gb-continuous.

Proof: Let G be nano clopen set in $(V,\tau_{R'}(X))$. Then $f^{-1}(G)$ is nano gb-open in $(U,\tau_R(X))$. Hence f is slightly nano gb-continuous.

Remark 4.9: The converse of the above theorem need not be true which can be seen from the following example.

Example 4.10: Let U = {x, y, z, w} with U/R = {{x}, {y}, {z}, {w}} and X = {x, w} then $\tau_R(X)$ = {U, ϕ, {x, w}}. Let V = {a, b, c, d} with V/R' ={{a}, {b}, {c, d}} and Y = {a, b, c}. Then $\tau_{R'}(Y)$ = {V, ϕ, {a, b}, {c, d}}. Define as f(x) = a, f(y)= d, f(z) = d and f(w) = b. Then the function f is slightly nano gb-continuous but not nano gb-continuous because $f^{-1}(\{a,b\})$ = {x, w} is not nano gb-closed in $(U,\tau_R(X))$.

Theorem 4.11: Let $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ and $g:(V,\tau_{R'}(X))\to(W,\tau_{R''}(Y))$ be functions.

(i) If f is nano gb-irresolute and g is slightly nano gb-continuous then $g\circ f:(U,\tau_{R'}(X))\to(W,\tau_{R''}(Y))$ is slightly nano gb-continuous.

(ii) If f is nano gb-irresolute and g is nano gb-continuous then $g\circ f:(U,\tau_{R'}(X))\to(W,\tau_{R''}(Y))$ is slightly nano gb-continuous.

(iii) If f is nano gb-irresolute and g is slightly nano continuous then $g\circ f:(U,\tau_{R'}(X))\to(W,\tau_{R''}(Y))$ is slightly nano gb-continuous.

Proof: (i) Let G be nano clopen set in $(W,\tau_{R''}(Y))$. Then $g^{-1}(G)$ is nano gb-open in $(V,\tau_{R'}(X))$. Since f is nano gb-irresolute $f^{-1}(g^{-1}(G))$ is nano gb-open in $(U,\tau_R(X))$. Since $f^{-1}(g^{-1}(G))=(g\circ f)^{-1}(G)$, $g\circ f$ is slightly nano gb-continuous.

(ii) Let G be nano clopen set in $\left(W,\tau_{R''}(Y)\right)$. Then $g^{-1}(G)$ is nano gb-open in $\left(V,\tau_{R'}(X)\right)$. Since f is nano gb-irresolute $f^{-1}(g^{-1}(G))$ is nano gb-open in $\left(U,\tau_{R}(X)\right)$. Since $f^{-1}(g^{-1}(G))=(g\circ f)^{-1}(G)$, $g\circ f$ is slightly nano gb-continuous.

(iii) Let G be nano clopen set in $\left(W,\tau_{R''}(Y)\right)$. Then $g^{-1}(G)$ is nano open in $\left(V,\tau_{R'}(X)\right)$ and any nano open set is nano gb-open. Since f is nano gb-irresolute $f^{-1}(g^{-1}(G))$ is nano gb-open in $\left(U,\tau_{R}(X)\right)$. Since $f^{-1}(g^{-1}(G))=(g\circ f)^{-1}(G)$, $g\circ f$ is slightly nano gb-continuous.

Theorem 4.12: Let $f:\left(U,\tau_R(X)\right)\to\left(V,\tau_{R'}(Y)\right)$ and $g:\left(V,\tau_{R'}(X)\right)\to\left(W,\tau_{R''}(Y)\right)$ be functions. If f is surjective and strongly nano gb-open and $g\circ f:\left(U,\tau_{R'}(X)\right)\to\left(W,\tau_{R''}(Y)\right)$ is slightly nano gb-continuous then g is slightly nano gb-continuous.

Proof: Let G be nano clopen set in $\left(W,\tau_{R''}(Y)\right)$. Then $f^{-1}(g^{-1}(G))$ is nano gb-open in $\left(U,\tau_R(X)\right)$. Since f is strongly nano gb-open and surjective we have $f(f^{-1}(g^{-1}(G)))=g^{-1}(G)$ is nano gb-open in $\left(V,\tau_{R'}(X)\right)$. Hence g is slightly nano gb-continuous.

Theorem 4.14: Let $f:\left(U,\tau_R(X)\right)\to\left(V,\tau_{R'}(Y)\right)$ and $g:\left(V,\tau_{R'}(X)\right)\to\left(W,\tau_{R''}(Y)\right)$ be functions. If f is surjective and strongly nano gb-open and and nano gb-irresolute, then $g\circ f:\left(U,\tau_{R'}(X)\right)\to\left(W,\tau_{R''}(Y)\right)$ is slightly nano gb-continuous if and only if g is slightly nano gb-continuous.

Proof: Let G be nano clopen set in $\left(W,\tau_{R''}(Y)\right)$. Then $f^{-1}(g^{-1}(G))$ is nano gb-open in $\left(U,\tau_R(X)\right)$. Since f is strongly nano gb-open and surjective we have $f(f^{-1}(g^{-1}(G)))=g^{-1}(G)$ is nano gb-open in $\left(V,\tau_{R'}(X)\right)$. Hence g is slightly nano gb-continuous. Conversely, let g be slightly nano gb-continuous and G be a nano clopen set in $\left(W,\tau_{R''}(Y)\right)$ then $g^{-1}(G)$ is nano gb-open in $\left(V,\tau_{R'}(X)\right)$. Since f is nano gb-irresolute, $f^{-1}(g^{-1}(G))$ is nano gb-open in $\left(U,\tau_R(X)\right)$. Hence $g\circ f$ is nano gb-continuous.

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