



A STUDY ON ENERGY OF HELM, CLOSED HELM, FLOWER AND BISTAR GRAPH USING MATLAB PROGRAM

R. Arundhadhi

Assistant Professor, Dept of Mathematics, D.G.Vaisnav College, Chennai, Tamil Nadu, India.

B. Megala*

Research Scholar, Dept of Mathematics, D.G.Vaisnav College, Chennai, Tamil Nadu, India. *Corresponding Author

ABSTRACT

The energy $E(G)$, of a simple graph G is the sum of the absolute values of the eigenvalues of its adjacency matrix $A(G)$. In this paper we have discussed the energy of Helm, Closed helm, Flower and Bistar graph for any values of n using MATLAB program.

KEYWORDS : Energy of graph, Helm graph, Flower graph, Bistar graph.

1.INTRODUCTION

Here all graphs under consideration are simple, finite and undirected. For notations and terminology see [2].

The energy of a graph was defined by Gutman [3] in 1978 in his paper "The energy of a graph". Balakrishnan [1] derived an upper bound for the energy of k -regular graph as $E(G) \leq k + k(n-1)(n-k)$ and this bound is sharp. He also shown that for any positive integer $n \geq 3$, there exist two equienergetic graphs of order $4n$ that are not cospectral. Kinkar Ch.Das et al [5] obtained some new upper bounds for energy of G in terms of number of vertices, number of edges, clique number, minimum degree and the first Zagreb index. A.Yu et al [7] presented two new upper bounds for energy of G in terms of number of vertices, edges, the degrees and 2-degrees of vertices. They also characterized the graphs for which the bounds are sharper. G.B.Sophia Shalini et al [6] done a detailed study on the bounds of energy of simple, connected and directed graphs of order less than 10. In this paper, some new families of hyper energetic graphs were identified and the relation between the energy of wheel and fan graph were also obtained. They observed that star graph possess the least energy among all the families of graphs under consideration. They also found the energy of certain families of graph with use of MATLAB program. Ivan Gutman [4], defined the laplacian energy of the graph as $LE(G) = \sum_{i=1}^n |\mu - \frac{2m}{n}|$ and there is a analogy between the properties of $E(G)$ and $LE(G)$.

The energy $E(G)$ of a graph G is the sum of absolute values of the eigenvalues of its adjacency matrix $A(G)$. That is,

$$E(G) = \sum_{i=1}^n |\lambda_i|$$

Where $\lambda_1, \lambda_2, \dots, \lambda_n$ are the eigenvalues for the adjacency matrix of the graph.

The Helm graph H_n is the graph obtained from an n -wheel graph by adjoining a pendant edge at each vertex of the cycle. The closed helm CH_n is the graph obtained from helm H_n by joining each pendant vertex to form a cycle.

The flower graph Fl_n is the graph obtained from a helm H_n by joining each pendant vertex to the apex of the helm.

Bistar graph $B_{m,n}$ is the graph obtained from K_2 by joining m pendant edges to one end and n pendant edges to the other end of K_2 .

2.Energy of Helm graph H_n

The energy of helm graph H_n for any values of n have been discussed in this section

2.1 MATLAB program to generate the energy of general Helm graph H_n

```
% 'A' is the adjacency matrix of a graph
% 'K' is the eigenvalue of the matrix
% 'E' is the energy of the graph
m= input (' Enter the number of vertices:');
A= Zeros(m);
for i=1:n-1
```

```
A(i,i+1)=1;
A(i+1,i)=1;
end
for i=1:n
A(i,i+n)=1;
A(i+n,i)=1;
A(i,m)=1;
A(m,i)=1;
end
A(1,n)=1;
A(n,1)=1;
A
K=eig(A);
E=sum(abs(k))
```

The energy of helm graph H_5 is obtained in the following example using above MATLAB program.

Example

```
% 'A' is the adjacency matrix of a graph
% 'K' is the eigenvalue of the matrix
% 'E' is the energy of the graph
m= input (' Enter the number of vertices:');
Enter the number of vertices:
11
A= Zeros(11);
for i=1:5-1
A(i,i+1)=1;
A(i+1,i)=1;
end
for i=1:5
A(i,i+5)=1;
A(i+5,i)=1;
A(i,11)=1;
A(11,i)=1;
end
A(1,5)=1;
A(5,1)=1;
A
A= 0 1 0 0 1 1 0 0 0 0 1
    1 0 1 0 0 0 1 0 0 0 1
    0 1 0 1 0 0 0 1 0 0 1
    0 0 1 0 1 0 0 0 1 0 1
    1 0 0 1 0 0 0 0 0 1 1
    1 0 0 0 0 0 0 0 0 0 0
    0 1 0 0 0 0 0 0 0 0 0
    0 0 1 0 0 0 0 0 0 0 0
    0 0 0 1 0 0 0 0 0 0 0
    0 0 0 0 1 0 0 0 0 0 0
    1 1 1 1 1 0 0 0 0 0 0
K=eig(A);
E=sum(abs(K));
E=14.6232
```

3. ENERGY OF CLOSED HELM GRAPH CH_n

This section deals with the energy of closed helm graph CH_n for any values of n using MATLAB program.

3.1 MATLAB program to generate the energy of general closed helm graph CH_n

```
% 'A' is the adjacency matrix of the graph
% 'K' is the eigenvalue of the matrix
% 'E' is the energy of the graph
m=input('Enter the number of vertices:');
A=zeros(m);
for i=1:n-1
A(i,i+1)=1;
A(i+1,i)=1;
end
for i=1:n
A(i,i+n)=1;
A(i+n,i)=1;
A(i,m)=1;
A(m,i)=1;
end
for i=n+1:2n-1
A(i,i+1)=1;
A(i+1,i)=1;
end
A(1,n)=1;
A(n,1)=1;
A(n+1,2n)=1;
A(2n,n+1)=1;
A
K=eig(A);
E=sum(abs(K))
```

The following example gives the energy of Closed Helm graph CH_4 using the above MATLAB program.

Example

```
% 'A' is the adjacency matrix of the graph
% 'K' is the eigenvalue of the matrix
% 'E' is the energy of the graph
m=input('Enter the number of vertices:');
Enter the number of vertices:
9
A=zeros(9);
for i=1:4-1
A(i,i+1)=1;
A(i+1,i)=1;
end
for i=1:4
A(i,i+4)=1;
A(i+4,i)=1;
A(i,9)=1;
A(9,i)=1;
end
for i=4+1:(2*4)-1
A(i,i+1)=1;
A(i+1,i)=1;
end
A(1,4)=1;
A(4,1)=1;
A(4+1,2*4)=1;
A(2*4,4+1)=1;
A
A= 0 1 0 1 1 0 0 0 1
    1 0 1 0 0 1 0 0 1
    0 1 0 1 0 0 1 0 1
    1 0 1 0 0 0 0 1 1
    1 0 0 0 0 1 0 1 0
    0 1 0 0 1 0 1 0 0
    0 0 1 0 0 1 0 1 0
    0 0 0 1 1 0 1 0 0
    1 1 1 1 0 0 0 0 0
K=eig(A);
E=sum(abs(K));
E=17.5454
```

4. ENERGY OF FLOWER GRAPH Fl_n

The energy of flower graph Fl_n for any values of n are discussed.

4.1 MATLAB program to generate the energy of general flower graph Fl_n

```
% 'A' is the adjacency matrix of a graph
% 'K' is the eigenvalue of the matrix
```

```
% 'E' is the energy of the graph
m=input('Enter the number of vertices:');
A=zeros(m);
for i=1:n-1
A(i,i+1)=1;
A(i+1,i)=1;
end
for i=1:n
A(i,i+n)=1;
A(i+n,i)=1;
A(i,m)=1;
A(m,i)=1;
end
A(1,n)=1;
A(n,1)=1;
A
K=eig(A);
E=sum(abs(k))
```

In the following example, we have obtained the energy of flower graph Fl_3 , by using the above MATLAB program.

Example

```
% 'A' is the adjacency matrix of a graph
% 'K' is the eigenvalue of the matrix
% 'E' is the energy of the graph
m=input('Enter the number of vertices:');
Enter the number of vertices:
7
A=Zeros(7);
for i=1:3-1
A(i,i+1)=1;
A(i+1,i)=1;
end
for i=1:3
A(i,i+3)=1;
A(i+3,i)=1;
A(i,7)=1;
A(7,i)=1;
A(i+3,7)=1;
A(7,i+3)=1;
end
A(1,3)=1;
A(3,1)=1;
A
A= 0 1 1 0 0 1
    1 0 1 0 1 0
    1 1 0 0 0 1
    1 0 0 0 0 1
    0 1 0 0 0 1
    0 0 1 0 0 1
    1 1 1 1 1 0
K=eig(A);
E=sum(abs(K));
E=10.1290
```

5. Energy of Bistar graph $B_{m,n}$

The energy of bistar graph $B_{m,n}$ for any values of n have been obtained in this section.

5.1 MATLAB program to generate the energy of general bistar graph $B_{m,n}$

```
% 'A' is the adjacency matrix of the graph
% 'K' is the eigenvalue of the matrix
% 'E' is the energy of the graph
P=input('Enter the number of vertices:');
A=zeros(p);
for i=1:m
A(1,i+1)=1;
A(i+1,1)=1;
end
for i=1:n
A(p,i+(m+1))=1;
A(i+(m+1),p)=1;
end
A(1,p)=1;
```

```
A(p,1)=1;
A
K=eig(A);
E=sum(abs(K))
```

By using the above MATLAB program the energy of bistar graph $B_{3,4}$ is obtained in the following example.

Example

```
% 'A' is the adjacency matrix of the graph
% 'K' is the eigenvalue of the matrix
% 'E' is the energy of the graph
P=input('Enter the number of vertices:');
Enter the number of vertices:
9
A=zeros(9);
for i=1:3
A(1,i+1)=1;
A(i+1,1)=1;
end
for i=1:4
A(9,i+(3+1))=1;
A(i+(3+1),9)=1;
end
A(1,9)=1;
A(9,1)=1;
A
A=0 1 1 1 0 0 0 0 1
    1 0 0 0 0 0 0 0 0
    1 0 0 0 0 0 0 0 0
    1 0 0 0 0 0 0 0 0
    0 0 0 0 0 0 0 0 1
    0 0 0 0 0 0 0 0 1
    0 0 0 0 0 0 0 0 1
    0 0 0 0 0 0 0 0 1
    1 0 0 0 1 1 1 1 0
K=eig(A);
E=sum(abs(K))
E=7.7274
```

CONCLUSION

In this paper, the following results are obtained by using MATLAB program.

1. Energy of Helm graph H_n
2. Energy of Closed helm graph CH_n
3. Energy of Flower graph Fl_n
4. Energy of Bistar graph $B_{m,n}$

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