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Statistics

CLASSIFICATION PRESERVATION OF EFFICIENT TOTAL MANUFACTURING SECTORS OF INDIAN STATES AND UNION TERRITORIES – A DEA STUDY BASED ON FREE DISPOSABLE HULL

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ABSTRACT For efficiency measurement one needs a frontier and a distance function. Frontier production function is envelopment surface of a production possibility set. Very widely used production possibility sets in Data Envelopment Analysis (DEA) are convex and non-convex, that are supported by the postulates of inclusion, free disposability and minimum extrapolation. Distance function helps to project inefficient production plan onto the envelopment surface, there by provides an efficiency score, efficient peer(s), and bench marks for inefficient Decision Making Units (DMUs). DMUs are primarily categorized into two subsets constituting efficient and inefficient production plans. This study aimed at identifying efficient total manufacturing sectors from 34 total manufacturing sectors of Indian States and Union Territories, and examine the stability of them to remain efficient under proportional input expansion and output contraction. The production possibility set whose envelopment surface is choosen as DEA frontier is Free Disposable Hull (FDH).

KEYWORDS : Data envelopment Analysis, Free Disposable Hull, Classification Stability.

1. INTRODUCTION:

Statistical fittings of familiar production functions such as the Cobb-Douglas,(1928) Constant Elasticity of Substitution (1961) and others often resulted in estimates falling out of their theoretically specified bounds. Empirical economic researchers, in their investigations believed that these erroneous messages were due to specifications error, which prompted them to seek alternative production function specifications. The emergence of wrong signs, wrong magnitudes of parametric estimates, was due mainly to the fact that, efficiency differences among production units' performance were grossly ignored.

With an exception of Transcendental Logarithmic Production Function (1971) may widely used and popular production functions had the ability to handle single output and many inputs. Difficulty arose in the case of multiple outputs production and multiple inputs application.

Piecewise linear frontier production functions and their dual cost functions came into practice since Farrell's (1957) seminal contribution that distinguished production units based on their efficiency differences. The piecewise linear production frontiers actively pursued are perceived as inner approximations of smooth production frontiers. Since derivatives do not exist for those production frontiers at join points, certain measurements such as returns to scale (in quantum), elasticity of substitution, input and output elasticity can neither be specified in closed form expressions nor their numerical values exist.

Farrell's approach was popularized by Sheppard (1970) who provided theoretical foundation. Charnes, Cooper and Rhodes (CCR, 1978) and Banker, Charnes and Cooper (BCC, 1984) provided empirical methodology to evaluate efficiency scores and suggested bench marks. CCR (1978) approach, provides efficiency scores, in which returns to scale differences are marked. In Economic Theory, optimal returns to scale are constant returns to scale. If a production unit is governed by increasing returns to scale, it is perceived that returns to scale advantages are not fully exploited, and expansion in size is suggested. Decreasing returns to scale suggest that the production unit suffers from scale disadvantage, and contraction in size is appropriate. BCC (1984) approach disentangled scale effects from efficiency scores of CCR and multiplicatively decomposed CCR efficiency into pure and scale efficiency. The efficiency scores of CCR and BCC are radial efficiency scores evaluated by radial contraction of inputs or radial expansion of outputs. These measures do not seek change in input mix or output mix.

Russell measures popularized by Fare et.al (1985) are non-radial measures. Under input orientation these seek input specific reduction and under output orientation they seek output specific expansion. This non-radial orientation has the ability to seek input specific contraction and output specific expansion simultaneously. In each of these approaches a certain mean is minimized.

Chambers et.al (1996) introduced a class of distance functions, which they called as the directional distance functions. It is a wide class that includes radial distance functions as special case. The efficiency scores derived out of directional distance functions are sensitive to the direction choosen to project inefficient production plan onto the envelopment frontier. While the radial measures are relevant to ex post production, the non-radial measures such as Russell and Directional are relevant to ex ante production.

Data Envelopment Analysis (DEA) studies predominantly were based on convex production possibility sets, (CCR, 1978, BCC, 1984; Fare et.al, 1985; Chambers et.al 1996) Deprins et.al (1984) introduced a non-convex production possibility set called Free Disposable Hull (FDH) which is based on the postulates of inclusion, free disposability and minimum extrapolation. For jth Decision Making Unit (DMUj), if the production plan is represented by (x_j, y_j) the collection of all input and output vector pairs that are dominated by (x_j, y_j) defines an orthant. The union of the orthants determined by the observed production plans defines a Free Disposable Hull (FDH). Linear programming problems formulated on convex production possibility sets can be extended to FDH by constraining the intensity parameters to be bivalent. Tulkens (1993) introduced an enumeration method to evaluate FDH based efficiency scores, which demonstrarates that there is no need to solve zero-one integer linear programming problems, to evaluate radial input/output technical efficiency scores.

The inefficient production units change their classification from an inefficient state to efficient state, by contracting inputs under input orientation or by expanding outputs under output orientation. They also experience change in classification by simultaneous contraction of inputs and expansion of outputs. An efficient DMU preserves its efficient status if its inputs are sufficiently expanded under input orientation or output are sufficiently contracted under output orientation so as to trancend certain input/output threshold. It also loses its efficient status if inputs are expanded and outputs are contracted simultaneously, sufficiently to exceed certain input and output thresholds.

To examine the stability of an efficient DMU to preserve its classification, Super Efficiency of the efficient DMU is helpful.

Anderson and Peterson (1993) in the frame work of CCR proposed input super efficiency model. Super efficiency is calculated only for efficient production units. The production plan of efficient test DMU is deleted, so that the envelopment frontier experiences modification. The deleted production plan falls outside the modified production possibility set. The modified frontier can be reached either by input expansion or output contraction or by both. If the frontier is reached by means of input expansion, input super efficiency can be obtained and such a score never be smaller than unity. On the other hand, if the frontier is reached by means of output contraction, output super efficiency can be deduced, which never exceeds unity. CCR input super efficiency problem is always feasible if input and output values are all positive. Classification stability of efficient DMUs depend upon the super efficiencies (Seiford, Zhu, 1998a, 1998b).

Super efficiency problems can be extended to BCC frame work, but unlike in CCR super efficiency problems infeasibility is a likely hindrance. Seiford and Zhu (1998) had shown that if input super efficiency problem is infeasible, then output super efficiency problem is feasible for the test DMU. Further, if output super efficiency problem is infeasible, then the input super efficiency problem is feasible.

Cooper et.al (2001) proposed optimization problems to find radius of efficiency stability for inefficient decision making units to remain inefficient and for efficient decision making units to remain efficient. These problems were originally postulated by Charnes, Haag, Jaska and Semple (1992) to study efficiency classification preservation. Seiford and Zhu (1999) generalized Charnes et.al (1992) problem, to study the ability of efficient decision making units to remain efficient, by seeking expansion of some of inputs and contraction of some outputs, but in the CCR frame work.

Zhu (2001) further generalized the problem confining to input specific expansion of some inputs and output specific expansion of some outputs, but in BCC frame work. The sum of the expansion and contraction parameters was minimized. While this problem was formulated it was assumed that the efficient rival DMUs maintained their status quo. Seiford and Zhu (1999) examined the efficiency stability of efficient decision making units by means of proportional input and / or output data perturbations. The efficient target DMU_{jo} expands inputs proportionally at the rate , simultaneously experiences outputs contraction at the rate δ^{-1} while its efficient rival DMUs contract inputs at the rate and expand outputs at the rate . Such perturbations affect subsets of inputs and outputs.

2. EFFICIENCY CLASSIFICATION STABILITY–PROPORTIONAL VARIATION OF INPUTS AND OUTPUTS:

Seiford and Zhu (1998b) formulated the following model in which directional inputs and outputs are proportionally perturbed, consequently the following problem is postulated:

$$\theta^* = Min\,\theta$$

such that

$$\sum_{\substack{j \neq j_0 \\ j=1}}^{n} \lambda_j \frac{x_{ij}}{\theta} \leq x_{ij_0}\theta,\ i \in I$$

$$\sum_{\substack{j \neq j_0 \\ j=1}}^{n} \lambda_j x_{ij} \leq x_{ij_0},\ i \in I^C \qquad \ldots\ldots (2.1)$$

$$\sum_{\substack{j \neq j_0 \\ j=1}}^{n} \lambda_j \theta y_{rj} \geq \frac{y_{rj_0}}{\theta},\ r \in O$$

$$\sum_{\substack{j \neq j_0 \\ j=1}}^{n} \lambda_j y_{rj} \geq y_{rj_0},\ r \in O^C$$

$$\sum_{\substack{j \neq j_0 \\ j=1}}^{n} \lambda_j = 1, \lambda_j \geq 0,\ j \in N,\ j \neq j_0$$

where $I \cup I^C = M$

$$O \cup O^C = S$$

DMU_{jo} remains to be efficient for simultaneous data variation for all such that,

$$1 \leq \theta \leq \theta^*$$

If all inputs and outputs are discretionary, and if the production possibility set is Free Disposable Hull (FDH), (2.1) can be modified and presented as follows:

$$Min\,\theta$$

such that

$$\sum_{\substack{j \neq j_0 \\ j=1}}^{n} \lambda_j \left(\frac{x_{ij}}{\theta}\right) \leq x_{ij_0}\theta,\ i \in M \qquad \ldots\ldots (2.2)$$

$$\sum_{\substack{j \neq j_0 \\ j=1}}^{n} \lambda_j \theta y_{rj} \geq \frac{y_{rj_0}}{\theta},\ r \in S$$

$$\sum_{\substack{j \neq j_0 \\ j=1}}^{n} \lambda_j = 1$$

$$\lambda_j \in \{0,1\}$$

$$j \in N,\ j \neq j_0$$

(2.2) may alternatively be expressed as,

$$Min\,\theta$$

such that

$$\sum_{j_p \in D_0}^{n} \lambda_{j_p} \left(\frac{x_{ij}}{\theta}\right) \leq \theta x_{ij_0},\ i \in M$$

$$\sum_{j_p \in D_0}^{n} \lambda_{j_p} \theta y_{rj_p} \geq \frac{y_{rj_0}}{\theta},\ r \in S \quad \ldots\ldots (2.3)$$

$$\sum_{j_p \in D_0}^{n} \lambda_{j_p} = 1$$

$$\lambda_{j_p} \in \{0,1\}$$

where $j_0 \in D_0, j_p \in D_0 \Rightarrow DMU_{j_p}$ is extremely efficient.

For some $j_p \in D_0$, $\lambda_{j_p} = 1$, then we have

$$x_{ij_p} \leq \theta^2 x_{ij_0},\ i \in M$$

$$\theta^2 \geq \frac{x_{ij_p}}{x_{ij_0}},\ i \in M \qquad \ldots\ldots (2.4)$$

$$y_{rj_p} \geq \frac{y_{rj_0}}{\theta^2}$$

$$\theta^2 \geq \frac{y_{rj_0}}{y_{rj_p}},\ r \in S \qquad \ldots\ldots (2.5)$$

Combining (2.4) and (2.5),

$$\theta^2 \geq \underset{i,r}{Max}\left\{\frac{x_{ij_p}}{x_{ij_0}}, \frac{y_{rj_0}}{y_{rj_p}}\right\}$$

$$\theta = \sqrt{\underset{i,r}{Max}\left\{\frac{x_{ij_p}}{x_{ij_0}}, \frac{y_{rj_0}}{y_{rj_p}}\right\}}$$

$$Min\,\theta = \underset{j_p \in D_0}{Min}\sqrt{\underset{i,r}{Max}\left\{\frac{x_{ij_p}}{x_{ij_0}}, \frac{y_{rj_0}}{y_{rj_p}}\right\}}$$

$$\delta_{FDH}^{j_o} = \sqrt{\underset{j_p \in D_0}{Min}\,\underset{i,r}{Max}\left\{\frac{x_{ij_p}}{x_{ij_0}}, \frac{y_{rj_0}}{y_{rj_p}}\right\}} \quad \ldots\ldots (2.6)$$

$$\delta_{FDH}^{j_o} > 1$$

For all θ such that $1 \leq \theta \leq \sqrt{\underset{j_p \in D_0}{Min}\,\underset{i,r}{Max}\left\{\frac{x_{ij_p}}{x_{ij_0}}, \frac{y_{rj_0}}{y_{rj_p}}\right\}}$ DMU_{j_0} preserves its efficiency classification.

3. THE STUDY:

This study is aimed at examining classification stability of efficient total manufacturing sectors of 28 Indian states and Six union territories. The data are secondary collected from Annual Survey of Industries (ASI) for the year 2015, published by the Govt. of India. Two DEA inputs and one output are involved in the computations.

DEA inputs:
(I) Fixed Capital (x_1),
(ii)Total Persons Engaged in Production (x_2)

DEA output: Net Value Added

2. PROPORTIONAL INPUT AND OUTPUT PERTURBATIONS - EFFICIENCY CLASSIFICATION STABILITY:

S.No	Total Manufacturing Sectors	(proportional input and output perturbations)	
1	Maharashtra	1.1394	0.8777
2	Gujarat	1.1350	0.8811
3	Tamil Nadu	1.2343	0.8102
4	Karnataka	1.0808	0.9252
5	Haryana	1.0826	0.9237
6	Uttar Pradesh	-	--
7	Uttarakhand	1.2542	0.7973
8	Rajasthan	--	--
9	Himachal Pradesh	1.2458	0.8027
10	Telangana	---	--
11	Andhra Pradesh	---	--
12	Madhya Pradesh	---	--
13	Jharkhand	1.0807	0.9253
14	Punjab	1.1139	0.8977
15	West Bengal	--	--
16	Chattisgarh	1.0790	0.9268
17	Odisha	--	--
18	Goa	1.3684	0.7308
19	Dadra & Nagar Haveli	1.2491	0.8006
20	Kerala	--	--
21	Assam	--	--
22	Delhi	1.1149	0.8969
23	Bihar	--	--
24	Daman & Diu	1.0493	0.9530
25	Jammu & Kashmir	1.0826	0.9237
26	Sikkim	2.1806	0.4586
27	Puducherry	--	--
28	Chandigarh	1.4605	0.6847
29	Meghalaya	--	--
30	Tripura	1.1784	0.8486
31	Arunachal Pradesh	2.0758	0.4817
32	Nagaland	--	--
33	Manipur	1.2984	0.7702
34	Andaman & Nicobar Islands	2.8989	0.3450

Andaman and Nicobar Island remain to be efficient under input expansion not exceeding 190 percent of the inputs that it employs while its rivals contract inputs upto 65 percent of their current application, simultaneously under output contraction upto 65 percent of current outputs and its rivals expand output upto 190 percent of their current production.

The total manufacturing sector of Sikkim can remain efficient under input expansion upto 118 percent of current application and output contraction upto 54 percent of present realization while its rivals contract their inputs upto 46 percent, expand outputs upto 118 percent.

The TMS of Arunachal Pradesh has the ability to remain efficient under input expansion upto 108 percent and output contraction upto 52 percent while its rivals contract their inputs upto 52 percent and expand their outputs upto 108 percent.

The TMS of Chandigarh remains to be efficient under input expansion upto 46 percent more than the input it consumes, under output contraction upto 32 percent of its current production, while its rivals contract their inputs upto 32 percent of their application and expands output upto 46 percent more than their current production.

Goa has the ability to remain efficient if its input expansion does not exceed 37 percent more than its current input consumption while its output contraction does not exceed 27 percent of the output that it produces currently. Its rivals, on the other hand, are expected to contract inputs upto 27 percent of their current consumption and expand outputs upto 37 percent more than present realization.

The TMS of Manipur is extremely efficient. It preserves efficiency classification under input expansion not exceeding 30 percent of their current input application, and if its output contraction leads output realization not falling short of 77 percent of the outputs that it produces at present. Its rival DMUs are expected to contract their inputs not exceeding 33 percent of their current consumption and expands outputs 30 percent more than they produce at present.

The TMS of Uttarakhund, Dadra & Nagar Haveli and Himachal Pradesh have the same ability for classification preservation. They all have the ability to remain efficient under output contraction not exceeding 20 percent of the output they produce at present, simultaneously expanding outputs not exceeding 25 percent beyond the inputs consumed at present. The rival states are expected to contract their inputs so that their application does not fall short of 80 percent of present consumption and expansion of outputs not exceeding 25 percent of current production.

Tamilnadu to preserve efficiency classification should be such that its input expansion must not be beyond 23 percent of the inputs that it consumes and output contraction should not be beyond 19 percent of present production. Simultaneously, its rival TMSs are expected to contract their inputs so that the consequent application levels will remain to be greater than 81 percent of current application and output expansion shall not exceed 24 percent of present production.

The TMS of Tripura, to preserve its efficiency classification, under input expansion application of inputs should not exceed 18 percent of current consumption and output contraction must not exceed 15 percent of current production. The rival TMSs that are efficient contract their inputs not exceeding 15 percent of current application, simultaneously expand their output, not greater than 18 percent of current production.

The TMS of Maharastra and Gujarat have the same ability to preserve efficiency classification. To remain efficient these two states should be such that their input expansion should not be greater than 14 percent of current consumption and output contraction less than 12 percentof current production. The rival DMUs participating in efficiency classification experience input contraction upto 12 percent and output expansion upto 14 percent of present application.

The TMSs of Delhi and Punjab are efficient. To preserve efficient status their input expansion should not go beyond 11 percent of present consumption and output contraction should fall below 10 percent of present production. The rival DMUs'input contraction and output expansion should not exceed 10 percent of present input consumption and 11 percent of current output production respectively.

The TMSs of Karnataka, Haryana, Jharkhand, Chattisgarh and Jammu & Kashmir have the same ability to remain efficient under input expansion not exceeding 8 percent of current input consumption and equal percentage reduction of present outputs. The input reduction and output expansion of rival DMUs must not go beyond 8 percent of present input consumption and output production respectively.

Inputs expansion and output contraction of Daman and Diu should be below 5 percent of present input consumption and output production respectively, in order to preserve efficiency classification. The other competing efficient TMSs' input contraction and output expansion should not exceed 5 percent of present input consumption and output expansion respectively.

5. CONCLUSIONS:

1. 20 out of 34 total manufacturing sectors of Indian States and Union territories are found efficient.

2. The ability of all efficient total manufacturing sectors of Indian States put together to remain efficient when their inputs are found expanding and outputs experience contraction is inferior to the ability of the TMSs of Union territories.

3. On the average, the efficient total manufacturing sectors of Indian States remain efficient under input expansion upto 29 percent, while the ability of the efficient total manufacturing sectors of Union territories to remain efficient under input expansion is upto 68 percent.

4. On the average, the input expansion capability of the efficient total manufacturing sectors of all India, to preserve efficiency classification is upto 35.77 percent of the inputs that are currently consumed.

5. The efficient total manufacturing sectors of Indian States and Union Territories put together have the ability to remain efficient if contraction of their outputs, on the average, does not exceed 21 percent of the outputs they produce.

6. The Indian efficient states, forfeit their efficiency status on the average, if output contraction go beyond 19 percent, where as the Indian efficient Union Territories lose their efficiency status if output contraction exceeds 28 percent.

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