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Mathematics

ON VAGUE VOLTERRA SPACES

Mariapresenti. L Department of mathematics,Nirmala College for women, Coimbatore- 641018, Tamilnadu, India

ABSTRACT In this paper a new class of vague Volterra space, vague Baire space and vague vague ε_r-Volterra space are defined and obtained several properties and their characterizations concerning vague volterra space.

KEYWORDS : Vague Set, vague topological space, vague Volterra space, vague ε_r-Volterra space and vague baire space.

1. Introduction

The theory of fuzzy set was introduced by Zadeh[8]. As a generalization of fuzzy set, Zadeh[9, 10, 11] introduced the concept of interval-valued fuzzy set, after that, some authors investigated the topic and obtained some meaningful conclusions. Chang[2] introduced and developed the theory of fuzzy topological spaces. In 1993 Gau and Buehrer [3] introduced the concept of vague set which was the generalization of fuzzy set with truth membership and false membership function. The vague set theory has been investigated by many authors and has been applied in different fields. In classical topology, the notion of generalized Volterra spaces was initiated and studied by Milan matejdes [5, 6].Later Thangaraj and Sundarajan[7] established the concept of fuzzy Volterra spaces. The purpose of this paper is to further extend the concept of vague set theory by introducing the concept of vague Volterra spaces along with some interesting properties.

2. Preliminaries

Definition2.1:[3] A vague set A in the universe of discourse U is characterized by two membership functions given by:

(i) A true membership function $t_A : U \to [0,1]$ and

(ii) A false membership function $f_A : U \to [0,1]$

where $t_A(x)$ is a lower bound on the grade of membership of x derived from the "evidence for x", $f_A(x)$ is a lower bound on the negation of x derived from the "evidence for x", and $t_A(x) + f_A(x) \leq 1$. Thus the grade of membership of u in the vague set A is bounded by a subinterval $[t_A(x), 1 - f_A(x)]$ of [0,1]. this indicates that if the actual grade of membership of x is μ(x), then, $t_A(x) \leq \mu(x) \leq 1 - f_A(x)$ .The vague set A is written as $A = \{\langle x, [t_A(x), 1 - f_A(x)] \rangle / u \in U\}$ where the interval $[t_A(x), 1 - f_A(x)]$ is called the vague value of x in A, denoted by $V_A(x)$.

Definition 2.2:[1] Let A and B be VSs of the form $A = \{\langle x, [t_A(x), 1 - f_A(x)] \rangle / x \in X\}$ and $B = \{\langle x, [t_B(x), 1 - f_B(x)] \rangle / x \in X\}$ Then

(i) $A \subseteq B$ if and only if $t_A(x) \leq t_B(x)$ and $1 - f_A(x) \leq 1 - f_B(x)$ for all x ∈ X

(ii) A=B if and only if $A \subseteq B$ and $B \subseteq A$

(iii) $A^c = \{\langle x, f_A(x), 1 - t_A(x) \rangle / x \in X\}$

(iv) $A \cap B = \{\langle x, \min(t_A(x), t_B(x)), \min(1 - f_A(x), 1 - f_B(x)) \rangle / x \in X\}$

(v) $A \cup B = \{\langle x, (t_A(x) \vee t_B(x)), (1 - f_A(x) \vee 1 - f_B(x)) \rangle / x \in X\}$

For the sake of simplicity, we shall use the notation $A = \langle x, t_A, 1 - f_A \rangle$ instead of $A = \{\langle x, [t_A(x), 1 - f_A(x)] \rangle / x \in X\}$.

Definition2.3: [4] A vague topology (VT in short) on X is a family τ of VSs in X satisfying the following axioms.

(i) 0, 1 $\in \tau$

(ii) $G_1 \cap G_2 \in \tau$, for any $G_1, G_2 \in \tau$

(iii) $\bigcup G_i \in \tau$ for any family $\{G_i / i \in J\} \subseteq \tau$.

In this case the pair (X, τ) is called a Vague topological space (VTS in short) and any VS in τ is known as a Vague open set(VOS in short) in X. The complement A^c of a VOS A in a VTS (X, τ) is called a vague closed set (VCS in short) in X.

Definition 2.4[4] Let (X, τ) be a VTS and $A = \langle x, t_A, 1 - f_A \rangle$ be a VS in X. Then the vague interior and a vague closure are defined by

Vint(A)= $\bigcup$ {G/G is an VOS in X and G $\subseteq$ A}

Vcl(A)= $\bigcap$ {K/K is an VCS in X and A $\subseteq$ K}

Note that for any VS A in (X,τ), we have $Vcl(A^c) = (V\operatorname{int}(A))^c$ and $V\operatorname{int}(A^c) = (Vcl(A))^c$.

3. Vague Volterra space

Definition 3.1:A vague set A in a vague topological space (X,τ) is called a vague dense if there exists no vague closed set B in (X,τ) such that $A \subset B \subset 1$.

Definition 3.2:A vague set A in a vague topological space (X,τ) is called a vague nowhere dense set if there exists no vague open set B in (X,τ) such that $B \subset Vcl(A)$. That is, $V\operatorname{int}(Vcl(A)) = 0$.

Theorem 3.3: If A is a vague dense and vague open set in a vague topological space (X,τ) then $\overline{A}$ is a vague nowhere dense set in (X,τ).

Proof: Let A be a vague dense and vague open set in (X,τ). Then we have $Vcl(A) = 1$ and $V\operatorname{int}(A) = A$. Now we have to show that $V\operatorname{int}(Vcl(\overline{A})) = 0$. Let $Vcl(\overline{A}) = \overline{V\operatorname{int}(A)} = \overline{A}$ which implies that $V\operatorname{int}(Vcl(\overline{A})) = V\operatorname{int}(\overline{A}) = \overline{Vcl(A)} = \overline{1} = 0$ That is, $V\operatorname{int}(Vcl(\overline{A})) = 0$. Hence $\overline{A}$ is vague nowhere dense set in (X,τ).

Theorem 3.4: Let A be a vague set. If A is a vague closed set in (X,τ) with $V\operatorname{int}(A) = 0$, then A is a vague nowhere dense set in (X,τ).

Proof: Let A is a vague closed set in (X,τ). Then $Vcl(A) = A$. Now $V\operatorname{int}(Vcl(A)) = V\operatorname{int}(A) = 0$ .and hence A is a vague nowhere dense set in (X,τ).

Theorem 3.5: Let A be a vague closed set in (X,τ), then A is a vague nowhere dense set in (X,τ) if and only if $V\operatorname{int}(A) = 0$.

Proof: Let A be a vague closed set in (X,τ), with $V\operatorname{int}(A) = 0$. Then by theorem 3.4, A is a vague nowhere dense set in (X,τ). Conversely, let A be a vague nowhere dense set in (X,τ) .Then $V\operatorname{int}(Vcl(A)) = 0$ which implies that $V\operatorname{int}(A) = 0$. Since A is a vague closed, $Vcl(A) = A$.

Theorem 3.6: If A is a vague nowhere dense set in a vague topological space (X,τ), then $Vcl(A)$ is also a vague nowhere dense set in (X,τ).

Proof: Let A be a vague nowhere dense set in (X,τ). Then, $V\operatorname{int}(Vcl(A)) = 0$. Now $Vcl(Vcl(A)) = Vcl(A)$. Hence $V\operatorname{int}(Vcl(Vcl(A))) = V\operatorname{int}(Vcl(A)) = 0$. Therefore $Vcl(A)$ is also a vague nowhere dense set in (X,τ).

Definition 3.7:A vague topological space (X,τ) is called a vague first category set if $A = \bigcup_{i=1}^{\infty}(A_i)$, where A_i's are vague nowhere dense sets in (X,τ). Any other vague set in (X,τ) is said to be of vague second category.

Definition 3.8:A vague set A in a vague topological space (X,τ) is called a vague G_δ-sets in (X,τ) if $A = \bigcap_{i=1}^{\infty}(A_i)$ where $A_i \in \tau$, for $i \in I$.

Definition 3.9:A vague set A in a vague topological space (X,τ) is called a vague F_σ-sets in (X,τ) if $A = \bigcup_{i=1}^{\infty}(A_i)$ where $\overline{A_i} \in \tau$, for $i \in I$.

Definition 3.10: A vague topological space (X,τ) is called a vague Volterra space if $Vcl(\bigcap_{i=1}^{N} A_i) = 1$, where A_i's are vague dense and vague G_δ-sets in (X,τ).

Example 3.11: Let X={a,b}. Define the vague sets A, B, C and D as follows, $A = \{<x,[0.2,0.5],[0.3,0.6]>\}$, $B = \{<x,[0.3,0.7],[0.2,0.6]>\}$, $C = \{<x,[0.1,0.8],[0.3,0.9]>\}$ and $D = \{<x,[0.3,0.6],[0.1,0.4]>\}$. Clearly $\tau = \{0,1,A,B,C,D\}$ is a vague topology on X. thus (X,τ) is a vague topological space. Let $E = \{A \cap B \cap C\}$, $F = \{A \cap B \cap D\}$ and $G = \{A \cap B \cap C \cap D\}$ where E, F and G are vague G_δ-sets in (X,τ). Also, we have $Vcl(E) = 1,\ Vcl(F) = 1\ \ Vcl(G) = 1$. Also we have $Vcl(E \cap F \cap G) = 1$. Therefore, is a vague Volterra space.

Theorem 3.12:A vague topological space (X,τ) is a vague Volterra space, if and only if $V\operatorname{int}(\bigcup_{i=1}^{N} \overline{A_i}) = 0$, where A_i's are vague dense and vague G_δ-sets in (X,τ).

Proof: Let (X,τ) be a vague Volterra space and A_i's are vague dense and vague G_δ-sets in (X,τ) .Then we have $Vcl(\bigcap_{i=1}^{N} A_i) = 1$ .Now $V\operatorname{int}(\bigcup_{i=1}^{N} \overline{A_i}) = \overline{Vcl(\bigcap_{i=1}^{N} A_i)} = 0$ .Conversely, let $V\operatorname{int}(\bigcup_{i=1}^{N} \overline{A_i}) = 0$, where A_i's are vague dense and vague G_δ-sets in (X,τ). Then $\overline{Vcl(\bigcap_{i=1}^{N} A_i)} = 0$ implies $Vcl(\bigcap_{i=i}^{N} A_i) = 1$. Hence (X,τ) is a vague Volterra space.

Theorem 3.13: Let (X,τ) be a vague topological space. If $V\operatorname{int}(\bigcup_{i=1}^{N} A_i) = 0$, A_i's are vague nowhere dense and a vague F_σ -sets in (X,τ), then (X,τ) is a vague Volterra space.

Proof: Let $V\operatorname{int}(\bigcup_{i=1}^{N} A_i) = 0$, then this implies that, $\overline{V\operatorname{int}(\bigcup_{i=1}^{N} A_i)} = 1$ (i.e.,) $Vcl(\bigcap_{i=1}^{N} \overline{A_i}) = 1$. A_i's are vague nowhere dense and vague F_σ -sets implies that $\overline{A_i}$'s are vague dense and a vague G_δ -sets in (X,τ), and also $Vcl(\bigcap_{i=1}^{N} \overline{A_i}) = 1$. Then (X,τ) is a vague Volterra space.

Definition 3.14 Let A be a vague first category set in a vague topological space (X,τ). Then $\overline{A}$ is called a vague residual sets in (X,τ).

Definition 3.15: A vague topological space (X,τ) is called a vague ε_r -Volterra space if $Vcl(\bigcap_{i=1}^{N} A_i) = 1$, where A_i's are vague dense and vague residual sets in (X,τ).

Theorem 3.16: Let (X,τ) be a vague ε_r -Volterra space, then $V\operatorname{int}(\bigcup_{i=1}^{N} A_i) = 0$, where A_i's are vague first category sets such that $V\operatorname{int}(A_i) = 0$ in (X,τ).

Proof: Let A_i's (i=1 to N) be vague first category set such that $V\operatorname{int}(A_i) = 0$ in (X,τ). Then $\overline{A}$ is vague residual sets such that $Vcl(\overline{A_i}) = 1$ in (X,τ). That is $\overline{A}$'s are vague residual and vague dense sets in (X,τ). Since (X,τ) is a vague ε_r -Volterra space, $Vcl(\bigcap_{i=1}^{N}(\overline{A_i})) = 1$ and hence therefore, we have $V\operatorname{int}(\bigcup_{i=1}^{N} A_i) = 0$ where A_i's are vague first category sets such that, $V\operatorname{int}(A_i) = 0$ in (X,τ).

Theorem 3.17 If each vague nowhere dense set is a vague closed set in a vague Volterra space in (X,τ), then (X,τ) is a vague ε_r -Volterra space.

Proof : Let A_i's (i=1 to N) be vague dense set and vague residual set in (X,τ). Since A_i's are vague residual set, $(\overline{A_i})'s$ are vague first category set in (X,τ). Now $\overline{A_i} = \bigcup_{i=1}^{\infty} B_{ij}$, where B_{ij}'s are vague nowhere dense set in (X,τ). By hypothesis, a vague nowhere dense set B_{ij}'s are vague closed sets and hence $(\overline{A_i})'s$ are vague F_σ sets in (X,τ). This implies that A_i's are vague G_δ -sets in (X,τ). Hence A_i's are vague dense and vague G_δ -sets in (X,τ). Since (X,τ) is a vague Volterra space, $Vcl(\bigcap_{i=1}^{N} A_i) = 1$. Hence $Vcl(\bigcap_{i=1}^{N} A_i) = 1$, where A_i's are vague dense and interval valued vague residual sets in (X,τ) implies that (X,τ) is a vague ε_r -Volterra space.

Definition 3.18: Let (X,τ) be a vague topological space. Then (X,τ) is called a vague baire space if $V\operatorname{int}(\bigcup_{i=1}^{\infty} A_i) = 0$ where A_i's are vague nowhere dense sets in (X,τ).

Theorem 3.19: Let (X,τ) be a vague topological space. Then the following are equivalent

(i) (X,τ) is a vague Baire space.

(ii) $V\operatorname{int}(A) = 0$, for every vague first category set A in (X,τ).

(iii) $Vcl(B) = 1$, for every vague residual set B in (X,τ).

Proof: (i)⟹(ii) Let A be a vague first category set in (X,τ). Then $A = \bigcup_{i=1}^{\infty} A_i$, where A_i's are vague nowhere dense sets in (X,τ). Now $V\operatorname{int}(A) = V\operatorname{int}(\bigcup_{i=1}^{\infty} A_i) = 0$. Since (X,τ) is a vague baire space. Therefore $V\operatorname{int}(A) = 0$.

(ii)⟹(iii) Let A be a vague residual set in (X,τ) . Then $\overline{B}$ is a vague first category set in (X,τ). By hypothesis $V\operatorname{int}(\overline{B}) = 0$ which implies that $\overline{Vcl(B)} = 0$. Hence $Vcl(B) = 1$.

(iii)⟹(i) Let A be a vague first category set in (X,τ). Then $A = \bigcup_{i=1}^{\infty} A_i$, where A_i's are vague nowhere dense sets in (X,τ). Now A is a vague first category set implies that $\overline{A}$ is a vague residual set in (X,τ). By hypothesis, we have $Vcl(\overline{A}) = 1$ which implies that $\overline{V\operatorname{int}(A)} = 1$. Hence $V\operatorname{int}(A) = 0$. That is, $V\operatorname{int}(\bigcup_{i=1}^{\infty} A_i) = 0$, where A_i's are vague nowhere dense sets in (X,τ). Hence (X,τ) is a vague baire space.

Theorem 3.20: If $\bigcup_{i=1}^{\infty} A_i$, is the vague set, A_i's are vague nowhere dense sets in a vague baire space in (X,τ), then (X,τ) is a vague ε_r-Volterra space.

Proof: Let (X,τ) be a vague baire space and A_i's (i=1 to N) be vague dense set and vague residual set in (X,τ). Since A_i's are vague residual set, $(\overline{A_i})'s$ are vague first category set in (X,τ). Now $\overline{A_i} = \bigcup_{i=1}^{\infty} B_{ij}$, where B_{ij}'s are vague nowhere dense set in (X,τ). By hypothesis $(\overline{A_i})$ is a vague nowhere dense sets in (X,τ). Let (B_α)'s be a vague nowhere dense sets in (X,τ) in which the first N vague nowhere dense sets be $(\overline{A_i})$. Since (X,τ) is a vague baire space, $V\text{int}(\bigcup_{i=1}^{\infty}(B_\alpha)) = 0$. But $V\text{int}(\bigcup_{i=1}^{N}\overline{A_i}) \le V\text{int}(\bigcup_{\alpha=1}^{\infty} B_\alpha)$ and $V\text{int}(\bigcup_{\alpha=1}^{\infty} B_\alpha) = 0$. Then $V\text{int}(\bigcup_{i=1}^{N}\overline{A_i}) = 0$. Therefore $Vcl(\bigcap_{i=1}^{N} A_i) = 1$ where A_i's (i=1 to N) are vague dense set and vague residual set in (X,τ). Therefore (X,τ) is a vague ε_r-Volterra space.

Theorem 3.21 If a vague ε_r-Volterra space is a vague baire space, then $Vcl(\bigcap_{i=1}^{N} A_i) = 1$, where where A_i's (i=1 to N) are vague residual set in (X,τ).

Proof: Let A_i's (i=1 to N) are vague residual set in (X,τ). Since (X,τ) is a vague baire space, (by theorem 3.19) $Vcl(A_i) = 1 \; \forall \; i$. Then A_i's are vague dense set and vague residual set in (X,τ). Since (X,τ) is a vague ε_r-Volterra space, $Vcl(\bigcap_{i=1}^{N} A_i) = 1$. Therefore, $Vcl(\bigcap_{i=1}^{N} A_i) = 1$ where A_i's (i=1 to N) are vague residual set in (X,τ).

Theorem 3.22 If $\bigcap_{i=1}^{N} A_i$, is a vague residual set in a vague baire space (X,τ), where A_i's (i=1 to N) are vague residual sets, then (X,τ) is a vague ε_r-Volterra space.

Proof: Let A_i's (i=1 to N) be vague dense and vague residual sets, then (X,τ). Then, by hypothesis, $\bigcap_{i=1}^{N} A_i$ is a vague residual set in (X,τ). Since (X,τ) is a vague baire space, therefore (by theorem 3.19) $Vcl(\bigcap_{i=1}^{N} A_i) = 1$. Hence $Vcl(\bigcap_{i=1}^{N} A_i) = 1$, where A_i's are vague dense set and vague residual set in (X,τ). Therefore (X,τ) is a vague ε_r-Volterra space.

REFERENCES

1. Borumandsaeid. A and Zarandi. A., Vague set theory applied to BM- Algebras. International journal of algebra, 5, 5 (2011), 207-222.
2. Chang. C. L., Fuzzy topological spacea, J. Math. Anal. Appl. 24(1968) 182-190.
3. Gau. W. L, Buehrer. D. J., Vague sets, IEEE Trans, Systems Man and Cybernet, 23 (2) (1993), 610-614.
4. Mariapresenti. L, Arockiarani. I., 2016. Vague generalized alpha closed sets in topological spaces, International journal of mathematical archive, 7(8), 23-29.
5. Milan matejdes., Generalized volterra spaces, Int. J. Pure Appl. Math. 85(5) (2013), 955-963.
6. Milan matejdes., Characterization of volterra spaces, Int. J. Pure Appl. Math. 88(3) (2013), 437-442.
7. Thangaraj. G and Soundararajan. S., A note on fuzzy volterra spaces, Annals of Fuzzy Mathematics and Informatics, 8(4) (2014), 505-510.
8. Zadeh. L. A, Fuzzy sets, Information and control, 8 (1965), 338-353.
9. Zadeh, L.A.: The concept of a linguistic variable and its application to approximate reasoning, Part 1, Infor. Sci. 8(1975), 199-249.
10. Zadeh, L.A.: The concept of a linguistic variable and its application to approximate reasoning, Part 2, Infor. Sci. 8(1975), 301-357.
11. Zadeh, L.A.: The concept of a linguistic variable and its application to approximate reasoning, Part 3, Infor. Sci. 9(1975), 43-80.