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Mathematics

THE FIRST AND SECOND ZAGREB INDICES OF DEGREE SPLITTING OF SOME GRAPHS

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ABSTRACT Let G be simple graph. The first Zagreb index is the sum of squares of degree of vertices and second Zagreb index is the sum of the products of the degrees of pairs of adjacent vertices. In this paper, we compute the first and second Zagreb index of degree splitting of some special types of graphs.

KEYWORDS : Zagreb Indices, Degree Splitting Of Graph

INTRODUCTION

Let G be a simple connected graphs, ie) connected graphs without loops and multiple edges. For a graph$G$, $V(G)$ and $E(G)$ denote the set of all vertices and edges respectively. For a graph$G$, the degree of a vertex v is the number of edges incident to v and denoted by $degG(v)$. The first Zagreb index $M_1(G)$ is equal to the sum of squares of the degrees of the vertices, and the second Zagreb index $M_2(G)$ is equal to the sum of the product of the degrees of pairs of adjacent vertices. The Zagreb indices have been introduced more than thirty years ago by Gutman and Trinajestic. They are defined as:

$$M_1(G) = \sum_{v \epsilon V(G)} d(v)^2$$

$$M_2(G) = \sum_{uv \in E(G)} d(u)d(v)$$

Let G = (V, E) be a graph with $V = S_1 \cup S_2 \cup \cdots, \cup S_t \cup T$ where each S_i is a set of vertices having at least two vertices and having the same degree and $T = V - \cup S_i$. The degree splitting graph of G is denoted by DS(G) is obtained from G by adding vertices $w_1, w_2, \cdots, w_t$ and joining w_i to each vertex of $S_i (1 \le i \le t)$.

Theorem 1: For a Grid Graph$P_m \times P_n$, $n \ge 3$,

$M_1(DS(P_m \times P_n)) = m^2n^2 + 8(m+n-2)^2 - (m+n-2)[4mn+34] + 25mn + 4,$

$M_2(DS(P_m \times P_n)) = 5m^2n^2 + 36m^2 + 36n^2 - 20mn(m+n) + 162mn - 261(m+n) + 428.$

Proof:

Let $V(P_m \times P_n) = \{v_{ij} / 1 \le i \le m, 1 \le i \le n\} \cup \{w_1, w_2, w_3\}$

Then, $d(v_{11}) = d(v_{1n}) = d(v_{n1}) = d(v_{nn}) = 3$,

$d(v_{ij}) = 4$ for (i =1, n and j ≠ 1, n) and (i ≠ 1, n and j = 1, n),

$d(v_{ij}) = 5$, otherwise

$d(w_1) = 4$, $d(w_2) = 2(m+n-2) - 4$, $d(w_3) = mn - 2(m+n-2)$

$$M_1(DS(P_m \times P_n)) = \sum_{i=1}^{m}\sum_{j=1}^{n} d(v_{ij})^2 + \sum_{i=1}^{2} d(w_i)^2$$
$$= (3^2)4 + 4^2[2(m+n-2)-4] + 5^2[mn - 2(m+n-2)] + 4^2 + [2(m+n-2)-4]^2 + [mn-2(m+n-2)]^2$$
$$= m^2n^2 + 8(m+n-2)^2 - (m+n-2)[4mn+34] + 25mn + 4.$$

$$M_2(DS(P_m \times P_n)) = \sum_{uv \in E} d(u)d(v)$$

$$= \sum(3)(4) + \sum(4)(4) + \sum(5)(4) + \sum(5)(5) + \sum(4)(2m+2n-12) + \sum(5)(4mn-5m-5n+12)$$
$$= 12[12] + 16[2m+2n-12] + 20[2m+2n-8] + 25[2mn-5m-5n+12] + 4[2m+2n-8]^2 + 5[(m-2)(n-2)]^2$$
$$= 5m^2n^2 + 36m^2 + 36n^2 - 20mn(m+n) + 162mn - 261(m+n) + 428.$$

Theorem 2: For a Tadpole graph $T_{m,n}$, $m \ge 3$, $n \ge 1$,

$M_1(DS(T_{m,n})) = m^2 + n^2 + 2mn + 5(m+n) + 8,$

$M_2(DS(T_{m,n})) = 9(m+n) + 6.$

Proof: Let $v_1, v_2, \ldots, v_m, v_{m+1}, v_{m+2}, \ldots, v_n$ be the vertices of $T_{m,n}$. Let w_1, w_2 and w_3 be the degree splitting vertices in which w_1 is adjacent to the pendant vertex, w_2 is adjacent to the 3 degree vertex and w_3 is adjacent to 2 degree vertices.

Then, $d(v_i) = \begin{cases} 2 & i = n \\ 4 & i = m \end{cases}$, $d(w_1) = 1, d(w_2) = m + n - 2, d(w_2) = 1.$
$\quad\quad\quad\quad 3 \;\; otherwise$

$$M_1\big(DS(T_{m,n})\big) = \sum_{i=1}^{m+n} d(v_i)^2 + \sum_{i=1}^{3} d(w_i)^2$$
$$= \sum_{i\neq m,n} d(v_i)^2 + d(v_m)^2 + d(v_n)^2 + d(w_1)^2 + d(w_2)^2 + d(w_3)^2$$
$$= \sum_{i\neq m,n} 3^2 + 4^2 + 2^2 + 1^2 + (m+n-2)^2 + 1^2$$
$$= m^2 + n^2 + 2mn + 5(m+n) + 8.$$

$$M_2\big(DS(T_{m,n})\big) = \sum_{uv\in E} d(u)d(v)$$
$$= \sum (3)(3) + (1)(2) + (3)(2) + \sum (3)(4) + (4)(1)$$
$$= 9[m+n-2] + 12 + 4[3]$$
$$= 9(m+n) + 6.$$

Theorem 3: For a Helm graph $H_{n,} n \geq 3$,
$M_1(DS(H_n)) = 3n^2 + 31n + 2$, $M_2(DS(H_n)) = 11n^2 + 29n + 1$.

Proof:
Let $v_1, v_2, \ldots, v_{2n}$ and u be the vertices of G_n. Let w_1, w_2 and w_3 be the degree splitting vertices.

Then $d(v_i) = \begin{cases} 5 & i = 1,2,\ldots,n \\ 2 & i = n+1,\ldots,2n \end{cases}$, $d(u) = n+1$, $d(w_1) = n$, $d(w_2) = n$ and $d(w_3) = 1$.

$$M_1\big(DS(H_n)\big) = \sum_{i=1}^{2n} d(v_i)^2 + d(u)^2 + \sum_{i=1}^{3} d(w_i)^2$$
$$= \sum_{i=1}^{n} 5^2 + \sum_{i=1}^{n} 2^2 + (n+1)^2 + n^2 + n^2 + 1^2$$
$$= 25(n) + 4(n) + 3n^2 + 2n + 2$$
$$= 3n^2 + 31n + 2.$$

$$M_2(DS(G_n)) = \sum_{uv\in E} d(u)d(v)$$
$$= \sum_{i=1}^{n} (5)(5) + \sum_{i=1}^{n} 5(n+1) + \sum_{i=1}^{n} 5(2) + \sum_{i=1}^{n} 2(n) + (n+1)$$
$$= 25(n) + 5n(n+1) + 10(n) + 2n^2 + n + 1$$
$$= 7n^2 + 41n + 1.$$

Theorem 4: For a Gear graph G_n $n \geq 4$,
$M_1(DS(G_n)) = 3n^2 + 27n + 2$, $M_2(DS(G_n)) = 11n^2 + 29n + 1$.

Proof:
Let $v_1, v_2, \ldots, v_{2n}$ and u be the vertices of G_n. Let w_1, w_2 and w_3 be the degree splitting vertices.
Then $d(v_{2i+1}) = 4, d(v_{2i}) = 3$ for all i $d(u) = n+1, d(w_1) = n, d(w_2) = n$ and $d(w_3) = 1$.

$$M_1\big(DS(G_n)\big) = \sum_{i=1}^{2n} d(v_i)^2 + d(u)^2 + \sum_{i=1}^{3} d(w_i)^2$$
$$= \sum_{i=1}^{n} 4^2 + \sum_{i=1}^{n} 3^2 + (n+1)^2 + n^2 + n^2 + 1^2$$
$$= 16(n) + 9(n) + 3n^2 + 2n + 2$$
$$= 3n^2 + 27n + 2.$$

$$M_2(DS(G_n)) = \sum_{uv\in E} d(u)d(v)$$
$$= \sum_{i=1}^{2n} 3(4) + \sum_{i=1}^{n} 4(n+1) + \sum_{i=1}^{n} 4(n) + \sum_{i=1}^{n} 3(n) + (n+1)$$
$$= 12(2n) + 4n(n+1) + 4n^2 + 3n^2 + n + 1$$
$$= 11n^2 + 29n + 1.$$

CONCLUSION

In this paper, I have computed the first and second Zagreb indices of some special types of graphs. Further work is going on molecular structures.

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