



APPLYING SPEA2 TO SOLVE MULTI-OBJECTIVE MANUFACTURING PROBLEM WITH DEMAND LEAKAGE

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ABSTRACT The focus of this paper is a manufacturing system producing two grades of a product. The manufacturing system produces defective items which are reworked and sold in the secondary market. The price of grade I product is greater than that for grade II product. The demand leakage is also being considered for the product of grade I. The demand leakage here means that the some of the customers who are willing to pay for product of grade I, are actually paying for product of grade II. The unsold units for both the grades are sold at giveaway price. Both the demand and sale are assumed to be fuzzy in nature. The objective of this research work is to study the behavior of the system under demand leakage and other conditions. A multi-objective problem has been proposed for the purpose and Strength Pareto Evolutionary Algorithm 2 (SPEA2) has been applied to study the behavior of the proposed problem. The variations of the objective values have been observed and analyzed. A numerical example shows the applicability of the proposed solution methodology.

KEYWORDS : Defective product, SPEA2, Demand leakage, Fuzzy demand, Fuzzy sale

I. INTRODUCTION

An effective manufacturing system depends on several factors that are required to be considered while solving a relevant problem. Both the demand and sales uncertainty causes uncertainty production variables. Besides, there are many factors like customer satisfaction, price, demand and profitability which are required to be taken account. This results in to complex manufacturing problems to solve. In this paper, a manufacturing system is being considered, which produces two grades of a product. Grade I is above the quality specification upper limit, grade II is between the upper quality specification limit and lower quality specification limit. Defective items can be defined as having quality lower than the lower specification limit. These items are assumed to be sold in the secondary market at a lower price. The price of grade II product is less than the price of grade I product. Both the demand and the sale quantities are assumed to follow uniform fuzzy distribution. This paper considers demand leakage which can be defined as a percentage of customers willing to buy grade I product, actually buys grade II product. In addition to this, this paper also considers that a portion of unsold products of grade I and grade II are sold at a lower price which is known as giveaway price. The relevant costs for grade I product, grade II product and repaired product are considered in this paper in order to calculate the total cost. The motivation for this paper comes from the research gap in the existing literature in which no such combination of objectives as presented in this paper has been considered before. A multi-objective formulation has been proposed in this paper for the manufacturing scenario. The scenario considers fuzzy demand and fuzzy sales. The remaining sections of this paper are organized as: Section II presents brief literature review; A multi-objective problem is formulated in Section III; The solution method and results are provided in Section IV; Concluding remarks are made in Section V.

II. LITERATURE REVIEW

Production inventory decision is a very popular area of research as observed in the existing literature. However the marketing aspects such as pricing, profits, customer satisfaction/dissatisfaction has not yet been integrated in any of the research article till now, as evident from the existing literature. However, the literature has established the price fencing very well, but the price fencing may fail since customers may shift from one product or brand to alternatives, which is termed as demand leakage (Zhang et al., 2010; Raza and Turiac, 2016; Raza et al., 2016). In this direction, the existing literature shows some significant publications considering multi-objective orientation. For example, Raza and Turiac (2016) considered multiple objectives on process mean, pricing and production decision. They considered price-dependent stochastic demand and demand leakage. Tao et al. (2018) investigated the problems of automated assembly lines and used multi-objective meta-heuristics. Zhang and Bell (2012) and Raza (2015) considered customers' willingness to pay which is the basis for demand leakage. Chen et al. (2004) studied customer behavior which included demand leakage situations. Some of the other significant and relevant research studies include the research studies of Talluri and Ryzin (2004) (studying customer behavior), Polotski et al. (2018)

(considered imperfect production process), Jeang (2009), Li et al. (2016), Rahim and Tuffaaha (2004), Hariga and Al-Fawzan (2005).

On the basis of the above literature review, this paper formulates a manufacturing problem with three objectives on sales return, profit and customer dissatisfaction. Such group of objectives has not yet been considered in the existing literature according to the best knowledge of the authors. The three objectives are maximization of the total expected gross sales return, maximization of expected profit and minimization of customer dissatisfaction. The customer dissatisfaction is assumed to be caused because of two factors – shortage of both the grades of the product and the defective items purchased. The costs considered in this paper are manufacturing cost, shortage cost, holding cost, repair cost, giveaway cost, and cost for dissatisfaction. The formulated problem has been solved by Strength Pareto Evolutionary Algorithm 2 (SPEA2) (Zitzler et al., 2001).

III. FORMULATION OF THE PROBLEM

This section shows the assumptions for the problem formulation followed by the formulated problem.

Assumptions:

1. Probability of selling decreases from grade I product to grade II product to repaired product.
2. Shortage cost is same for all types of products.
3. Holding cost for grade II product is higher than that for grade I product.
4. There is no holding cost for the repaired products.
5. Production amount depends on the demand forecast.
6. A portion of unsold products for both grade I and grade II products are sold at a lower price called giveaway price.

Notations - Parameters:

- p_1 : Sale price per unit of product of grade I
- p_2 : Sale price per unit of product of grade II
- p_3 : Sale price per unit of repaired product
- p' : Probability of selling product of grade I
- p'' : Probability of selling product of grade II
- p''' : Probability of selling repaired product
- c_s : Shortage cost per unit of shortage
- c_{h1} : Holding cost per unit per unit time for grade I product
- c_{h2} : Holding cost per unit per unit time for grade II product
- c : Cost of producing an unit of product
- c_R : Repair cost per unit
- c_d : Cost of dissatisfaction per unit
- c_g : Cost per unit of giveaway product of grade I
- c_g : Cost per unit of giveaway product of grade II
- θ_1 : Fraction for giveaway products of grade I
- θ_2 : Fraction for giveaway products of grade II

Notations – Decision Variables

- x_i : Total production of product with grade I

- x_{II} : Total production of product with grade II
- x_R : Total number of defective products produced
- D_I : Demand for product of grade I
- D_{II} : Demand for product of grade II
- ϕ : Fraction of customers willing to pay for grade I but purchasing grade II products (demand leakage)
- τ : Total sale of repaired products

The formulation for the problem is next presented below.

Expected gross sales:

$$Z_1 = p_1(1 - \phi)\min(x_I, D_I) + p_2\{\phi\min(x_I, D_I) + \min(x_{II}, D_{II})\} + p_3x_R \tag{1}$$

Expected profit:

$$Z_2 = p'[p_1x_I - (D_I - x_I)C_z - (x_I - D_I)C_{h1} - \theta_1(D_I - x_I)C_g - x_I C] + p''[p_2x_{II} - (D_{II} - x_{II})C_z - (x_{II} - D_{II})C_{h2} - \theta_2(D_{II} - x_{II})C_g - x_{II} C] + p'''[p_3x_R - (C_R + C)\tau] \tag{2}$$

$$Z_3 = (D_I - x_I)C_d + (D_{II} - x_{II})C_d + \tau C_d \tag{3}$$

Constraints:

- $d_l \leq D_I \leq d_u$ (Upper and lower limits of demand for grade I product)(4)
- $d_l \leq D_{II} \leq d_u$ (Upper and lower limits of demand for grade II product)(5)
- $S_l \leq x_I \leq d_u$ (Upper and lower limits of production for grade I product)(6)
- $S_l \leq x_{II} \leq d_u$ (Upper and lower limits of production for grade II product)(7)
- $S_l \leq x_R \leq d_u$ (Upper and lower limits of production for repaired product)(8)

The first objective (1) maximizes the expected gross sales return. The first, the second and the third terms represent the expected sales for products of grade I, II and repaired products respectively. The symbol ϕ represents the fraction of demand leakage and is generated randomly using random numbers. The first term shows the remaining fraction of customers purchasing product of grade I $(1-\phi)$. The part of the term $\min(x_I, D_I)$ represents the fact that if the demand is more than the produced number then sold product equals the produced products and if the demand is less than produced number, then sold products equals the demand. That means that the actual sold product equals the minimum of the demand and produced products. The first component of the second term represents the customers who are willing to pay for product of grade I but has purchased product of grade II (demand leakage). The second component is the number of original customers purchasing product of grade II. The third term represents the expected sales of the repaired products.

The second objective (2) maximizes the expected profit. The first, the second and the third terms represent the expected profit from the sales of products of grade I, II and repaired products respectively. The first component is obtained by subtracting shortage cost, holding cost, giveaway cost and manufacturing cost from the expected sales for product of grade I. The second component is also obtained in similar way. The third term is obtained by subtracting the total repair and manufacturing cost of the repaired items from the total expected sale of the repaired items. p' , p'' , p''' are the respective probabilities for the three terms representing the respective probabilities of sales.

The third objective (3) minimizes the expected customer dissatisfaction. The first and the second components are the dissatisfactions arising of the shortages of grade I product and grade II product. The third term is the dissatisfaction due to the defective products. The constraints ((4)–(8)) specify the upper and lower limits of the demands and sales for both the grades of the product and the repaired product. The next section shows the application of NSGA-II to solve the above formulated problem.

I. SOLUTION METHOD AND RESULTS

In this section, the formulated problem has been solved by the

application of Strength Pareto Evolutionary Algorithm 2 (SPEA2). The SPEA2 algorithm as applied in this paper is shown Figure 1. The parameters for the algorithm for the experimentation purpose are shown in Table 1.

The population size has been taken to be 80 chromosomes. The structure of each chromosome is shown in Figure 2. Both demands and the productions as shown in the structure have been generated by application of uniform fuzzy numbers. The algorithm has been coded in Matlab 2014b in a Pentium V based processor and 4 GB memory based PC. The algorithm has been run a total of 20 times. Table 2 shows a glimpse of Pareto optimal solutions. Figure 3 shows a sample Pareto optimal solution through a 3D graph. The values of each pair of objectives among the three objectives have also been compared to show the variation of each objective values with respect to the other objectives' values. Therefore the comparison between objectives 1 (maximizing expected sales) and objective 2 (maximizing expected profit), between objective 2 and objective 3 (minimizing customer dissatisfaction), between objective 1 and objective 3 are shown in Figure 4, Figure 5 and Figure 6 respectively. Figure 5 and Figure 6 shows similar patterns which indicate similar variations between sales-dissatisfaction and profit-dissatisfaction. The Pareto optimal solutions in Figure 3 show an increasing trend in Pareto optimal solutions which has also been observed from the values obtained.

1. Generate population of chromosomes.
2. Evaluate objectives for the generated chromosomes.
3. Determine the strength value of each chromosome by counting the number of chromosomes that each chromosome dominates and is dominated by.
4. Determine the fitness of each chromosome.
5. Sort the population of chromosomes in the descending order of strength values.
6. Fill the external archive with nondominated individuals.
7. For each generation, perform step 8 to step 14.
8. Generate mating pool by electing the chromosomes from the population randomly.
9. Perform crossover or mutation on pairs of chromosomes in the mating pool based on the crossover probability. Thus a group of offsprings is generated.
10. Evaluate objectives for each offspring.
11. Determine strength and fitness values of each offspring.
12. Combine the main population with the offspring population to generate intermediate population.
13. Sort the intermediate mediate population based on strength values.
14. Modify the archive by choosing the best chromosomes to replace the worse chromosomes.

Fig. 1. SPEA2 Algorithm as Applied

Table I. Parameters Of The Algorithm

Parameters	Value
Population size	80
Selection type	Tournament
Crossover	Average of gene values
Mutation	Random change in any one gene value
Generations	50
Crossover probability	0.7
Mutation probability	0.3

Demand of grade I product	Demand of grade II product	Production of grade I product	Production of grade II product	Number of defective products produced	Demand leakage	Total sale of repaired products
233	156	233	167	42	0.369627	69.25
252	132	235	165	53	0.383119	55.75
250	158	242	160	40	0.450079	70.21875
250	162	247	159	50	0.509714	75.4375
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250	162	247	159	50	0.509714	75.4375
250	162	247	159	50	0.509714	75.4375
250	162	247	159	50	0.509714	75.4375
266	141	248	145	40	0.326291	54.25
258	157	241	146	34	0.515526	57.59375
239	156	236	162	42	0.371976	69.5625

Fig. 2. Structure Of Chromosome

Table II. Sample Pareto Optimal Solutions

D_I	D_{II}	x_I	x_{II}	x_R	ϕ	τ	Objective 1	Objective 2	Objective 3	Fitness values
233	156	233	167	42	0.369627	69.25	10077.19	5861	413	295
252	132	235	165	53	0.383119	55.75	8471.916	6073	427	3720
250	158	242	160	40	0.450079	70.21875	9858.802	7138	350	275
250	162	247	159	50	0.509714	75.4375	10472.3	5486	273	3798
250	162	247	159	50	0.509714	75.4375	10472.3	5486	273	3798
250	162	247	159	50	0.509714	75.4375	10472.3	5486	273	3798
250	162	247	159	50	0.509714	75.4375	10472.3	5486	273	3798
250	162	247	159	50	0.509714	75.4375	10472.3	5486	273	3798
266	141	248	145	40	0.326291	54.25	10136.24	5066	378	3821
258	157	241	146	34	0.515526	57.59375	8899.284	5521	427	3622
239	156	236	162	42	0.371976	69.5625	10149	5331	301	3702

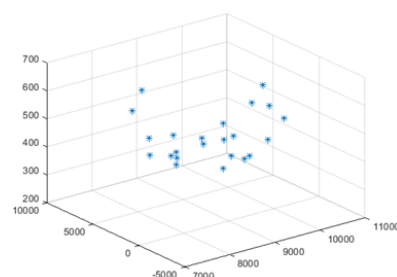
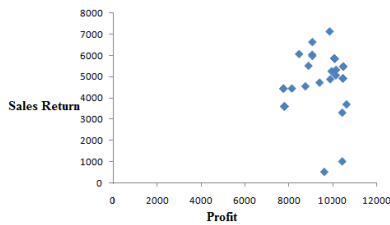
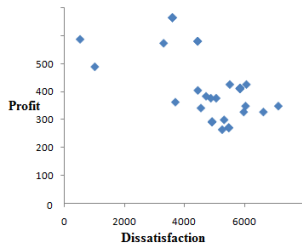
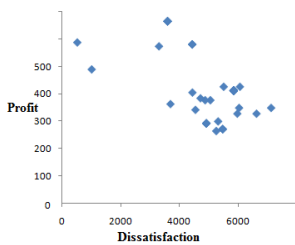


Fig. 3. 3D Plot of Pareto Optimal Solutions**Fig. 4. Comparison between Objective 1 and Objective 2****Fig. 5. Comparison between Objective 2 and Objective 3****Fig. 6. Comparison between Objective 1 and Objective 3**

CONCLUSION

A novel multi-objective formulation for a manufacturing system has been proposed in this paper. The manufacturing system as considered in this paper two grades of products along with some defective products. The defective products are sold at a low price in a secondary market. The price of product of grade II is less than that for grade I product. The paper also considers demand leakage. The unsold products of both the grades of the product are sold at a lower giveaway price. The unsold repaired products are treated as scrap and as a result, no inventory is kept for the unsold repaired products.

The paper has considered three objectives for the above problem – maximizing expected gross sales return, expected profit and expected customer dissatisfaction. This multi-objective problem has been solved by Strength Pareto Evolutionary Algorithm 2 (SPEA2) and has been implemented in Matlab. Numerical example has been shown in order to show the applicability of the proposed problem.

This paper can be extended in future by adding more number of objectives and by solving the resulting problem by a graph-based method which can truly visualize the solution space. Thus the further investigation of the authors is towards the direction of finding a novel method of solving multi-objective problem

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