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Mathematics

ON TERNARY QUADRATIC DIOPHANTINE EQUATION $9(x^2+y^2)-17xy+x+y+1=39z^2$

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ABSTRACT The ternary quadratic equation in place of non-homogeneous cone given by $9(x^2+y^2)-17xy+x+y+1=39z^2$ is analyzed for its non-zero discrete integer points on it. The different patterns of integer points fulfilling the cone under consideration are obtained. A few interesting dealings between the solutions and special number patterns are offered.

KEYWORDS : Ternary, Non-homogeneous, Quadratic, Integral solutions.

INTRODUCTION

The ternary quadratic Diophantine equations offer an indefinite field for research due to their range [1]. For a general review of various problems, one may refer [2-16]. The communication concerns with yet another interesting ternary quadratic equation $9(x^2+y^2)-17xy+x+y+1=39z^2$ representing a cone for determining its considerably many non-zero integral points. Also, a few interesting relations among the solutions are presented.

NOTATIONS

$T_{m,n}$ – Polygonal number of rank n with size m.
P^m_n - Pyramidal number of rank n with size m.
$Ct_{m,n}$ – Centered Polygonal number of rank n with size m.
Gno_n – Gnomonic number of rank n.
Pr_n – Pronic number of rank n.

METHOD OF ANALYSIS

The Ternary quadratic Diophantine equation to be solved for its non-zero discrete integral solution is $9(x^2+y^2)-17xy+x+y+1=39z^2$ — (1)

The replacement of linear transformation
$x=u+v, y=u-v$ ---(2)
In 1 leads to
$(u+1)^2+35v^2=39z^2$---(3)
Special patterns of solutions of (1) exist below.
Pattern I:
Mark 39 as
$39=(2+i\sqrt{35})\ (2-i\sqrt{35})$ --(4)

Imagine that $z=a^2+35b^2$ ---(5)

Where a, b are non - zero discrete integers.

Use (4) & (5) in (3) and applying the method of factorization, identify
$(u+1)+i\sqrt{35}\,v=(2+i\sqrt{35})\ (a+i\sqrt{35}\,b)^2$--- (6)

Equating the real and imaginary parts, we have
$u=u(a,b)=2a^2-70ab-70b^2-1$
$v=v(a,b)=a^2+4ab-35b^2$

Substituting the above values of u and v in equation (2), the value of x and y are given by

$$\left.\begin{aligned} x &= x(a,b) = 3a^2-66ab-105b^2-1\\ y &= y(a,b) = a^2-74ab-35b^2-1\\ z &= z(a,b) = a^2+35b^2 \end{aligned}\right\} \quad --- \quad (7)$$

Thus equation (7) stand for a non-zero discrete integral solutions of (1) in two parameters.

A few interesting properties are given below:
1. $x(1,b)-3y(1,b)-78G_{nob}=80$
2. $y(a,1)+z(a,1)-T_{6,a}\equiv-1(mod-73)$
3. z(1,b) is a nasty number
4. x(1,b)+3z(1,b)-12T_(13,b)=31

Pattern II: As an alternative of (4) we mark as

Mark 1 as $1=(1+i\sqrt{35})(1-i\sqrt{35})$ ---(9)

Imagine that $z=a^2+35b^2$ ---(10)
equation (3) can be write as
$(u+1)^2+35v^2=39z^2*1$ ---(11)

use (8) & (9) in (11) and applying the method of factorization define

$$(u+1)^2+i\sqrt{35}v=\frac{1}{12}(4+2i\sqrt{35})(1+i\sqrt{35})(a+i\sqrt{35}b)^2 \quad --- \quad (12)$$

Equating real and imaginary part, we have

$$\left.\begin{aligned} u &= u(a,b) = \tfrac{1}{12}[-66a^2+2310b^2+420ab-12]\\ v &= v(a,b) = \tfrac{1}{12}[6a^2-210b^2+132ab] \end{aligned}\right\} \quad --- \quad (13)$$

Change a by 12A and b by 12B in (10) and (13)

$$\left.\begin{aligned} u &= u(a,b) = -792A^2+27720B^2+5040AB-1\\ v &= v(a,b) = 72A^2-2520B^2+584AB\\ z &= z(a,b) = 144A^2+5040B^2 \end{aligned}\right\} \quad --- \quad (14)$$

Substituting the values of u and v in equation (2), the value of x and y are given by

$$\left.\begin{aligned} x(A,B) &= -720A^2+25200B^2+5624AB-1\\ y(A,B) &= -864A^2+30240B^2+4456AB-1\\ z(A,B) &= 144A^2+5040B^2 \end{aligned}\right\} \quad --- \quad (15)$$

Thus the equation (15) represents non-zero discrete integral solutions of (1) in two parameters.

A few interesting properties are given below:
1. $x(A,1)-5z(A,1)+2812G_{noA}+5624=0$
2. $y(1,B)+6z(1,B)-60480Pro_B+28012_{GnoB}=28013$
3. $z(A,A)$ is a perfect square number
4. $x(A,1)-y(A,1)-144T_{4,A}\equiv-368(mod\ 1168)$

Pattern III:
Equation (3) can be write as
$(u+1)^2=39z^2-35v^2$ ---(16)

Introducing the linear transformation
$Z=X+35T$ and $V=X+39T$ ---(17)
in (16) we get

$$X^2=1365T^2+\left(\frac{u+1}{2}\right)^2 \quad --- (18)$$

Which is satisfied by

$$\left.\begin{aligned} T(p,q) &= 2pq\\ u(p,q) &= 2730p^2-2q^2-1\\ x(p,q) &= 1365p^2+q^2 \end{aligned}\right\} \quad --- \quad (19)$$

Substituting the values of (19) in (17) and using (2), the corresponding integer solutions of (1) are given by

$$\left.\begin{aligned} x &= x(p,q) = 4095p^2 - q^2 + 78pq - 1 \\ y &= y(p,q) = 1365p^2 - 3q^2 - 78pq - 1 \\ z &= z(p,q) = 1365p^2 + q^2 + 70pq \end{aligned}\right\} \quad --- \quad (20)$$

Thus the equation (20) represent a non-zero discrete integral solutions of (1) in two parameters.

A few interesting properties are given below:
1. $x(P,1)-3y(P,1)\equiv 10 (mod\, 312)$
2. $y(1,q)-z(1,q)+4T_{4,q}+74_{Gnoq}=75$
3. $x(a,a)+3z(a,a)$ is a nasty number
4. $x(P,1)-3z(P,1)+66_{Gnop}=71$

Note: In addition to (17) one may also consider the linear transformation $Z=X-35T$ and $V=X-39T$ following the method presented above different set of solutions are obtained.

CONCLUSION:

In this paper, we have obtainable three singular patterns of non-zero discrete integer solutions of the non-homogeneous cone given by $9(x^2+y^2)-17xy+x+y+1=39z^2$. To end, one may search for patterns of non-zero discrete integer solutions and their corresponding properties for other choices of ternary quadratic diophantine equation.

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