



APPLICATIONS OF KELKAR'S MODIFIED EQUATIONS OF GENERAL RELATIVITY TO PROBLEMS OF QUASISTATIC, ROTATING AND RADIALLY MOVING STARS. ASPECT OF POTENTIAL ENERGY DENSITY, COMPARISON WITH EINSTEIN

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ABSTRACT Prof. Dr.Kelkar proposed a new field equation in the theory of general relativity and was introduced in the Ph.D thesis of Dr. Deshpande in the year 1982. The new field is different from Einstein. $R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R = 4\pi\rho_0 g^{\mu\nu} + \eta^{\mu\nu}$ where $\eta^{\mu\nu}$ is a small term which makes the field equation more general and can be applied to non-uniform density

KEYWORDS :

Rotation, radial motion and stability of stars and planets depend upon the interior field i.e. field inside the source. The work done by Prof.Dr. Kelkar and his four earlier students at the Institute of science show that Einstein's theory i.e. his field equation predicts results which are simply absurd according to Newton's theory and of course do not agree with observations.

- 1) Einstein's theory predicts that a star at rest and in equilibrium can expand to infinity with uniform velocity where as Kelkar et.al. Papers published show that the star is accelerated inwards according to Newton.
- 2) A rotating star can have any shape (any ellipticity including $\varepsilon = 0$ meaning spherical shape) according to Einstein but Newton predicts a unique ellipticity i.e. unique shape [described in "figure de la terra" by Clairaut]
- 3) A cluster of stars [which is pressure less distribution of matter technically called 'DUST'] with uniform density can remain as a static sphere according to Einstein. According to Newton such a sphere of dust of uniform density must collapse to a point.

The anomalies mentioned in the introduction give a very strong motivation for a new theory of GR. Kelkar et al (private communication) have shown that there are three equations in GR for Newtonian approximation and of course for agreement with observations.

A) The new field equation is given as $R_{\mu\nu} = -4\pi\rho_0 g_{\mu\nu} + \eta_{\mu\nu} = K_{\mu\nu}$ (Kelkar)

B) With $\eta^{\mu\nu} = 4\pi P \left[4 \frac{dx^\mu}{ds} \frac{dx^\nu}{ds} - g^{\mu\nu} \right]$ so that $g_{\mu\nu} \eta^{\mu\nu} = 0$

Equation A can also be written as $R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 4\pi\rho_0 g_{\mu\nu} + \eta_{\mu\nu}$. It is easy to see that $\eta_{\mu\nu} \ll \rho_0$ and is only a correction term.

C) The equation of state which avoids collapse of a star.

In the present investigation the potential energy of a star is calculated using isotropic metric $-e^{\mu} (dx^2 + dy^2 + dz^2) + e^{\nu} dt^2 = ds^2$ where $\mu = \mu(r, t)$ and $\nu = \nu(r, t)$

It is shown that the total energy is conserved though the potential energy and kinetic energy are not individually conserved. Both the P.E and K.E have double the Newtonian value. Since kinetic energy is always positive and potential energy is negative in the gravitational field, their sum will have Newtonian value if initially the value is zero. But the value 0 can be adjusted by adding a suitable constant to potential energy, since addition of a constant to potential energy does not affect physics. The anisotropic line element gives a potential energy which is negligible in comparison with Newton hence it is concluded that it is not a suitable metric in GR. The negligible potential energy explains the anomalous motion of Ross star.

Using new theory and new field equation the radial motion of DUST is studied in some details using isotropic coordinate system.

The radial motion of a star by using Einstein's equation has also been studied by Naraliker et al¹⁰ using the condition of isotropy of pressure and isotropic coordinates and is compared to results given by the present investigation using new theory. Naraliker's assumption of

isotropy of pressure will not affect the field equation significantly but it has significance in the hydrodynamic equation since the leading term of the field equation is ρ_0 which is 10^{18} times greater than the leading term $\rho \frac{v^2}{c^2}$ of the hydrodynamic equation. Naraliker has only obtained the solution but has not interpreted his solution.

It is the interpretation of the present investigation that Naraliker's solution will give either static solution of no motion or arbitrary motion since the solution contains an arbitrary function $f(t)$. On the other hand our theory and its application to the radial motion has a Newtonian approximation for both the field and hydrodynamic equation and one can get unique solutions which are approximately Newtonian.

Radial motion in isotropic coordinate system according to our theory can be studied by perturbation method but the method will not be like that of Ross. Ross has used Einstein's equation so that e^{μ}, e^{ν} and P are analytical functions of $u = \frac{dr}{dt}$ and because of this he can start with static solution of any radius and can consider the star to be moving with uniform radial velocity $u = \frac{dr}{dt}$. According to Newton's theory the star experiences acceleration towards the centre. Ross gets a result which is simply absurd according to Newton's theory. Newton gives unique acceleration. In our theory e^{μ}, e^{ν} and P are not analytical functions of u but they can be regarded as analytical functions of time. Hence the field equation and hydrodynamic equation give approximately same result as Newton. We are sure to get unique solution like Newton. However we have not used perturbation method to obtain solutions because our aim is to show that our theory gives Newtonian approximation in all physical situations in which there is an interior field such as radial motion and rotation. However there can be quasistatic sphere of fluid in the new theory if the observation time is in milliseconds (because unit of time is $\frac{1}{c}$ seconds) and the fluid starts from rest. But actually the observation time has been over geological ages from the beginning of the heavenly bodies and is of the order like millions of years. Hence we see static spheres of fluid in which equation of state avoids further contraction. There is no static sphere of fluid in Newton's theory and in our theory provided gravitation is stronger than molecular forces. In other words pressure is too small and mean free path is large enough and probability of collision is negligible. For instance our Sun has contracted to the present size in millions of years and further contraction has not been possible due to equation of state. But at the beginning when sun was born it probably had a diameter up to 10^{10} (predicted by Rawal and confirmed by observations) planet and pressure must have been low.

In the present investigation the rotation of a star is studied by using the new field equation. It is concluded that there is a Newtonian approximation.

The rotation of a star of non uniform density and time dependent rotation (i.e. rotation as well as radial motion) of a star are studied to the extent of showing that the field and hydrodynamic equations have Newtonian approximation.

The geodesic motion using isotropic coordinate system is studied in this investigation since isotropic coordinate system gives Newtonian Approximation as shown in the introduction. In the anisotropic system the perturbation is a second order effect, the equation being

$\frac{d^2u}{dr^2} + u = \frac{m}{r^2} + 3mu^2$ the perturbation is $3mu^2$ because the rest of the equation is just the Newtonian equation of the orbit.

In the isotropic coordinate system the geodesic equation has been obtained and given as $\frac{d^2u}{ds^2} + \alpha^2 u = \frac{m}{h^2} + \frac{\beta m^3 u^2}{h^2}$ where $\alpha^2 = \left(1 - \frac{6m^2}{h^2}\right)$ and is slightly different from 1.

$-\frac{6m^2}{h^2}$ is the main perturbation. The term $\frac{\beta m^3 u^2}{h^2}$ contributes little to the perturbation. The anisotropic metric gives precession of perihelion of mercury only if there is an elliptical orbit but does not predict any difference from Newton's theory if the orbit is circular. The precession of perihelion depends only slightly upon ellipticity of the orbit in the case of isotropic metric and motion is non Newtonian even in the case of circular orbit. Since the orbit of a planet is in the field exterior to the source there is no difference between Einstein's theory and our theory as regards orbits but as stated earlier the prediction of isotropic and anisotropic metrics are materially different. This result can be verified by observing an artificial satellite made to revolve round the earth in suitable orbits.

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