

KEYWORDS : Non-uniform Heat Source, Cylindrical Annulus, Radiation, Soret Effect

INTRODUCTION:

The study of flow and heat transfer in the annular region between the concentric cylinders has applications in nuclear waste disposal research. It is known that canisters filled with radioactive rays be buried in earth so as to isolate them from human population and is of interest to determine the surface temperature of these canisters. This surface temperature strongly depends on the buoyancy driven flows sustained by the heated surface and the possible moment of ground water past it. This phenomenon is ideal to the study of convection flow in a porous medium contained in a cylindrical annulus [17,19].

Free convection in a vertical porous annulus has been extensively studied by Prasad and Kulacki [17] and Prasad et. al, [19] both theoretically and experimentally. Caltagirone [3] has published a detailed theoretical study of free convection in a horizontal porous annulus including possible three dimensional and transient effects. Convection through annulus region under steady slate conditions has also been discussed with two cylindrical surface kept at different temperatures [12]. This work has been extended in temperature dependent convection flow [7,8] as well as convection flows through horizontal porous channel whose inner surface is maintained at constant temperature while the other surface is maintained at circumferentially varying sinusoidal temperature [15,22]. Free convection flow and heat transfer in hydromagnetic case is important in nuclear and space technology [14,21,24]. In particular, such convection flow in a vertical annulus region in the presence of radial magnetic field has been studied by Sastry and Bhadram [23]. Whitehead [27], Leppinen et al[11] examined free convection in a shallow annular cavity filled with a porous medium. Jha[9] studied free convection flow through an annular porous medium. Non-Darcian thermal stability of a heat generating fluid in a porous annulus was investigated by Saravanan and Kandaswamy[22]. Charrier Mojtabi[4] studied numerical simulation of two-and three dimensional free convection flows in a horizontal porous annulus using a pressure and temperature formulation. Chmaissem et al[5] reported numerical study of a Boussinesq model of natural convection in an annular space having a horizontal axis bounded by circular and elliptical isothermal cylinders.

In many problems, there may be plausible temperature difference between the surface and the ambient fluid. This obligates the consideration of temperature dependent heat sources/sinks which may exert strong effect on the heat transfer characteristics. The study of heat source/sink in MHD fluid flow is gaining attention because of its increasing applications to many engineering problems. Several researchers [1,6,10,16, 18,20,25] have studied the effects of nonuniform heat source on convective heat and mass transfer in fluid saturated porous medium. Bhuvanavijaya et al [2] have studied double diffusive convection of a rotating fluid over a vertical plate embedded in Darcy-Forchheimer porous medium with non-uniform heat sources. Sudarsana Reddy et al[26] have discussed the convective heat and mass transfer flow of a viscous fluid in a concentric cylindrical annulus with Soret and Dufour effect. Mallikarjuna et al[13] have investigated the mixed convective heat and mass transfer flow through a porous medium in a vertical cylindrical annulus with Soret and Dufour effects



FORMULATION OF THE PROBLEM

We consider the free and forced convection flow in a vertical circular annulus through a porous medium under the influence of radial magnetic field, whose walls are maintained at a constant heat and concentration. The flow is a mixed convection flow taking place under thermal and molecular buoyancies and uniform axial pressure gradient.

The governing equations with Rosseland approximation in the presence of non-uniform heat source are

$$\begin{aligned} & -\frac{\partial p}{\partial z} + \frac{v}{\delta} \left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} \right) - (\frac{\sigma \mu_s^2 H_o^2}{r^2}) u - (\frac{v}{k}) u - \frac{\delta F}{\sqrt{k}} u^2 \end{aligned} \tag{1} \\ & +\rho\beta(T-T_l) + \rho\beta^*(C-C_l) = 0 \\ & \rho C_p u \frac{\partial T}{\partial z} = k_f \left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right) + (\frac{k_f}{a^2 v}) \\ & (A^*(T_w - T_0) u + B^*(T - T_0) \\ & + \frac{16\sigma^* T_o^3}{3\beta_R} \left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right) \\ & u \frac{\partial C}{\partial z} = D_B \left(\frac{\partial^2 C}{\partial r^2} + \frac{1}{r} \frac{\partial C}{\partial r} \right) - k_c' C + \frac{D_B K_T}{T_m} \left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right) \end{aligned}$$

where $u, T, C, k, k_D, F, D_{B_{u}}, \beta, Q_H, Cp, g, \bullet A, \bullet Band pare the axial velocity, temperature, concentration, permeability of porous medium, thermal diffusivity, function that depends on Reynolds number, the microstructure of the porous medium, molecular diffusivity, coefficient of the thermal expansion, heat source coefficient, radiation absorption coefficient, specific heat, density, g gravity coefficients of space dependent and temperature dependent internal heat generation or absorption respectively. It is noted that the case <math>\bullet A > 0$ and $\bullet B > 0$,

INDIAN JOURNAL OF APPLIED RESEARCH

21

corresponds to internal heat generation and that $\bullet A < 0$ and $\bullet B < 0$, the case corresponds to internal heat absorption case.

The relevant boundary conditions are
$$0=u, T=T_w, C=C_w$$
 at $r=a\&a+s$ (4)

Following Tao(47),we assume that the temperature and concentration of the both walls is $T_w = T_0 + Az$, $C_w = C_0 + Bz$ where A and B are the vertical temperature and concentration gradients which are positive for buoyancy –aided flow and negative for buoyancy –opposed flow, respectively, 0*T* and 0*C* are the upstream reference wall temperature and concentration, respectively. For the fully developed laminar flow in the presences of radial magnetic field, the velocity depend only on the radial coordinate and all the other physical variables except temperature, concentration and pressure are functions of r and z, z being the vertical co-ordinate . The temperature and concentration inside the fluid can be written as $T = T^*(r) + Az$, $C = C^*(r) + Bz$ (5)

We now define the following non-dimensional variables

$$z^* = \frac{z}{a} , \qquad r^* = \frac{r}{a} , \qquad u^* = \left(\frac{a}{v}\right)u$$
$$p^* = \frac{pa\delta}{\rho v^2} , \quad \theta^*(r^{\bullet}) = \frac{T^{\bullet} - T_0}{Aa} , \qquad C^*(r^{\bullet}) = \frac{C^{\bullet} - C_0}{Ba} ,$$

Introducing these non-dimensional variables, the governing equations in the non-dimensional form are (on removing the stars)

$$\left| \begin{array}{c} \left(\frac{\partial^{2} u}{\partial r^{2}} + \frac{1}{r} \frac{\partial u}{\partial r} \right) = 1 + \delta \left(\frac{M^{2}}{r^{2}} + D^{-1} \right) u \\ + \delta^{2} (D^{-1/2}) - \Lambda u^{2} - \delta G(\theta + NC) \\ \left(\frac{1}{\Pr} \right) \left(1 + \frac{4Rd}{3} \right) \left(\frac{\partial^{2} \theta}{\partial r^{2}} + \frac{1}{r} \frac{\partial \theta}{\partial r} \right) = u + (A l u + B l \theta) \\ \left(\frac{\partial^{2} C}{\partial r^{2}} + \frac{1}{r} \frac{\partial C}{\partial r} \right) - \gamma C + ScSr \left(\frac{\partial^{2} \theta}{\partial r^{2}} + \frac{1}{r} \frac{\partial \theta}{\partial r} \right) = Sc u \\ \end{array} \right|$$
(11)

where

 $\Lambda = FD^{-1}$ (Inertia parameter or Forchhimer number)

$$G = \frac{g\beta(T_e - T_i)a^3}{v^2} \qquad (Grashof number)$$

$$M^2 = \frac{\sigma\mu_e^2 H_o^2}{av} \qquad (Magnetic parameter)$$

$$D^{-1} = \frac{a^2}{k} \qquad (Inverse Darcy parameter)$$

$$P_r = \frac{\mu C_p}{k_f} \qquad (Prandtl number)$$

$$Sc = \frac{v}{D_B} \qquad (Schmidt number)$$

$$Sr = \frac{D_B K_T (C_w - C_o)}{T_m (T_w - T_o)} \qquad (Soret parameter)$$

$$\gamma = \frac{k_c^1 a^2}{DB} \qquad (Chemical Reaction parameter)$$

$$r_a = \frac{A^3 v}{D_b} \qquad (Space dependent heat source parameter)$$

$$Rd = \frac{4\sigma^* T_e^3}{\beta_R k_f}$$
(Radiation parameter)

The corresponding non-dimensional conditions are 0=u, 0=0, C=0 at r=1 and 1+s (12)

METHOD OF SOLUTION

The finite element method is powerful technique for solving ordinary or partial differential equations. The steps involved in the finite element analysis are as follows;

- Discretization of the domain into elements
- Derivation of element equations
- Assembly of element equations
- Imposition of boundary conditions
- Solution of assembled equations

The whole flow domain is divided into 1000 quadratic elements of equal size. Each element is three-noded and therefore the whole domain contains 2001 nodes. We obtain a system of equations contains 8004 equations. The obtained system is non-linear. Therefore an iterative scheme is utilized in the solution. After imposing the boundary conditions the remaining system contains 7997 equations, which is solved by the Gauss elimination method while maintaining an accuracy of 10^{-5} .

SHEAR STRESS, NUSSELT NUMBER AND SHERWOOD NUMBER

The shear stress (τ) is evaluated using the formula $\tau = (\frac{du}{dr})_{r=1,1+s}$

The rate of heat transfer (Nusselt number) is evaluated using the formula

$$Vu = -(\frac{d\Theta}{dr})_{r=1,1+s}$$

The rate of mass transfer (Sherwood number) is evaluated using the formula

$$Sh = -\left(\frac{dC}{dr}\right)_{r=1,1+1}$$

COMPARISON:

In the absence of thermo-diffusion(Sr=0),magnetic field(M=0) the results are in good agreement with *Madhusudan et al* [13].

Table – 1

Parameters			Madhusudana et al 13]					
Ν	Rd	γ	Nu(1)	Nu(2)	Sh(1)	Sh(2)		
1	0.5	0.5	0.10673	1.7335	12.3727	14.5232		
2	0.5	0.5	0.11549	1.74131	12.3734	14.5233		
-0.5	0.5	0.5	0.01152	1.74152	12.3718	14.5237		
-1.5	0.5	0.5	0.01156	1.74162	12.3712	14.5138		
1	1.5	0.5	0.2225	3.3518	12.3721	14.5237		
1	5.0	0.5	0.30165	4.5736	12.3713	14.5239		
1	0.5	1.5	0.11552	1.74131	13.0332	14.4276		
1	0.5	-0.5	0.11554	1.741525	13.3226	14.4516		
1	0.5	-1.5	0.115432	1.741625	12.1728	14.4526		
Parameters			Present results(Sr=0)					
Ν	Rd		Nu(1)	Nu(2)	Sh(1)	Sh(2)		
1	0.5	0.5	0.10669	1.7336	12.36987	14.52289		
2	0.5	0.5	0.11538	1.741299	12.37299	14.52319		
-0.5	0.5	0.5	0.011492	1.741499	12.3714	14.52299		
-1.5	0.5	0.5	0.01151	1.74155	12.3706	14.5136		
1	1.5	0.5	0.22239	3.35168	12.3719	14.5231		
1	5.0	0.5	0.30159	4.57345	12.3709	14.52301		
1	0.5	1.5	0.115499	1.74131	13.03289	14.4266		
1	0.5	-0.5	0.115035	1.741499	13.32189	14.4516		
1	0.5	-1.5	0.115437	1.741587	12.17187	14.4522		

RESULTS AND DISCUSSION

In order to get physical Insight into the problem we have carried out numerical calculations for non-dimensional velocity, temperature and spices concentration, skin-friction, Nusselt number and Sherwood number by assigning some specific values to the parameters entering into the problem

Fig.1 represents u with radiation parameter (Rd).It can be seen from the profiles that higher the radiative heat flux larger the magnitude of the axial velocity, owing to the fact that the thickness of the momentum boundary layer enhances with increase in Rd.The temperature and Concentration increase in the presence of thermal radiation throughout the boundary layer(fig.7&13).The effect of thermo-diffusion (So) on u can be seen from fig.2. From the profiles that we find that u increases in the flow region. Higher the thermodiffusion effect larger the thickness of the thermal boundary layer which results in a rise in the fluid temperature in the entire flow region(fig.8). The concentration reduces with increasing values of Soret parameter(S_0) in the entire flow region(fig.14). The effect of chemical reaction parameter(γ) on u is exhibited in fig.3.It is found that the magnitude of the axial velocity enhances with increase in γ in both degenerating/generating chemical reaction cases in the entire flow region.

the temperature depreciates and the concentration enhances with increase in in both the degenerating /generating chemical reaction cases(fig.9&15). Fig.4 & 5 shows the variation of u with heat source parameter (A1&B1). It is found that an increase in the strength of the space dependent heat source/sink(A1>0 or A1<0) smaller the velocity in the entire flow region. From fig.5, we find that the presence of the heat generating/absorbing source ,larger the temperature in the flow region. This may be attributed to the fact that the heat energy is generated in the boundary layer due to the presence of heat sources/sinks. the presence of the heat generating/absorbing source energy is absorbed in the flow region and as a result the temperature falls.The influence of temperature dependent heat source/sink (B1>0or B1<0) the temperature reduces due to the absorption of thermal energy in the flow region.Fig.6 show the variation of u with Forrchimer parameter(Λ). It is found that higher the values of smaller Λ the magnitude of the velocity and the temperature and larger the concentration in the entire flow region(Fig.12&18).

Effects of parameters on Skin friction, Nusselt number and Sherwood number:

The Skin friction ,the rate of heat and mass transfer on the inner and outer cylinder r=1&2 is exhibited in table.2. With reference to the chemical reaction parameter(γ) we find that the skin friction enhances in the degenerating chemical reaction case and reduces in the generating chemical reaction case on both the cylinders. Higher the thermo-diffusion effects/thermal radiation effects larger the skin friction .An increase in A1>0 enhances skin friction on both the cylinders while decreasing values of A1<0, reduces the skin friction on the inner cylinder and enhances on the outer cylinder. An increase in strength of the heat generating/absorbing heat source increases the skin friction on both the cylinders. As the Prandtl number increases the skin friction enhances on r=1 & 2.

The rate of heat transfer(Nusselt number) enhances on both the cylinders with So or Rd. With respect to the chemical reaction parameter (γ) we find that the magnitude of Nu enhances in the degenerating chemical reaction case and reduces in the generating chemical reaction case on both the cylinders. An increasing in A1>0 enhances Nu on both the cylinders while for decreasing values of A1<0, it enhances on r=1 and reduces on the outer cylinder r=2. An increase in B1>0 or B1<0 leads to an enhancement in the rate of heat transfer on both the cylinders.

The rate of mass transfer (Sherwood number), the chemical reaction parameter (γ) we find that the rate of mass transfer reduces on r=1&2 in both the degenerating/generating chemical reaction cases. An increase in Rd reduces Sh on r=1 and increases on r=2.An increase in A1>0enhances Sh on r=1 and reduces on r=2 while foe decreasing values of A1<0, it reduces on both the cylinders. An increase in B1>0 or for decreasing values of B1<0, enhances Sh on r=1 and reduces on r=2.





INDIAN JOURNAL OF APPLIED RESEARCH





Fig. 16 : Variation of C with A1 Rd=0.5, S_0 =0.5, γ =0.5, B1=0.1,

23

Volume-9 | Issue-1 | January-2019 | PRINT ISSN - 2249-555X





Fig. 18 : Variation of C with Λ A1=0.1,B1=0.1, γ =0.5, Rd=0.5, Λ =2

Table : 2 : Skin Friction(τ), Nusselt Number (Nu), Sherwood Number (Sh) at r = 1,2

Parameter		τ(1)	τ(2)	Nu(1)	Nu(2)	Sh(1)	Sh(2)
γ	0.5	0.46898	-0.46907	-0.787071	-0.134627	0.590826	1.49995
	1.5	0.73517	-0.66406	-1.39801	-0.216173	0.310262	1.28467
	-0.5	0.73714	-0.66566	-1.40187	-0.216772	-0.923066	-0.95456
	-1.5	0.73617	-0.66488	-1.39993	-0.216457	-0.312254	-0.78785
So	0.5	0.468982	-0.346907	-0.870716	-0.134627	0.590826	1.499953
	1.0	0.734128	-0.66322	-1.39599	-0.215858	0.950445	1.516874
	1.5	0.744129	-0.67334	-1.39622	-0.215966	0.950299	1.596554
Rd	0.5	0.46898	-0.46907	-0.870716	-0.13462	0.590826	1.49995
	1.5	0.734171	-0.66324	-2.69903	-0.41733	0.950524	1.65567
	3.0	0.734199	-0.66355	-3.68051	-0.56909	0.951414	2.16567
	5.0	0.934187	-0.66387	-4.47388	-0.69186	0.952311	2.89862
A1	0.1	0.46898	-0.46907	-0.870716	-0.134627	0.590826	1.4999
	0.3	0.73451	-0.66323	-2.01651	-0.311802	0.950596	1.39567
	-0.1	0.73413	-0.66305	-0.77495	-0.119548	0.951072	1.05643
	-0.3	0.73411	-0.66321	-1.55108	-0.023983	0.950775	0.96799
B1	0.1	0.468982	-0.46907	-0.870716	-0.134627	0.590826	1.49995
	0.3	0.734159	-0.66308	-1.38438	-0.214226	0.950531	1.20976
	-0.1	0.734136	-0.66322	-1.40821	-0.217743	0.950643	1.10345
	-0.3	0.734137	-0.66323	-1.42064	-0.219666	0.950655	0.09543

CONCLUSIONS:

The important conclusions of the analysis are

- The velocity, the concentration reduces and the temperature enhances with increase in A1>0 while with decreasing values of A1<0,the velocity, concentration enhances while the temperature reduces in the flow region.. The skin friction ,Nusselt and the Sherwood numbers enhance with increase in the strength of the heat generating/absorption source.
- The velocity, concentration reduces and the temperature enhances in the flow region in the degenerating /generating chemical reaction cases. The skin friction and Nusselt number enhance on the cylinders with increase in $\gamma > 0$ and for an increase in $\gamma < 0$, the skin friction and Nusselt number reduces on the cylinders. The rate of mass transfer reduces on both the cylinders with increase in $\gamma > 0$ and $\gamma < 0$.
- Higher the thermo-diffusion effects smaller the velocity, concentration and larger the temperature in the flow region. Higher the thermal radiation larger the velocity, temperature and concentration in the flow region. The skin friction, Nusselt and Sherwood numbers enhances on the cylinders with increasing Sr and Rd
- Higher the values of Forrcheimer parameter (Λ) smaller the velocity, temperature and larger the concentration in the flow region.

REFERENCES

24

- Antonio Barletta : Combined forced and free convection with viscous dissipation in a [1]. vertical duct, Int. J. Heat and mass transfer v. 42, pp. 2243–2253 (1999). Bhuvanavijaya,R and Mallikarjuna,B :Double diffusive convection of a rotating fluid
- over a vertical plate embedded in a Darcy-Forchheimer porous medium with nonuniform heat sources., Int. J.Emerging Trends In Engineering and Development, V.2(3),pp415-431(2013)
- Caltagirone J.P, Thermo convective instabilities in a porous medium bounded by two [3]. concentric horizontal cylinders, J. fluid Mech, v. 76, p. 337-362 (1976)
- Charrier-Mojtabi,M.C:Numerical simulation of two and three dimensional free [4]. convection flows in a horizontal porous annulus using a presseure and temperature formulation., Int.J.Heat Mass Transfer, v.40, pp.1521-1533(1997) Chmaissem, W., Suh, S.J., Degueene, M; Numerical study of Boussinesq model of natural
- [5]. convection in an annular space: Having a horizontal axis by circular and elliptical isothermal cylinders, Appl. Thermal eng., V.22, pp.1013-1025(2002). Dulal Pal and Hiranmoy Mondal: Effect of variable viscosity on MHD non-Darcy
- [6]. mixed convective heat transfer over a stretching sheet embedded in a porous medium with non-uniform heat source/sink., Communcations in Non-linear Science and Numerical Simulation, V.15, pp.1553-1564(2010)
- Emad M Abo-Eldahab and Mohamed A El Aziz:Blowing/Suction effect on [7].

INDIAN JOURNAL OF APPLIED RESEARCH

hydromagnetic heat transfer by mixed convection from an inclined continuous stretching surface with internal heat generation/absorption., Int. J.Termal Sciences, V.43(7), pp.709-719 (2004).

- Faces, N and Faroup, B. ASME, J. Heat transfer, v. 105, p. 680 (1983).
- Jha,B.K:Free convection flow through an annular porous medium., heat Mass Transfer, [9]. V.41, pp.675-679(2005).
- [10]. Kumar,D and Singh,A.K:Effect of induced magnetic field on natural convection with Newtonian heating/cooling in vertical concentric annuli., Procedia Eng., V.127, pp.568-574(2015)
- Leppinen, D.M., Pop, I.Rees, D.A and Storesletten, L: Free convection in a shallow annular cavity filled with a porous medium., J.Porous Media, V.7, pp.929-938(2004).
 Mallikarjuna, B, Ali J Chamkha and R, Bhuvana Vijaya: Soret and Dufour effects on
- double diffusive convective flow through a non-Darcy porous medium in a cylindrical annular region in the presence of heat sources., J. Porous Media, V.17(7), pp.623-636(2014)
- [13]. Madhusudha Reddy, P: Mixed convective heat and mass transfer flow through a porous medium in channel with Radiation effect. Ph. D. Thesis (2010).
- [14]. Neeraja G : Ph. D thesis, S.P. Mahila University, Tirupathi, India (1993).
- [14] Foreigu G. Lin, Darosty, J. Mullin, M.G. Robilard and Vasseur, P. ASME, The American Society of Mechanical Engineers, Paper. No: 85–WA/HT–8, New York (1985).
 [16]. Poots, G.: Int. J. Heat and mass transfer, v. 3, p. (1961).
- 7]. Prasad V, and Kulacki, F.A : Int. J. Heat and mass transfer, v. 27, p. 207 (1984). [18]. Prasad, M.V.S.N. : Hydromagnetic mixed convective heat and mass transfer flow through a porous medium in channels, pipes with heat sources. Ph.D Thesis,
- S.V.University, Tirupati, pp:73-93 (2009) [19]. Prasad, V. Kulacki, F.A and Keyhani, M : Natural convection in a porous media., J. Fluid Mech, V. 150, pp. 89-119 (1985).
- [20]. Rahman, M.M., Uddin M.J and Aziz, A: Effects of variable electric conductivity and nonuniform heat source/sink on convective micropolar fluid flow along an inclined flat plate with surface heat flus.,Int.J.Thermal Sciences,V.48,pp.2331-2340(2009)
- [21]. Rani, A : Unsteady convection heat and mass transfer flow through a porous medium in wavy channels, Ph. D thesis S.K. University Anantapur, India (2003). [22]. Saravannan, S and Kandaswamy, P:non-darcian thermal stability of a heat generating
- fluid in a porous annulus., Int.J.Heat Mass transfer, V.46, pp.4863-4875 (2003)
- [23]. Sastry, V. U.K and Bhadram, C.V.V: APP, Sci. Res, v. 34, 213, p. 117 (1978).
 [24]. Shaarawi, EL. M.A. I and AL. Nimir, M.A: Fully developed laminar natural convection in open ended vertical concentric annuli, Int. J. Heat and mass transfer pp. 1873-1884 $(19\hat{9}9)$
- [25]. Singh,S.K Jha,B.K and Singh,A.K: Natural convection in vertical concentric annuli under a radial magnetic field., Heat Mass Transfer, V.32, pp.399-401(1997)
- [26]. Sudarsana Reddy,P, Sreenivas,G, Sreenivasa Rao,P and Prasada Rao, D.R.V: Non-Darcy mixed convective heat and mass transfer flow of a viscous electrically conducting fluid through a porous medium in a circular annulus in the presence of temperature dependent heat sources with Soret and Dufour effects-A-finite element study.Int.J.Comp and Appl.Math,V,5(5),pp.595-609(2010).
 [27]. Whitehead, J.A.: Observations of rapid means flow produced mercury by a moving
- heater, Geophys Fluid dynamics, v. 3, pp. 161-180 (1972).