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Communication

A STUDY ON GAME THEORY AND IN COMMUNICATION NETWORKS

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ABSTRACT Due to its capability to solve the Situations of Conflict and competition, Game Theory has been used as a mathematical tool in economics , politics, biology human and psychology. Nash equilibrium, being the solution of a non cooperative game, gives a stable state in a sense that no agent have any positive incentive to deviate from its current adopted strategy, when all other players of the game stick to their current moves. To Cope with the selfish and competitive behaviour of the network entities , game theory provides a feasible solution for resource utilization and service provisioning. This paper presents the detailed overview of the game theory concepts and its applications in the communications networks, both from cooperative and non-cooperative.

KEYWORDS : Game Theory , Nash Equilibrium, cooperative games, Non-cooperative games and communication networks.

1.INTRODUCTION

Game theory is the study of mathematical models which are used in a strategic when multiple entities interact with each other in a strategy setup. Game Theory took a revolutionary leap when john Nash Presented the models for non-cooperative games in 1951.his proposed solution for non-cooperative games ,later on called Nash Equilibrium is still considered as a standard for any conflicting situation's outcome. Any Game , when played, consists of the participants called players or agents of the game, each having his own preference or goal. For each player of the game, the choices available to them are called strategies.table1 where two players P_1 and P_2 come in a strategic interaction to play this game. For each, the game is represented by a matrix called payoff matrix.if P_1 player chooses X,then his payoff depends upon which choice the player if P_2makes.if P_1choose X and P_2 also chooses X, if P_1 payoff will be2,otherwise 10.similarly,if P_1 chooses the Y,Then Depending Upon the choices of P_2he will happen for player P_2 as well. here in the game, the players are P_1 and P_2 ,the strategies for each are X and Y and the benefits or outcome of the game are represented in the form of matrix where each entry the payoffs in the form (payoff of P_1, payoff of P_1)duple. The numerical outcome for each player depends on the utility function used by each in the situation in which this game is played.

	P_1	P_2
	X	Y
X	(3,9)	(9,2)
Y	(2,9)	(4,4)

II. INTRODUCTION

Prisoners 'Dilemma

The well known game of prisoner's dilemma was presented by professor tucker of Princeton university in 1950.this game depicts an imaginary situation where two person are arrested under the suspicion of their involvement in a crime .the police place both the suspects in separate rooms so they are not able to communicate with each other during the interrogation process. Each suspect is informed of the following payoffs based on their strategies separately:

- if both suspects confess the crime, each will serve in jail for 7 years .
- If both deny the crime , both will serve in jail for 5 years.
- If one confees and the other deny, the confessor will be set free and the denier will be sent to jail for 13 years.

The choices available to the suspects and their corresponding outcomes when they play this game are given in the table2.

	Suspect 1	Suspect 2
	confess	deny
Confess	(7,7)	(0,13)
Deny	(13,0)	(5,5)

the (Deny, Deny) strategy giving both a maximum payoff of 5 years jail sentences. since players of the game are unware of the decision of each other due to no (confess, confess) = 7 years ,(confess, Deny) = 0, (Deny, confess) = 13 , (Deny, Deny) = 5 years. in the game above ,suspect 1 will always go for confess, irrespective of suspect 2 decisions ,similarly, in the prospective of suspect 2 ,confess in the dominant strategy, irrespective of suspect 1's adopted strategies. Assuming that both players are rational ,the solution of this game is (confess, confess) i.e.,both players play their dominant strategies.

B.Battle of the Sexes

Another famous game in the field of game theory is called the battle of the sexes and was introduced by R. Duncan Luce .The game is played between and both have to decide between two independent and simultaneous accruing events to attend .the assumes that there are two events , a hockey match and a dance, and both the husband and wife have different payoffs from each .the payoff matrix and the strategy set for each player is shown in the table 3.

both player's prospective. However, husband prefers hockey more than dance based on his utility function while the wife has high preference for music based on her

	Husband	Wife
Husband	(2,4)	(0,0)
Wife	(0,0)	(4,2)

Utility function .In this particular game, there is no dominant strategy for both players and hence the solution space is either (hockey, hockey) or (dance, dance).This particular examples shows that a game might have more than one solutions i.e., multiple nash equilibrium.

III .SOLUTION CONCEPTS IN GAME THEORY

A. Nash Equalibrium

Definition: Nash Equilibrium for any game is the set of strategies of all players, called the strategy Profile, where no player can increase its payoff by changing his current strategy, assuming that all players keep their current strategies intact. Mathematically, a Nash equilibrium of a game is the strategy profile A of all players such that :

$$U_i(A_i, A_{-i}) \geq U_i(A'_i, A_{-i}) \geq \forall A'_i \in A \qquad (1)$$

Where A_i is the current strategy of player I against all other strategies of other (A_{-}). A'_i are all other strategies of player i. In the examples given in the sections game.

B.Pareto Efficiency

make at least one person better off without making anyone worse off. Pareto efficiency is the measure the performance a game outcome. Mathematically a strategy profile A will be pareto Efficient if and only if there is no other profiles $A'_i \in A'_j$, such that for any players I,j:

$$U_i(A'_i) \geq U_i(A_i) \geq \forall\, i \in N, \quad \forall A'_i \in A \qquad (2)$$

And for any j € N, for any A'j € A :

$$U_i(A'_i) \leq U_i(A_i) \qquad (3)$$

A.Pure And Mixed Strategy Nash Equilibrium

When players are playing a game, the strategies can be pure and mixed in a pure strategy game ,all the players are taking moves in discrete values refer to the example game in the section III(A)a game in the table4,with two players P_1 And P_2 .The Set of strategies for both{X,Y}.Let us assume that player P_1 Picks X With probability of 0.8 times and Y with a probability of 0.2 times. It is assumed that player P_2 play with pure strategies ,either Xor Y for this game ,the player P_1

will have an expected payoff for playing X or Y in terms of player P_1's randomization .i.e.,:

Expected payoff of P_2 for playing X=[(2*0.8)+(4*0.2)=2]and for playing Y=[(4*0.8)+(2*0.2)=2]

	X	Y
	P_1	P_2
X(probability)=0.8	(2,2)	(5,6)
Y(probability)=0.2	(5,6)	(9,8)

The mixed strategy are normally used in the repeated where players know the history of each other's preference over the strategy set. There might be situation that a player plays with mixed strategy when he is indifferent towards his all pure strategies or when the game is of a pure guess or when the players can guess the next move of each other. There might be situation for a player to play with mixed strategies. There might be situation where a pure strategy game does not converge to the Nash Equilibrium. The mixed strategy games always have a Nash Equilibrium solution.

In order to calculate the expected utilities in mixed strategies applied by the players of the battle of sexes game, lets assume that the women want to music and hockey events equally likely as shown in the table 5.for this assumption .let the husband want to randomize his moreover his pure strategies by 1/5 and2/5.he will want 1/5nof the time to go to the hockey event.th expected utilities of wife in terms of the mixed strategies of her husband can be calculated as follows:

$$E(U)_F^W = (\text{Hockey}, \emptyset)$$

$$= [(,\emptyset * payoff_h^W) + (1-\emptyset) * payoff_h^W] \quad (4)$$

$$= (\text{Dance}, \emptyset)$$

$$= [(\emptyset * payoff_d^W) + (1-\emptyset) * payoff_d^W] \quad (5)$$

Where $E(U)_D^W$ IS the expected value of wife's payoff when the husband is going to the music event 1/3($\emptyset$) times in the game . Similarly $E(U)_H^W$ is the expected payoss of wife when the husband is going (1- $\emptyset$) times to the football match. Payoff $\square_d^W$ is the wife payoff in pure strategies when her husband is opting for music event .similarly ,the wife can also randomize her moves over her pure strategy and husband can derive his expected value of utility from the game of mixed strategies.

	Hockey(4/5)	Dance(1/5)
Hockey(1/5)	(2,4)	(4,2)
Dance(4/5)	(0,0)	(0,0)

Table5: battle of the sexes with mixed strategy

IV. Games types

A.non cooperative and cooperative games

In non cooperative games, each participants player acts in his own interest and the unit of analysis is always the individual player instead of group of player in these types of games, the players are always selfish-i.e., they always try to increase their own individual payoffs without taking care of other player's payoffs in the game .so, non cooperative game theory studies the competitive nature of individual Players where come into contact with the sole aim to increase their own benefits from the strategic situation.

In cooperative games, the group of players are the unit of analysis and the players tend to increase their group payoffs as well as their own. a cooperative games can be considered as a competition among the groups in a game rather than individual players . The application of cooperative game theoretical models are in the situations where players form groups ,called coalitions ,and the individual or group of player's contribution towards the game depends on the actions of their agents in the game.

Most of the problems in communication networks have been modeled as Non cooperative games, where each node is considered to be a selfish self maximize without taking care of the benefit of other nodes in a conflicting situation .however, there are some studies where the coalitations games have been modelled to study the individual nodes behaviour in a network each contribution to a coaliation.

VI. CONCLUSION

This paper presents a detailed study of game theory and the associated concepts the classification of games based on the information ,rules, moves and rationality has been discussed .the two famous games in the literature of game theory, prisoner's dilemma and the battle of sexes have been investigated in detail with the associated concepts of pure and mixed strategies. Nash equilibrium along with pareto efficiently has been covered in details along with their properties . Finally , the paper gives an insight to the use of game theory in the problem of communication networks both from cooperative and non-cooperative perspectives.

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