

INTRODUCTION

The concept of fuzzy sets was introduced by Zadeh [11] and later Atanassov [1] generalized this idea to intuitionistic fuzzy sets using the notion of fuzzy sets. On the other hand Coker [4] introduced intuitionistic fuzzy topological spaces using the notion of intuitionistic fuzzy sets. Dontchev [5] has introduced contra continuous functions in topological spaces. Erdal Ekici and Etienne [6] have introduced fuzzy contra continuities in fuzzy topological spaces. In this paper, we introduced intuitionistic fuzzy contra β generalized continuous mappings and studied some of their basic properties. We arrived at some characterizations of intuitionistic fuzzy contra β generalized continuous mappings.

2 PRELIMINARIES

Definition 2.1: [1] Let X be a non empty fixed set. An intuitionistic fuzzy set (IFS in short) A in X is an object having the form $A = \{x, \mu_A(x), \nu_A(x) / x X\}$ where the functions $\mu_A(x)$: X [0, 1] and $\nu_A(x)$: X [0, 1] denote the degree of membership (namely $\mu_A(x)$) and the degree of non -membership (namely $\nu_A(x)$) of each element x X to the set A, respectively, and $0 \le \mu_A(x) + \nu_A(x) \le 1$ for each x. Denote the set of all intuitionistic fuzzy sets in X by IFS (X).

Definition 2.2: [1] Let A and B be IFS's of the form

A= { $x, \mu_A(x), \nu_A(x) / x X$ } and

 $B = \{x, \mu_B(x), \nu_B(x) / x X\}.$ Then

(a) A B if and only if $\mu_A(x) \le \mu_B(x)$ and $\nu_A(x) \ge \nu_B(x)$ for all x X,

(b) A=B if and only if A B and B A,

(c) $A^{c} = \{x, v_{A}(x), \mu_{A}(x) / x X\},\$

(d) A B = { $x, \mu_A(x) \ \mu_B(x), \nu_A(x) \ \nu_B(x) / x \ X },$

(e) A B = { x, $\mu_A(x)$ $\mu_B(x)$, $\nu_A(x)$ $\nu_B(x)$ /x X }.

For the sake of simplicity, we shall use the notation $A = x, \mu_A, \nu_A$ instead of $A = \{x, \mu_A(x), \nu_A(x) / x X\}$. Also for the sake of simplicity, we shall use the notation $A = x, (\mu_A, \mu_B), (\nu_A, \nu_B)$ instead of $A = x, (A/\mu_A, B/\mu_B), (A/\nu_A, B/\nu_B)$. The intuitionistic fuzzy sets $0 = \{x, 0, 1 / x X\}$ and $1 = \{x, 1, 0 / x X\}$ are respectively the empty set and the whole set of X.

Definition 2.3: [4] An intuitionistic fuzzy topology (IFT in short) on a non empty X is a family τ of IFS in X satisfying the following axioms: (a) $0_{-}, 1_{-} \tau$

(b) $G_1 G_2 \tau$, for any $G_1, G_2 \tau$

(c) $G_i \tau$ for any arbitrary family $\{G_i/iJ\} \tau$.

In this case the pair (X, τ) is called an intuitionistic fuzzy topological space (IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS for short) in X.

The complement A° of an IFOS A in an IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS for short) in X.

Definition 2.4: [4 Let (X, τ) be an IFTS and $A = x, \mu_A, \nu_A$ be an IFS in X. Then the intuitionistic fuzzy interior and an intuitionistic fuzzy closure are defined by int(A) = {G/G is an IFOS in X and G A},

 $cl(A) = \{K/K \text{ is an IFCS in } X \text{ and } A K \}.$

Note that for any IFSA in (X, τ) , we have $cl(A^c) = [int(A)]^c$ and $int(A^c) = [cl(A)]^c$.

Definition 2.5:[7] An IFS $A = \langle x, \mu_A, \nu_A \rangle$ in an IFTS (X, τ) is said to be a

- intuitionistic fuzzy semi closed set (IFSCS for short) if int(cl(A)) ⊆ A,
- (ii) intuitionistic fuzzy pre-closed set (IFPCS for short) if cl(int(A)) ⊆ A,
- (111) intuitionistic fuzzy α -closed set (IF α CS for short) if $cl(int(cl(A))) \subset A$,
- (iv) intuitionistic fuzzy γ -closed set (IF γ CS for short) if $cl(int(A)) \cap int(cl(A)) \subseteq A$.

The respective complements of the above IFCSs are called their respective IFOSs.

The family of all IFSCSs, IFPCSs, IF α CSs and IF β CSs (respectively IFSOSs, IFPOSs, IF α OSs and IF γ OSs) of an IFTS (X, τ) are respectively denoted by IFSC(X), IF α C(X), IF α C(X) and IF γ C(X) (respectively IFSO(X), IF α O(X), IF α O(X) and IF β O(X)).

Definition 2.6:[8] An IFS A in an IFTS (X, τ) is an intuitionistic fuzzy generalized closed set (IFGCS in short) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in X.

Definition 2.6:[8] An IFS A in an IFTS (X, τ) is an intuitionistic fuzzy generalized closed set (IFGCS in short) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in X.

Definition 2.7:[7] An IFS A in an IFTS (X, τ) is said to be an intuitionistic fuzzy generalized semi closed set (IFGSCS in short) if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in (X, τ) .

Definition 2.8:[8] An IFS A is said to be an intuitionistic fuzzy generalized semi open set (IFGSOS in short) in X if the complement A^c is an IFGSCS in X.

The family of all IFGSCSs (IFGSOSs) of an IFTS (X, τ) is denoted by IFGSC(X) (IFGSO(X)).

Definition 2.9:[7] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be intuitionistic fuzzy continuous (IF continuous in short) if $f^{-1}(B) \in IFO(X)$ for every $B \in \sigma$.

Definition 2.10: [7] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ) . Then f is said to be an

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- (i) intuitionistic fuzzy semi continuous mapping (IFS continuous mapping for short) if $f^{-1}(B) \in IFSO(X)$ for every $B \in \sigma$
- (ii) intuitionistic fuzzy α -continuous mapping (IF α continuous mapping for short) if $f^{-1}(B) \in IF\alpha O(X)$ for every $B \in \sigma$
- (iii) intuitionistic fuzzy pre continuous mapping (IFP continuous mapping for short) if $f^{-1}(B) \in IFPO(X)$ for every $B \in \sigma$
- (iV) intuitionistic fuzzy γ continuous mapping (IF γ continuous mapping for short) if $f^{-1}(B) \in IF\gamma O(X)$ for every $B \in \sigma$.

Definition 2.11: [10] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an intuitionistic fuzzy generalized continuous mapping (IFG continuous mapping for short) if f ⁻¹(B) \in IFGC(X) for every IFCS B in Y.

Definition 2.12:[10] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an intuitionistic fuzzy semi-pre continuous mapping (IFSP continuous mapping for short) if f⁻¹(B) \in IFSPO(X) for every B $\in \sigma$.

Result 2.13:[9]Every IF continuous mapping is an IFG continuous mapping.

Definition 2.14:[8]A mapping f: $(X, \tau) \rightarrow (Y,\sigma)$ is called an *intuitionistic fuzzy generalized semi continuous* (IFGS continuous in short) if f⁻¹(B) is an IFGSCS in (X, τ) for every IFCS B of (Y, σ) .

Definition 2.15:[8]An IFS A is said to be an intuitionistic fuzzy alpha

generalized open set (IF α GOS in short) in X if the complement A^c is an IF α GCS in X.

The family of all IF α GCSs (IF α GOSs) of an IFTS (X, τ) is denoted by IF α GC(X) (IF α GO(X)).

Definition 2.16:[7] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be intuitionistic fuzzy continuous (IF continuous in short) if $f^{-1}(B) \in IFO(X)$ for every $B \in \sigma$.

Definition 2.17:[3] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ) . Then f is said to be an intuitionistic fuzzy contracontinuous (IF C continuous in short) mapping if $f^{-1}(B)$ is an IFOS in X for each IFCS B in Y.

Definition 2.18: [3] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ) . Then f is said to be an

- (1) intuitionistic fuzzy contra continuous mapping (IFC continuous mapping for short) if f⁻¹(B)∈IFO(X) for each IFCS B in Y
- (ii) intuitionistic fuzzy contra α -continuous mapping (IFC α -continuous mapping for short) if f⁻¹(B) \in IF α O(X) for each IFCS B in Y
- (iii) intuitionistic fuzzy contra pre-continuous mapping (IFCP continuous mapping for short) if f⁻¹(B)∈IFPO(X) for each IFCS B in Y
- (iV) intuitionistic fuzzy contra γ-continuous (IFCγ continuous for short) mapping if f⁻¹(B) is an IFγOS in X for each IFCS B in Y.

Result 2.19: [3] Every IF contra-continuous mapping is an IFC α continuous mapping and IFCP continuous mapping.

Definition 2.20: [8] An IFTS (X, τ) is said to be an intuitionistic fuzzy $\beta a T_{1/2}$ (IF $\beta a T_{1/2}$ in short) space if every IF β GCS in X is an IFCS in X.

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3. Intuitionistic fuzzy contra β generalized continuous mappings In this section we have introduced intuitionistic fuzzy contra β generalized continuous mappings. We investigated some of its properties.

Definition 3.1: A mapping : X Y is said to be an intuitionistic fuzzy contra β generalized continuous mapping (IFC β G continuous mapping) if ⁻¹(A) is an IF β GOS in X for every IFCS A in Y.

Example 32:

Let $X = \{a, b\}, Y = \{u, v\}$ and $G_1 = x, (0.1, 0.1), (0.5, 0.6), G_2 = y, (0.3, 0.1), (0.5, 0.6)$. Then $\tau = \{0_{-}, G_{1, 1_{-}}\}$ and $= \{0_{-}, G_{2, 1_{-}}\}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau)$ (Y, by f(a) = u and f(b) = v. Then f is an IFC β G continuous mapping.

Theorem 3.3: Every IF contra continuous mapping is an IFC β G continuous mapping but not conversely.

Proof: Let $f: (X, \tau)$ (Y,) be an IF contra continuous mapping. Let A be an IFCS in Y. By hypothesis, $f^{-1}(A)$ is an IFOS in X. Since every IFOS is an IF β GOS, $f^{-1}(A)$ is an IF β GOS in X. Hence f is an IFC β G continuous mapping.

Example 3.4: Let $X = \{a, b\}$, $Y = \{u, v\}$ and $G_1 = x$, (0.2, 0.1), (0.5, 0.6), $G_2 = y$, (0.3, 0.1), (0.4, 0.3). Then $\tau = \{0_{-}, G_{1,1_{-}}\}$ and $= \{0_{-}, G_{2,1_{-}}\}$ are IFTs on X and Y respectively. Define a mapping f: (X, τ) (Y,) by f(a) = u and f(b) = v. Then f is an IFC β G continuous mapping. But f is not an IF contra continuous mapping since $G_2^{\circ} = y$, (0.4, 0.3), (0.3, 0.1) is an IFCS in Y but f⁻¹(G_2°) = x, (0.4, 0.3), (0.3, 0.1) is not an IFOS in X.

Theorem 3.5: Every IFC continuous mapping is an IFC β G continuous mapping but not conversely.

Proof: Let $f: (X, \tau)$ (Y,) be an IFC continuous mapping. Let A be an IFCS in Y. Then by hypothesis, f-1(A) is an IFOS in X. Since every IFOS is an IF β GOS, f-1(A) is an IF β GOS in X. Hence f is an IFC β G continuous mapping.

Example 3.6: Let $X = \{a, b\}$, $Y = \{u, v\}$ and let the IFS $G_1 = x$, (0.3, 0.1), (0.5, 0.6), $G_2 = x$, (0.7, 0.7), (0.1, 0.1) and $G_3 = y$, (0.4, 0.5), (0.3, 0.3). Then $\tau = \{0_{-1}, G_{-1}, 1_{-2}\}$ and $= \{0_{-2}, G_{-1}, 1_{-2}\}$ are IFTs on X and Y respectively. Define a mapping $f: (X, \tau)$ (Y,) by f(a) = u and f(b) = v. Then f is an IFC β G continuous mapping But f is not an IFC continuous mapping since $G_3^c = y$, (0.3, 0.3), (0.4, 0.5) is an IFCS in Y but $f^{-1}(G_3^c) = x$, (0.3, 0.3), (0.4, 0.5) is not an IFOS in X.

Remark 3.7: IFC continuous mapping and IFC β G continuous mapping are independent of each other.

Example 3.8: Let $X = \{a, b\}, Y = \{u, v\}$ and $G_1 = x, (0.4, 0), (0.4, 0.8), G_2 = y, (0.3, 0.2), (0.5, 0.1)$. Then $\tau = \{0_{-3}, G_{1,}1_{-}\}$ and $= \{0_{-3}, G_{2,}1_{-}\}$ are IFTs on X and Y respectively. Define a mapping $f: (X, \tau)$ (Y, σ) by f(a) = u and f(b) = v. Then f is an IFC continuous mapping. But f is not an IFC β G continuous mapping since $G_2^{c} = y, (0.5, 0.1), (0.3, 0.2)$ is an IFCS in Y but $f^{-1}(G_2^{c}) = x, (0.5, 0.1), (0.3, 0.2)$ is not an IF β GOS in X.

Example 3.9: Let $X = \{a, b\}, Y = \{u, v\}$ and $G_1 = x, (0.4, 0), (0.4, 0.8), G_2 = y, (0.6, 0.7), (0.1, 0.1). Then <math>\tau = \{0_{-}, G_{1,1_{-}}\}$ and $= \{0_{-}, G_{2,1_{-}}\}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau)$ (Y,) by f(a) = u and f(b) = v. Then f is an IFC β G continuous mapping. But f is not an IFC continuous mapping, since $G_2^{e} = y, (0.1, 0.1), (0.6, 0.7)$ is an IFCS in Y but $f^{-1}(G_2^{e}) = x, (0.1, 0.1), (0.6, 0.7)$ is not an IFOS in X.

Remark 3.10: IFCP continuous mapping and IFC β G continuous mapping are independent of each other.

Example 3.11: Let $X = \{a, b\}, Y = \{u, v\}$ and $G_1 = x, (0, 0.8), (0.4, 0), G_2 = y, (0, 0.2), (0.7, 0.7). Then <math>\tau = \{0_{-}, G_{1,1}\}$ and $= \{0_{-}, G_{2,1}\}$ are IFTs on X and Y respectively. Define a mapping $f: (X, \tau)$ (Y,) by f(a) = u and f(b) = v. Then f is an IFCP continuous mapping. But f is not an IFC β G continuous mapping since $G_2^{\circ} = y, (0.7, 0.7), (0, 0.2)$ is an IFCS in Y but $f^{-1}(G_2^{\circ}) = x, (0.7, 0.7), (0, 0.2)$ is not an IFGOS in X.

Example 3.12: Let $X = \{a, b\}, Y = \{u, v\}$ and $G_1 = x, (0.2, 0.2), (0.5, 0.6), G_2 = y, (0.3, 0.2), (0.4, 0.5). Then <math>\tau = \{0_{-}, G_{1,1}\}$ and $= \{0_{-}, G_{2,1}\}$ are IFTs on X and Y respectively. Define a mapping $f: (X, \tau)$ (Y, σ) by f(a)

= u and f(b) = v. Then f is an IFC β G continuous mapping. But f is not an IFCP continuous mapping since $G_2^c = y, (0.4, 0.5), (0.3, 0.2)$ is an IFCS in Y but $f^{-1}(G_2^{\circ}) = x, (0.4, 0.5), (0.3, 0.2)$ is not an IFPOS in X.

Theorem 3.13: A mapping f: X Y is an IFC β G continuous if and only if the inverse image of each IFOS in Y is an IF β GCS in X.

Proof: Necessity: Let A be an IFOS in Y. This implies Ac is an IFCS in Y. Since f is an IFC β G continuous mapping, $f^{(A)}$ is an IF β GOS in X. Hence f is an IFC β G continuous mapping.

Theorem 3.14: Let $f: (X, \tau)$ (Y, σ) be a mapping and let $f^{-1}(A)$ be an IFROS in X for every IFCS A in Y. Then f is an IFC β G continuous mapping.

Proof: Let A be an IFCS in Y. Then by hypothesis, $f^{-1}(A)$ is an IFROS in X. Since every IFROS is an IF β GOS, f⁻¹(A) is an IF β GOS in X. Hence f is an IFC β G continuous mapping.

Theorem 3.15: Let $f: (X, \tau)$ (Y, σ) be an IFC β G continuous mapping, then f is an IF contra continuous mapping if X is an IF β aT_{1/2}space.

Proof: Let A be an IFCS in Y. Then f - 1(A) is an IF β GOS in X, by hypothesis. Since X is an IF β aT_{1/2} space f⁻¹(A) is an IFOS in X. Hence f is an IF contra continuous mapping.

Theorem 3.16: If $f: (X, \tau)(Y, \sigma)$ is an IFC β G continuous mapping and $g: (Y, \sigma) (Z, \eta)$ is an IF contra continuous mapping, then $g \circ f: (X, \tau)$ (Z, η) is an IF β G continuous mapping.

Proof: Let A be an IFCS in Z. Then $g^{-1}(A)$ is an IFOS in Y, by hypothesis. Since f is an IFC β G continuous mapping, f'(g'(A)) is an IF β GCS in X. That is (g o f)⁻¹(A) is an IF β GCS in X. Hence g o f is an IF β G continuous mapping.

Theorem 3.17: If $f: (X, \tau)$ (Y, σ) is an IF β G continuous mapping g: $(Y, \sigma)(Z, \eta)$ is an IF contra continuous mapping, then g o f: $(X, \tau)(Y, \eta)$ σ) is an IFC β G continuous mapping.

Proof: Let A be an IFCS in Z. Then $g^{-1}(A)$ is an IFOS in Y, by hypothesis. Since f is an IF β G continuous mapping, f¹(g⁻¹(A)) is an IF β GCS in X. That is (g o f)⁻¹(A) is an IF β GCS in X. Hence g o f is an IFC β G continuous mapping.

Theorem 3.18: If $f: (X, \tau)$ (Y, σ) is an IFC β G continuous mapping g: (Y, σ) (Z, η) is an IF continuous mapping, then g o f: (X, τ) (Z, η) is an IF β G continuous mapping.

Proof: Let A be an IFCS in Z. Then $g^{-1}(A)$ is an IFCS in Y, by hypothesis. Since f is an IFC β G continuous mapping, f'(g'(A)) is an IF β GCS in X. That is $(g \circ f)^{-1}(A)$ is an IF β GCS in X. Hence g o f is an IFC β G continuous mapping.

Theorem 3.19: Let $f: (X, \tau)$ (Y, σ) and $g: (Y, \sigma)$ (Z, δ) be a mapping. Then the following conditions are equivalent if X is an IF β aT_{1/2} space.

- (I) gofisan IFC β G continuous mapping
- (ii) $cl(int(cl(g f)^{-1}(B))) \subseteq (g f)^{-1}(B)$ for every IFOS B in Z.

Proof: (I) (ii): Let B be any IFOS in Z. Then (g f)-1(B) is an IF β GCS in X by hypothesis. Since X is an IF β aT_{1/2} space, (g f)¹(B) is an IFCS in X. Therefore, $cl((g \circ f)^{-1}(B)) = (g \circ f)^{-1}(B)$. Now $cl(int(cl((g \circ f)^{-1}(B)))) \subseteq cl(cl$ $(cl((g \circ f)^{-1}(B)))) = cl((g \circ f)^{-1}(B))$. This implies $cl(int(cl((g \circ f)^{-1}(B))))) \subseteq (g \circ f)^{-1}(B))$ $f)^{-1}(B).$

(ii) (I): Let B be an IFCS in Z. Then its complement B^c is an IFOS in Z. By hypothesis $cl(int(cl((g \circ f)^{-1}(B^{\circ})))) \subseteq (g \circ f)^{-1}(B^{\circ})$. Hence $(g \circ f)^{-1}(B^{\circ})$ is an IFCS in X. Since every IFCS is an IF β GCS (g o f)⁻¹(B^e) is an IF β GCS in X. Therefore, (g f)-1(B) is an IF β GOS in X. Hence g of is an IFC β G continuous mapping.

Theorem 3.20: Let $f : (X, \tau) \rightarrow (Y, \sigma)$ and $g : (Y, \sigma) \rightarrow (Z, \eta)$ be a mapping. Then the following conditions are equivalent if X is an IF β aT_{1/2} space.

(i) g o f is an IFC β G continuous mapping.

(ii) $(g \circ f) - 1(B) \subseteq int(cl(int((g \circ f) - 1(B))))$ for every IFCS B in Z.

Proof: (I) (ii): Let B be an IFCS in Z. By hypothesis, $(g \circ f)^{-1}(B)$ is an IF β GOS in X. Since X is an IF β aT_{1/2} space, (g o f)⁻¹(B) is an IFOS in X. Therefore, $(g \circ f)^{-1}(B) = int((g \circ f)^{-1}(B))$. But $int((g \circ f)^{-1}(B)) \subseteq int(cl(int((g \circ f)^{-1}(B))))$ f)⁻¹(B)))). This implies (g o f)⁻¹(B) int(cl(int((g o f)⁻¹(B)))) for every IFCS B inZ.

(ii) (I): Let B be an IFCS in Z. By hypothesis $(g \ f)^{-1}(B) \subseteq int(cl(int)(g \ f))^{-1}(B)$ $f^{-1}(B)$)). This implies $(g f)^{-1}(B)$ is an IFOS in X and hence $(g f)^{-1}(B)$ is an IF β GOS in X. Therefore f is an IFC β G continuous mapping.

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