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Statistics

CONSTRUCTION OF NEW SERIES OF SECOND ORDER ROTATABLE DESIGNS

Kanna E. Department Of Statistics, University College Of Science, Osmania University, Hyderbad-7.

Ameen Saheb Sk. School Of Mathematics And Statistics, University Of Hyderabad, Hyderabad-500046, India.

Bhatra Charyulu, N. Ch.* Department Of Statistics, University College Of Science, Osmania University, Hyderbad-7. *Corresponding Author

ABSTRACT A new series of Second Order Rotatable Design is constructed using Balanced Ternary Design of Bhatra charyulu (2006). The existence of the method is proved and illustrated with a suitable example.

KEYWORDS : Balanced Ternary Design, Second Order Rotatable Designs.

1. INTRODUCTION

Let $\underline{Y}$ be the vector of 'N' responses observed on the combination of 'v' factors with specified levels, where x_{ui} be the level of i^{th} factor in the u^{th} factors combination (i = 1, 2, ... v; u = 1, 2 … N). Assume each factor has 's' levels. The average mathematical relationship between factors and response, i.e. $E(Y_u) = f(x_{u1}, x_{u2}, \dots, x_{uv})$ is called the response surface. If the functional form of the response surface is polynomial of degree two, i.e. second order response surface design model, at the u^{th} design point is

$$Y_u = \beta_0 + \sum_{i=1}^{v} \beta_i x_{ui} + \sum_{i=1}^{v} \beta_{ii} x_{ui}^2 + \sum_{i<j=1}^{v} \beta_{ij} x_{ui} x_{uj} + \varepsilon_u \qquad (1.1)$$

where, Y_u is the response at the u^{th} design point, $\underline{\beta} = (\beta_0, \beta_1, \beta_2, \dots \beta_v, \beta_{11}, \beta_{22}, \dots \beta_{vv}, \beta_{12}, \dots \beta_{v-1v})'$ be the vector of parameters and ε_u be the random error corresponding to Y_u.

The estimated values for the parameters and responses can be obtained using least squares method as $\boldsymbol{\beta}\hat{} = (XX)^{-1}XY$ and $Y\hat{} = X(XX)^{-1}XY$. The variance of the estimated response is $V(Y\hat{}) = (XX)^{-1} \sigma^2$. Design matrix X is said to be orthogonal, if its XX matrix is a diagonal matrix. But, for second order designs a diagonal moment matrix is impossible. When the restrictions are imposed on the moment matrix towards reaching to orthogonality, that is, maximum number of off diagonal elements in XX matrix elements making to zero, i.e. δ4ulδ3ukδ2ujδ1uixxxx∑= 0 if any value is odd. Assume $x^2_{ui} = N\lambda_2$; $x^4_{ui} = CN\lambda_4$; $x^2_{ui} x^2_{uj} = N\lambda_4$ and let $^2 = \sum=vi1 x_{ui}^2$ and $=$ [$_4$ (c+v–1) – $v\lambda_2^2$]. Then the variance of estimated response will be in the form

$$V(\hat{Y}_u) = V(\beta_0) + [\, V(\beta_i) + 2\,Cov(\beta_0, \beta_{ii})\,]\, \rho^2 + V(\beta_{ii})\, \rho^4 + \left[\frac{(c-3)}{(c-1)N\lambda_4}\right] \Sigma\, x^2_{ui} x^2_{uj} \qquad (1.2)$$

In particular, Box and Hunter (1957) noted that "Reasons are advanced for preferring designs having a spherical or nearly spherical variance function. Such designs ensure that the estimated response has a constant variance at all points, which have the same distance from the center of the design and designs having this property are called rotatable designs". When c = 3, the term containing $x^2_{ui} x^2_{uj}$ in (2.1.5) vanishes, then the variance of estimated response can be expressed in the form of a function of ρ^2 as

$$V(\hat{Y}_u) = \alpha\, \rho^4 + \beta\, \rho^2 + \gamma \qquad (1.3)$$

Where $\alpha = \frac{\sigma^2}{N\Delta}\left[\frac{\Delta - \lambda_2^2}{\lambda_4(c-1)}\right]$; $\beta = \frac{\sigma^2}{N\Delta}\left[\frac{\Delta - 2\lambda_2^2}{\lambda_2}\right]$; $\gamma = \frac{\sigma^2}{N\Delta}\left[\Delta + v\lambda_2^2\right]$ and $\rho^2 = \sum_{i=1}^{v} x_{ui}^2$

2. NEW SERIES OF SECOND ORDER ROTATABLE DESIGN

In this section, a new series of Second Order Rotatable Design using Balanced Ternary Design Bhatra Charyulu (2006) is constructed and presented with a suitable example. The detailed step by step procedure is presented below.

Step 1: Let N_1 be the incidence matrix of a Balanced Incomplete Block Design with parameters v, b, r, k, such that v > k+1. The size of the block is increased by adding a treatment which is absent in that block twice to that block. The resulting is a Balanced Ternary Designs with parameters V = v, B = b(v-k), R = 2(b-r)+r(v-k), K = k+2 and = (v-k)+4(r-).

Step 2: Replace the elements 2 with α and 1 with β, then associate each block with an appropriate fraction of factorials (say $12k$) with levels 1 such that no lower order interaction effects are confounded.

Step 3: Add n_0 ($n_0 > 0$) central design points (0, 0, … 0) to the resulting design, then total number design points in the respective designs are: $n = (v-k)b + n_0$

Step 4: The levels 'α' and 'β' can be obtained such that $t = {}^2/{}^2$. Where the real roots for t are

$$t = \frac{6(r-\lambda) \pm \sqrt{36(r-\lambda)^2 - 4(b-r)(r-6\lambda)(v-k)}}{2(b-r)}$$

Choose the value for then = t^2. The resulting design provides a v-dimensional Second Order Rotatable Design with five levels (, , 0).

THEOREM 2.1:

A Second Order Rotatable Design exists with five levels (α, β, 0) using a Balanced Ternary Design with parameters V = v, B = b(v-k), R = 2(b-r)+r(v-k), K = k+2 and = (v-k)+4®-).

Proof :

Let N_{BXV} be the incidence matrix of a balanced Ternary Design with parameters V = v, B = b(v-k), R = 2(b-r)+r(v-k), K = k+2 and = (v-k)+4(r-). Each column of the incidence matrix has the elements 2's, 1's and 0's which are repeated b-r, r(v-k) and (v-k-1)(b-r) times respectively. Every pair of columns contains the pairs (2,2), (1,1) and ((2,1) or (1,2)) occurs 0 (zero), (v-k) and 2(r-) times respectively. Replace the elements in N_{BxV} as 1 with β and 2 with α and choose an appropriate fraction of factorials (say $12k$) for v factors, with levels 1. After augmenting n_0 central points, the resulting design has N=b(v-k) + n_0 design points. Then we have,

$$\sum_{u=1}^{N} x^4_{ui} = 2^{k_1} \{(b-r)\alpha^4 + (vr-rk)\beta^4\} \text{ and } \sum_{u=1}^{N} x^2_{ui} x^2_{uj} = 2^{k_1} \{ 2(r-\lambda)\alpha^2 \beta^2 + 2\lambda(v-k)\beta^4\} \qquad (2.1)$$

From the rotatable condition, we obtain

$(b\text{-}r)\,^4 + \{ (vr\text{-}rk)\text{-}6\lambda(v\text{-}k)\}\,^4 \text{-}6(r\text{-}\lambda)\,^{22} = 0$ (2.2)

Let $t = {}^2/{}^2$, then (3.2.20) can be expressed in the quadratic form as

$(b\text{-}r)\, t^2 – 6(r\text{-}\lambda)\, t + (r\text{-}6\lambda)(v\text{-}k) = 0$ (2.3)

$$t = \frac{6(r-\lambda) \pm \sqrt{36(r-\lambda)^2 - 4(b-r)(r-6\lambda)(v-k)}}{2(b-r)}$$

The roots are real if k)(v)6r)(r4(bλ)9(r2−−−≥−λ. Choose any real value for ", and obtain value for " so that $^2 = t^2$. The resulting design provides a v-dimensional Second Order Rotatable Design in five levels.

EXAMPLE 2.1:

Let N_1 be the incidence matrix of a Balanced Incomplete Block Design with parameters v = 4, b = 6, r = 3, k = 2 and λ = 1.adding a treatment twice to the BIBD provides Balanced Ternary Design is with parameters V = 4, B = 12, R = 12, K = 4 and π = 10. The Second Order

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Rotatable Design with four factors is presented in. Table 2.1

Table 2.1 Construction of Second Order Rotatable Design

N_1	N	SORD
$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 1 & 2 & 0 \\ 1 & 1 & 0 & 2 \\ 1 & 2 & 1 & 0 \\ 1 & 0 & 1 & 2 \\ 1 & 2 & 0 & 1 \\ 1 & 0 & 2 & 1 \\ 2 & 1 & 1 & 0 \\ 0 & 1 & 1 & 2 \\ 2 & 1 & 0 & 1 \\ 0 & 1 & 2 & 1 \\ 2 & 0 & 1 & 1 \\ 0 & 2 & 1 & 1 \end{bmatrix}$	$\begin{bmatrix} \pm\beta & \pm\beta & \pm\alpha & 0 \\ \pm\beta & \pm\beta & 0 & \pm\alpha \\ \pm\beta & \pm\alpha & \pm\beta & 0 \\ \pm\beta & 0 & \pm\beta & \alpha \\ \pm\beta & \pm\alpha & 0 & \pm\beta \\ \pm\beta & 0 & \pm\alpha & \pm\beta \\ \pm\alpha & \pm\beta & \pm\beta & 0 \\ 0 & \pm\beta & \pm\beta & \pm\alpha \\ \pm\alpha & \pm\beta & 0 & \pm\beta \\ 0 & \pm\beta & \pm\alpha & \pm\beta \\ \pm\alpha & 0 & \pm\beta & \pm\beta \\ 0 & \pm\alpha & \pm\beta & \pm\beta \end{bmatrix}$

The Second Order Rotatable Design is existed for t= 4.4, a can be evaluated by choosing b arbitrarily.

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