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Music

A MATHEMATICAL STUDY OF RAIN DROP SOUND

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ABSTRACT In this work we first download the recorded sound of drizzling rain and convert them to digits using Octave. Then we analyze this time series formed to interpret the rain data. We find that his data is nonlinear and yet have the similarity of being a linear data. We perform both linear and nonlinear time series analysis here in our work using the computational tool TISEAN. We present our results in this paper.

KEYWORDS : Rain Sound, Time Series, Power Spectrum, Correlation, Lyapunov Spectrum, Arima, False Nearest Neighbors.

INTRODUCTION

When we observe rain we see that it has a specific rhythm and pattern, whenever I used to hear it, it made me more and more inquisitive. In the study of signal analysis we know that there are several linear tools like Fourier series, Laplace Transform, Wavelet Transform, Z Transform etc. Which converts the time domain function to frequency domain function. But when they are plotted and if they have a specific pattern they would just give a single line showing the power spectral density. From Fourier Transform with the sum of the squares of the Fourier coefficients a_n and b_n gives the intensity of the signal. The Power Spectral Density is the square root of this summation. In more of the Literature we observe that the Power Spectral Density plot vs the Frequency plot gives mainly Peaks and Subpeaks representing Harmonics and Sub Harmonics respectively of the data. That shows that the Signal which has been processed contains mainly multiple patterns of these Harmonics and Sub harmonics generated in terms of peak, Harmonic Data point. Beyond this for a Non Linear Data until Fourier we could not analyze further. With the advent of the nonlinear Time Series Analysis tools in recent years people could analyze EEG(Electro Encephalogram Data) to predict Epilepsy Seizures at least around half an hour before the attack. The utility of these tools has given optimism to the medical field as one could try and predict heart attacks. However in our work we try to analyze a simple sound data through these techniques and try to discuss about the pattern so obtained. The tools involved in this work are Audacity, Octave and Tisean. Audacity is a software with which we convert the Mp3 form of the signal to the wave format, Audacity processes this data to 16 bit, 24 bit, 34 bit, 64 bit. The Frequency can be adjusted upto the desired level, that is, we could either get the frequency of 220500 Hz or 441000 Hz of output from Audacity. As a note of caution we mention that even though Audacity converts the data to ASCII we differ from using that as it is not reliable. However this is the simplest tool which is freely available for our Analysis. The Second Tool we used was Octave which is also a freely available software and a substitute for MATLAB. There are Signal Processing commands like audioread and audiowrite which reads this audio as ASCII Data points and write as Audio from ASCII points respectively. We use save commands to save the data in the format required. The save command used saves the ASCII file as the text file and the text file as the audio file. The output of this process is the intensity of the sound normalised and with the frequency of the audacity, that is, suppose we choose 44100 Hz then per second we would eventually have 44100 data points for a one second time noise. We note here that the Mp3 file when converted to wave becomes bulky as the compressions is removed and the wave file gives the exact data of the sound and text file produced will become even more bulkier, Hence we took only first and last pattern for our data analysis. This could be of a maximum of 30 seconds of data.

The final Tool used for our Analysis was TISEAN which is an Acronym for TIme SEries ANalysis. In this app we have commands for both Linear and Non Linear Analysis of the Time Series. For our work we use the following Linear and Non Linear Data Analysis.

1. Stationary Analysis
2. Correlation Analysis
3. Spectral Analysis
4. Estimation of Time delay and Embedding Dimension
5. Arima Model
6. State Space Reconstruction

The Non Linear Analysis includes finding the following
1. Average Mutual Information
2. False Nearest Neighborhood estimation
3. Renyl Entropy
4. Correlation Dimension and Sum
5. Lyapunov Spectrum

With these Analysis we conclude the theory of the sound of the rain data.

Data Preparation

Any Function of Time t is a signal and the conversion of this signal to data is converting the signal to Time Series which is evident as a Signal is a Function of Time. A signal which is periodic and follows the Dirichlet's conditions can be converted in terms of Fourier coefficients using Euler's Formula. That is, If f(x) is periodic between the [c, c+2l] with the period 'l', then the signal f(t) can be expressed as

$f(t) = a_0/2 + \sum_{n=1}^{\infty} a_n \cos(\frac{n\pi t}{l})$, where a_0, a_n, b_n are given by Euler's formula as,

$$a_0 = \frac{1}{l}\int_c^{c+2l} f(x)dx$$

$$a_n = \frac{1}{l}\int_c^{c+2l} f(x)\cos\frac{n\pi x}{l}\,dx$$

$$b_n = \frac{1}{l}\int_c^{c+2l} f(x)\sin(\frac{n\pi x}{l})dx \quad (1)$$

A Detailed observation of this procedure tells us that f(t) is converted into a Product of two Infinite Tuples consisting of Fourier coefficients a_i, b_i for i=0 to ∞ and cos $(\frac{i\pi t}{l})$ and sin $(\frac{i\pi t}{l})$ for i=0 to ∞.

$$f(t) = [a_0, a_1, \ldots.]\begin{bmatrix} \cos\frac{0\pi t}{l} \\ \cos\frac{\pi t}{l} \\ \ldots\ldots\ldots \end{bmatrix} + [b_0, b_1, \ldots.]\begin{bmatrix} \sin\frac{0\pi t}{l} \\ \sin\frac{\pi t}{l} \\ \ldots \end{bmatrix} \quad (2)$$

Lets make a small comparison,
Any vector can be represented as $\vec{a} = xi + yj + zk$

in three dimensions, This is nothing but (x,y,z) $\begin{pmatrix} i \\ j \\ k \end{pmatrix}$

f(t) is a signal in (1)
This when transformed into Fourier coefficients, is, Equation (2), is the product of infinite tuples with respect to their orientation in the signal with unit vectors cos $\frac{i\pi t}{l}$, sin $\frac{i\pi t}{l}$ with max (cos, sin)=1. However this is possible only if a signal is linear which means it is superimposable, suppose the signal is non-linear then this conversion leads to loss of information. A signal is nonlinear if it has variable standard deviation and variance and hence not stationery and not superimposable. One cannot be sure whether the data is linear or nonlinear, as among the signal processing tools we never had until Tisean any Non Linear Tools, Hence our Analysis is unique.

In this work we present the Analysis of four data as given below:-
1. The recording of Initial droplets of sound
2. Mid recording of droplets of sound
3. The recording of the Initial droplets of the different type of data
4. Mid recording of the second source data.

Source: An Mp3 audio file which was originally a video on Youtube and was converted into Mp3 format using the freely available websites. This audio file is read in Audacity app and with the frequency of 44100Hz. This is converted into .wav file. Hence we convert the rain data into .wav file and name them as r1.wav, r2.wav, r11.wav, r12.wav. These audio files are then fed into Octave, The audio files are read in Octave as

x=audioread('fname.wav'),

this is saved as

save f1.txt x;

where f1.txt is the name given to the left and right audio signals obtained in ASCII. f1.txt consists of two columns with 44100 data points per second and showing the normalised intensity of the second pattern , hence we obtain four files namely :-
1. f1.txt
2. f2.txt
3. f11.txt
4. f12.txt

Which consists of rain noise for the first data of one minute cut into 30 seconds of two parts through audacity and the second data of three minutes cut into first 30 seconds part and mid 30 seconds part through audacity , Hence f1 consists of 44100×30 data points and similarly f2 , f11 and f12. Therefore the data is prepared through the above procedure and the audio is converted to the text 132300 data points.

The Analysis
We Analyse the four files namely f1.txt, f2.txt, f11.txt and f12.txt using the linear tools to begin with which is included in the TISEAN Software. TISEAN version 3.0.0 is the software which consists of both linear and nonlinear Time Series Analysis Tools developed by Holger Kantz and Thomas Schrieber [1], The two Mathematicians are now developing the Python and R codes for the same, in our work we have used the executable files developed by them on windows which works on the command prompt. Initially they developed C codes and Fortran codes, these codes are available on the lib folder of TISEAN. The following are the Linear Tools which we use for our Initial Time Series Analysis
a. d2
b. spectrum
c. arima model
d. stp
e. mutual
f. fn
g. lyap_spec

The basis for linear analysis is the Fourier analysis which depends on superposition and scalability of the time series. The basic property that categorize the stationarity of the time series is the constant mean, variance and covariance of the time series. To find this we predict the time series using arima-model. Arima is the acronym for auto regressive integrated moving average model, which finds the trend in the time series and predicts the next or the future value of the time series. We note that in the above data of rain sounds the time series had no trend and hence its mean and variance had no significant change in the course of time. Hence the data considered was stationary. Using arima model we predicted the future patterns of the rain data showed remarkable accuracy. We may conclude that this data is linear and Fourier analysis is applicable. Thus we performed the power spectral analysis using spectrum.

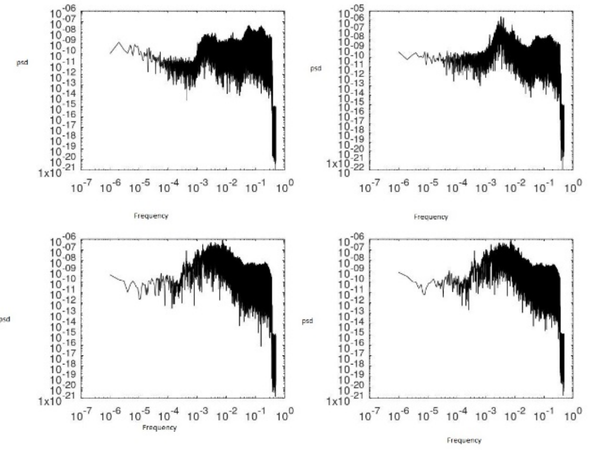

Figure 1: The plot showing the power spectral density of f1.txt, f2.txt, f11.txt, f12.txt respectively.

The power spectral plots shows that the intensity is very high and concentrated around 10^{-6}. The f1.txt and f2.txt has similar patterns whereas f11.txt and f12.txt have similar patterns which is as expected, due to the extracts are of same data. This will be even more evident with the correlation plots which eventually is 1. That is that the data is highly correlated as shown in the figure below.

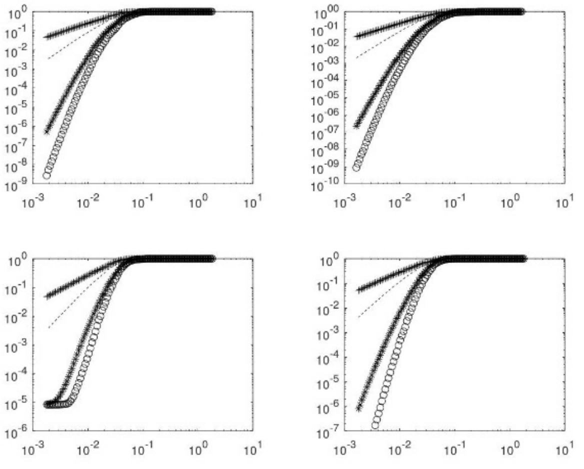

Figure 2: Shows The Correlation Amongst The Rain Data.

The above figure shows that the correlation dimension eventually settles at 1 at around or between 10-2 and 10-1. This shows that the embedding dimension is also very small and is close to 3. The literature about the embedding dimensions and time delay can be found in [1].

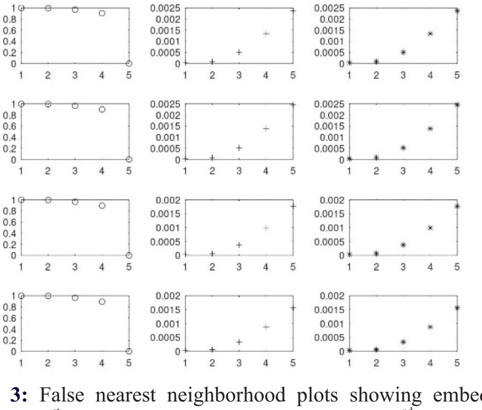

Figure 3: False nearest neighborhood plots showing embedding dimension (1st column), fraction of false neighbors (2nd column) and average square size of neighbors (3rd column).

We also plot the phase plots by taking into consideration the time delay of 44100 hertz. The plots are as follows.

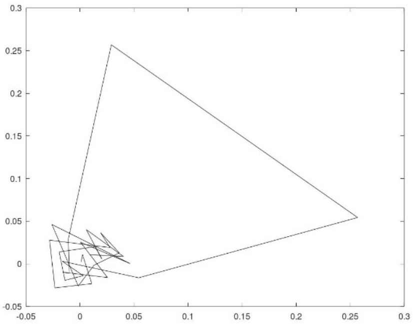

Figure 4: Phase plot of f1.txt

Figure 4 shows the phase diagram shows that the normalized frequency of the rain data largely ranges from -0.05 to +0.05, only two data points diverge largely. This might be due to the thunderstorm present in the rain data.

Figures 5 to 7 are phase plots of f2.txt, f11.txt and f12.txt. no significant change in the range was observed.

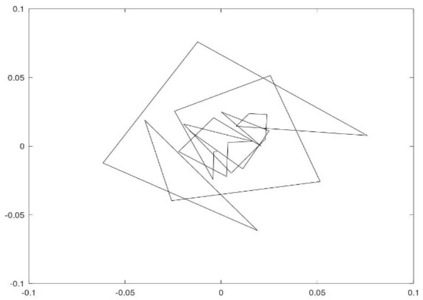

Figure 5: Phase plot for f2.txt

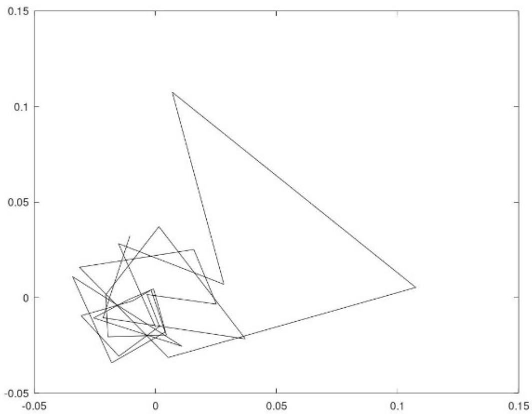

Figure 6: Phase plot for f11.txt

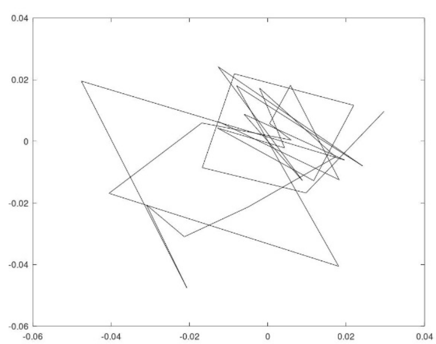

Figure 7: Phase plots for f12.txt

The same information can be obtained from average mutual information plot figure8 below. This information might be due to the initial rain data taken as it is slow and drizzling.

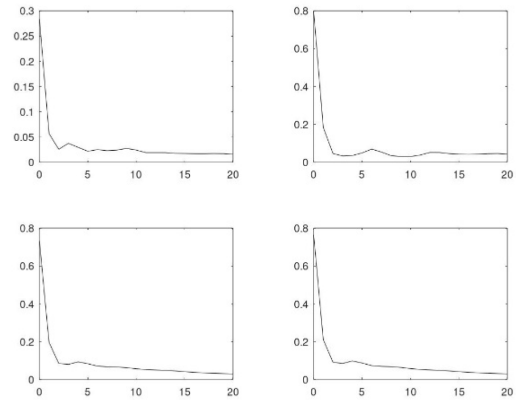

Figure 8: Average Mutual information plot

We observe that if we remove the points (0.05, 0.25) and (0.25, 0.05) from figure4 we get the same plot as that of figure5. This shows that the second part of the data does not have thunderstorms. The last part of the second recording shows that amplitude is comparatively small and it lies between -0.02 and 0.02, when compared to -0.05 and 0.05. At 2:30 minutes rain starts pouring very slowly.

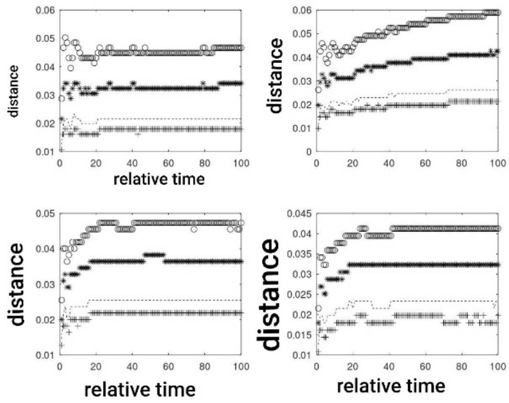

Figure 9: Space time separation plot.

Space time Separation plot shows how large the temporal distance between points should be so that they form independent samples according to the invariant measure. In this figure there is a saturation of temporal distance between the points as seen in the figures for different embedding dimensions.

To conclude the behavioral pattern of the rain data we found the Lyapunov spectrum using TISEAN. From the literature we know that the Lyapunov spectrum if is negative then the time series is asymptotically stable hence the rain data has no room for chaos or randomness.

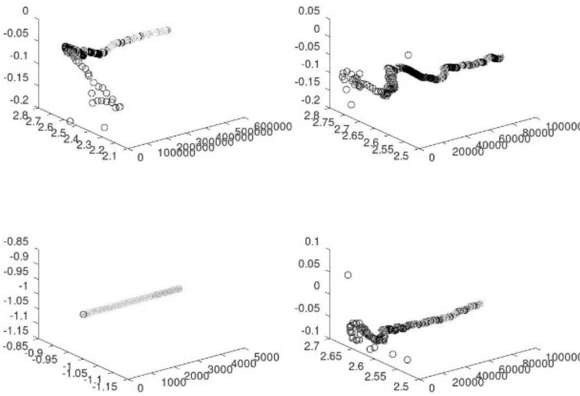

Figure 10: Lyapunov spectrum for the rain data.

CONCLUSION

From the above study we conclude the following:

1. Rain drops on ground that is the drizzling sound is very rhythmic
2. Rain sound is linear and stationary.
3. The Lyapunov spectrum shows the stability of these sounds and the rain sound is asymptotically stable
4. This work can be extended on any other forms of sounds and music.

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1. Kantz H, Scrieber T. (2003): *Nonlinear Time series* Analysis, second edition, Cambridge University Press.