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Statistics

AN OPTIMAL REPLACEMENT PROBLEM FOR A PRODUCTION SYSTEM USING PREVENTIVE MAINTENANCE MODEL

Dr. B. Venkata Ramudu

Associate Professor of Statistics, S.S.B.N Degree College (Autonomous), Anantapuramu-515001 A.P., India.

ABSTRACT This paper studies a replacement model for a repairable system with preventive maintenance in which the interval of preventive repair is fixed and preventive repair is 'as good as new'. While, the failure repair is alpha series process repair. The replacement policy N is the number of failures of the system, The objective function is minimizing cost efficiency (i.e the lon-run average cost rate per unit working time). In this model he proved that the optimal policy with preventive repair is better than the policy without preventive maintenance (PM). The optimal replacement policy N* can be found both under PM and without PM such that the average cost rate is minimum. Finally, numerical results are given to highlights the theoretical results.

KEYWORDS : Renewal process, replacement policy, geometric process, alpha series process, long run average rate.

1. INTRODUCTION

In the past, many replacement models are invented to the maintenance problems of production system under certain criteria. Obviously, maintenance models for minimizing cost, maximizing profit, increasing reliability and availability of the systems have been studied under the assumption that the system after repair 'as good as new'. But, for a deteriorating system it is reasonably assume that the system after repair is not 'as good as new'. In order to increase the availability or to decrease the operating costs of repairable systems, preventive maintenance (PM) has been widely used. The minimal repair model proposed by Barlow and Hunter [1] and the imperfect repair model considered by Brown and Proschan [6] have been used to detect unexpected repairs, needed between scheduled PM activities. A minimal repair does not change the failure rate of the system. An imperfect repair is equivalent to perfect repair with probability p and minimal repair with probability 1 p ($0 < p < 1$). Many research studies on this topic have been reported (see, for example, Nakagawa[12], Kijima [9], Black e.t al [4], Brown and Proschan [2,3] and others)

Lam[10,11] proposed a repair and replacement framework for a repairable deteriorating system with one repair man. In his model, the successive working times become shorter and shorter while the consecutive failure repair times of the system become longer and longer. Eventually, either the length of working time will become too short or the failure repair time will become too long and, as a result, the system will have to be replaced. Such a model is called a `geometric process' model. Two replacement policies have been developed: one based on the accumulated working time T of the system, and the other based on the accumulated number N of failures of the system. Explicit expressions for the long-run average cost per unit time (i.e. the average cost rate) under each replacement policy are evaluated, and the corresponding optimal replacement time, expressed in terms of T^* or N^*, can be found analytically or numerically. Under some mild conditions, Lam has proved that the optimal policy N^* is better than the optimal policy T^*. Since the geometric process is a special monotonic process, Zhang[14] combined the two replacement policies used by Lam [10,11] and introduced a bivariate replacement policy ($T;N\dagger$) under which the system is replaced either when the accumulated working time reaches T or after the Nth failure, whichever occurs ® rst. Under certain conditions, Zhang showed that the optimal ($T;N\dagger^*$) policy is better than the optimal N^* policy and the optimal T^* policy. In most reported studies of deteriorating repairable systems with minimal repair, imperfect repair or geometric process degradation, the average cost rate has been used as the objective function to be minimized. The average cost rate is de® ned as the long-run average cost per unit of time.

Zhang et,al[16] studied a replacement model for a deteriorating production system with preventive maintenance in which the interval of preventive repair is fixed and preventive repair is 'as good as new'.While, the failure repair is not.The replacement policy N is the number of failures of the system,The objective function is cost efficiency i.e the lon-run average cost rate per unit working time. The optimal replacement policy N* can be found such that the average cost rate is minimum. Finally, numerical results are given to highlights the theoretical results.

Zhang [15] considered a replacement model for a repairable system with preventive maintenance in which the interval of preventive repair is fixed and preventive repair is 'as good as new' while, the failure repair is geometric process repair. The replacement policy N is the number of failures of the system, The objective function is minimizing cost efficiency (i.e the lon-run average cost rate per unit working time). In this model he proved that the optimal policy with preventive repair is better than the policy without preventive maintenance (PM). The optimal replacement policy N* can be found both under PM and without PM such that the average cost rate is minimum. Finally, numerical results are given to highlights the theoretical results.

Wang and Zhand [7] investigated a bi-variate replacement model (L,N) for a deteriorating production system with preventive maintenance in which the interval length is L between two successive preventive repairs and N is the number of preventive repairs .The replacement policy (L*,N*) is can be found such that the cost efficiency is minimum.(i.e the lon-run average cost rate per unit working time is minimum). Finally, numerical results are given to highlights the theoretical results. Later Wang and Zhang [8] developed a bi-variate replacement model (R,N) for a deteriorating production system with critical reliability is R and N is the number of preventive repairs .The replacement policy (R*,N*) is can be found such that the cost efficiency is minimum.(i.e the lon-run average cost rate per unit working time is minimum). Finally, numerical results are given to highlights the theoretical results. Braun et.al [5] presented that both the increasing geometric process and the α-series process have a finite first moment under certain general conditions. Thus the decreasing -series process may be more appropriate for modeling system working times while the increasing geometric process is more suitable for modeling repair times of the system.

On this understanding, this paper considered a replacement model for a repairable system with preventive maintenance in which the interval of preventive repair is fixed and preventive repair is 'as good as new'. While, the failure repair is alpha series process repair. The replacement policy N is the number of failures of the system, The objective function is minimizing cost efficiency (i.e the lon-run average cost rate per unit working time). In this model he proved that the optimal policy with preventive repair is better than the policy without preventive maintenance (PM). The optimal replacement policy N* can be found both under PM and without PM such that the average cost rate is minimum. Finally, numerical results are given to highlights the theoretical results.

2. The Model

To formulate the deteriorating production system we make use the following assumptions.

1) Assume that the system is installed at time t=0.
2) The repair man will be conducted repair immediately whenever the system fails.
3) The system after repair is “not as good as new”.
4) The time duration from initial time o to completion of the first repair breakdown is called first repair cycle.
5) After the system is repaired, immediately put the system into operation again, it enters its second cycle which ends at second

repair failure is completed.

6) The time interval between the completion of (n-1)th repair and nth repair is called nth cycle.
7) After each failure, the operating time stochastically decreasing and form a decreasing alpha series process.

A preventive maintenance is made after the time duration τ. The effect of preventive maintenance is as good as new. That is, the PM is restore the system to state of working condition.

Assume that Assume that $\{X_n^{(m)}, m=1,2,......, \upsilon_n\}$ are independent and identically distributed random variables. Here, $X_n^{(0)}$ represents the life of the system from beginning of the cycle n. If the system does not fail within the time τ preventive maintenance is conducted. The number of preventive interventios in acycle n is υn which is a random variableand may take the values 0,1,2,….. As a result, $\{X_n^{(m)}, m=1,2,......., \upsilon_n\}$ are all truncated at the length τ. In otherwords, the reliazation of $x_n^{(i)}$ of $X_n^{(i)}$ is equal to τ for i=1,2,….υ_{n-1}. While, $x_n^{(\upsilon_n)}$ is less than or equal to τ. Based on the given model decription, the total operating time and preventive maintenance time is given by

$$X_n = \upsilon_n \tau + X_n^{(\upsilon_n)}{}_{\chi_{\{x_n^{(\upsilon_n)}<\tau\}}} \tag{2.1}$$

$$Z_n = Z_n^{(1)} + Z_n^{(2)} + ... + Z_n^{(n)} \tag{2.2}$$

Where $Z_n^{(i)}$ for each fixed n are i.i.d random variables because PM is As good as new within the cycle. Assuming that the system working time form an alpha series process, the the distribution function of $X_n^{(0)}, X_n^{(m)}, Z_n^{(i)}$, and Y_n are given below:

$$K_n(t) = K(n^{\alpha_1}, t), \tag{2.3}$$

$$F_n(t) = F(n^{\alpha_2}, t), \tag{2.4}$$

$$H_n(t) = H(n^{\beta_2}, t), \tag{2.5}$$

$$G_n(t) = G(n^{\beta_1}, t), \tag{2.6}$$

where, $\alpha_i > 0,\ \beta_i < 0,\ i = 1,2.$

In the above $X_n^{(0)}$ and $X_n^{(m)}$ are stochastically decreasing and form a decreasing alpha series process while $Z_n^{(i)}$ and Y_n are stochastically increasing alpha series process.

We also assume that, $X_n^{(0)}$, $X_n^{(m)}$, $Z_n^{(i)}$ and Y_n are all independent each other for fixed n.

3.The Model Analysis

Let T_1 be the first replacement time and let T_n(n>=2) be the time duration between (n-1)[th] replacement and n[th] replacement .Obviously, $\{T_1, T_2, ….\}$ will form renewal process. Here the interarival time between two consecutive replacements is called a renewal cycle.

The cost efficiency of the system under policy N is given by see (Ross [13])

$$D(N) = \frac{\text{The expected cost in a renewal cycle}}{\text{Expected working time in a renewal cycle}} \tag{3.1}$$

$$D(N) = \frac{C_f\left(\sum_{n=1}^{N} E(Y_n)\right) + C_p\left(\sum_{n=1}^{N} E\left(\sum_{j=1}^{\upsilon_n} Z_n^{(j)}\right)\right) + C - C_w E(L)}{E(L)} \tag{3.2}$$

Now we evaluate the expected length of operating time under policy N as follws:

$$E(L) = \sum_{n=1}^{N}\left[E(\upsilon_n)\tau + E(X_n^{(\upsilon_n)}{}_{\chi_{\{x_n^{(\upsilon_n)}<\tau\}}})\right] \tag{3.3}$$

Where, $$E(\upsilon_n) = \frac{p_n}{q_n} = \left(\frac{1-\exp(-n^{\alpha_2}\tau/\lambda_2)}{\exp(-n^{\alpha_2}\tau/\lambda_2)}\right) \tag{3.4}$$

And

$$E(X_n^{(\upsilon_n)}{}_{\chi_{\{x_n^{(\upsilon_n)}<\tau\}}}) = \int_0^{\tau} t\ dF(n^{\alpha_2} t) \tag{3.5}$$

$$E(X_n^{(\upsilon_n)}{}_{\chi_{\{x_n^{(\upsilon_n)}<\tau\}}}) = \frac{\lambda_2}{n^{\alpha_2}}\left[1-\exp(-\frac{\tau n^{\alpha_2}}{\lambda_2}) - \frac{\tau n^{\alpha_2}}{\lambda_2}\exp(-\frac{\tau n^{\alpha_2}}{\lambda_2})\right] \tag{3.6}$$

From equations (3.4) and (3.6) we have

$$E(L) = \sum_{n=1}^{N}\left(\frac{1-\exp(-n^{\alpha_2}\tau)}{\exp(-n^{\alpha_2}\tau)}\right)\tau + \sum_{n=1}^{N}\frac{\lambda_2}{n^{\alpha_2}}\left[1-\exp(-\frac{\tau n^{\alpha_2}}{\lambda_2}) - \frac{\tau n^{\alpha_2}}{\lambda_2}\exp(-\frac{\tau n^{\alpha_2}}{\lambda_2})\right] \tag{3.7}$$

$$E(L) = \tau l_1 + \lambda_2 l_2 \tag{3.8}$$

Where $l_1 = \sum_{n=1}^{N}\left(\frac{1-\exp(-n^{\alpha_2}\tau/\lambda_2)}{\exp(-n^{\alpha_2}\tau/\lambda_2)}\right)$ and

$$l_2 = \sum_{n=1}^{N}\frac{1}{n^{\alpha_2}}\left[1-\exp(-\frac{\tau n^{\alpha_2}}{\lambda_2}) - \frac{\tau n^{\alpha_2}}{\lambda_2}\exp(-\frac{\tau n^{\alpha_2}}{\lambda_2})\right]$$

$$E\left(\sum_{j=1}^{\upsilon_n} Z_n^{(j)}\right) = E\left(E\sum_{j=1}^{\upsilon_n} Z_n^{(j)} / \upsilon_n\right) \tag{3.9}$$

$$E\left(\sum_{j=1}^{\upsilon_n} Z_n^{(j)}\right) = \frac{\mu_2}{\beta_2^{n-1}}\frac{p_n}{q_n}, \beta_2 > 0, \mu_2 > 0 \tag{3.10}$$

$$E\left(\sum_{j=1}^{\upsilon_n} Z_n^{(j)}\right) = \frac{\mu_2}{\beta_2^{n-1}}\left(\frac{1-\exp(-n^{\alpha_2}\tau/\lambda_2)}{\exp(-n^{\alpha_2}\tau/\lambda_2)}\right) \tag{3.11}$$

$$\text{Let } \sum_{n=1}^{N} E\left(\sum_{j=1}^{\upsilon_n} Z_n^{(j)}\right) = \sum_{n=1}^{N}\frac{\mu_2}{\beta_2^{n-1}}\left(\frac{1-\exp(-n^{\alpha_2}\tau/\lambda_2)}{\exp(-n^{\alpha_2}\tau/\lambda_2)}\right) = l_3 \tag{3.12}$$

$$E(Y_n) = \int_0^{\infty} t\ d\,G(\beta_1^{\,n} t) \tag{3.13}$$

$$E(Y_n) = \int_0^{\infty} t\, dG(\beta_1^{\,n}) = \frac{\mu_1}{\beta_1^{n-1}}, \mu_1 > 0, \beta_1 > 0. \tag{3.14}$$

$$\text{Let } \sum_{n=1}^{N} E(Y_n) = \sum_{n=1}^{N}\frac{\mu_1}{\beta_1^{n-1}} = l_4 \tag{3.15}$$

$$E(X_n') = \int_0^{\infty} t\, dK(n^{\alpha_1} t) = \frac{\lambda_1}{n^{\alpha_1}}, \lambda_1 > 0, \alpha_1 > 0 \tag{3.16}$$

$$E(X_n'') = \int_0^{\infty} t\, dF(n^{\alpha_2} t) = \frac{\lambda_2}{n^{\alpha_2}}, \lambda_2 > 0, \alpha_2 > 0 \tag{3.17}$$

Using equations (3.8) to (3.17) the equation (3.2) becomes:

$$D(N) = \frac{C_f l_4 + C_p l_3 + C - C_w(\tau l_1 + \lambda_2 l_2)}{\tau l_1 + \lambda_2 l_2} \tag{3.18}$$

4 Numerical results

Table:4.1

For the given hypothetical values of α_2=0.75, β_2=-0.95, β_1=-0.85, λ2=10, μ_1=30, μ_2=20, C=5000, C_p=1.5 ,C_f=5, C_w=50, τ-750,

N	D(N)	N	D(N)
1	465	13	359.0888
2	291.1369	14	388.3056
3	238.4839	15	419.2242
4	219.2064	16	451.7471
5	214.9954	17	485.7939
6	219.4628	18	521.2977
7	229.7331	19	558.2014
8	244.2914	20	596.4557
9	262.2483	21	636.0179
10	283.0378	22	676.8501
11	306.2773	23	718.9187
12	331.6947	24	762.1936

Graph 4.1: Lon-run Average cost Rate against N

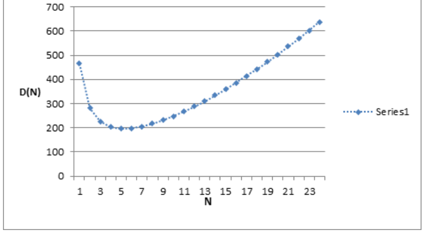
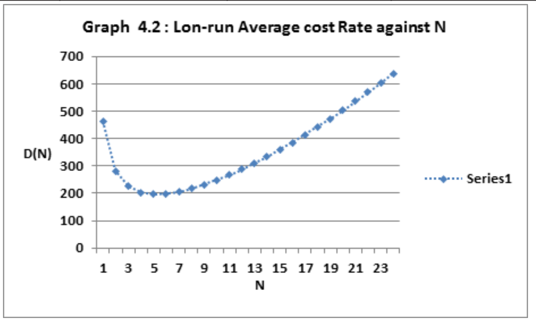


Table:4.2

For the given hypothetical values of α_2=0.65, β_2=-0.95, β_1=-0.85, $\lambda 2$=10, μ_1=30, μ_2=20, C=5000, C_p=1.5 ,C_f=5, C_w=50, τ-750,

N	D(N)	N	D(N)
1	465	13	310.2647
2	282.2449	14	334.2113
3	225.7859	15	359.5307
4	203.6694	16	386.1269
5	196.7662	17	413.9218
6	198.4207	18	442.8502
7	205.6476	19	472.8574
8	216.881	20	503.8962
9	231.2056	21	535.926
10	248.0426	22	568.9111
11	267.0026	23	602.8201
12	287.8106	24	637.6249



5 CONCLUSIONS

From the table 4.1 and graph 4.1, it is observed that the long-run average cost per unit time C(5)= 214.9954 is minimum for the given α_1=0.75,$\beta 1$= -0.95, $\beta 2$= -0.85. We should replace the system at the time of 5th failure.From the table 4.2 and graph 4.2, it is observed that the long-run average cost per unit time C(5)= 196.7662 is minimum for the given α_1=0.65, $\beta 1$= -0.95, $\beta 2$= -0.85. We should replace the system at the time of 5th failure.Similar conclusions may be drawn from table 2.4.3 and 2.4.4 .it can be concluded that as .α' decreases an increase in the number of failure, which coincides with the practical analogy and helps the decision maker for making an appropriate decision.

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