Original Research Paper

Volume -10 | Issue - 5 | May - 2020 | PRINT ISSN No. 2249 - 555X | DOI : 10.36106/ijar

Mathematics

HIGHER DIMENSION CHAPLYGIN GAS MODEL OF THE UNIVERSE WITH VARIABLE G AND Λ

R. S. Rane	Department of Mathematics, Y. C. Science & Arts college, Mangrulpir, Washim, Maharashtra-444403, India.
G. C. Bhagat*	Department of Mathematics, S.P.M. Science and Gilani Arts and Commerce College, Ghatanji, Yavatmal, Maharashtra-445301, India.*Corresponding Author

ABSTRACT In this paper we have studied thehybrid exponential lawcosmological model of the universe representing a Chaplygin gas equation of state i.e.$p = A\rho - \frac{B}{\rho^m}$ with variable gravitational and cosmological constant terms in the context of higher dimensional space-time. We have used the relation of energy density with hybrid exponential law for scale factor to obtain the solution of field equations. Here we have also discussed some physical behavior of the model.

KEYWORDS : Higher dimensional space-time;Chaplygin gas;Hybrid Exponential Law.

Introduction

The higher dimensional cosmological model is widely studied in order to describe the dynamics of our universe and the geometrical structure of the physical world both at a large and at a small scale for unifying gravity with other fundamental forces in nature. In last few years, many attempts was made on cosmological models in which space-time has more than four dimensions. The most famous theory is due to Kaluza [1] and Klein [2]. They have introduced five-dimensional space-time and have shown that the gravitation and electromagnetism could be unified in a single geometrical structure. Chodos and Detweiler [3] studied the higher dimensional cosmological model in which an extra dimension contracts to a very small scale and pointed out that this contraction of extra dimension is a consequence of an evolution of the large scale structure of the universe. Many authors [4-20] obtained the solution of Einstein's field equations for higher dimensional space-time containing the energy-momentum tensor of matter generated by a perfect fluid. In their analysis, some authors have shown that extra dimension contracts or remains constants.

In general relativity the Newtonian constant of gravitation Ggravitation G plays the role of a coupling constant between geometry of space-time and matter in the Einstein's field equations. In 1937, Dirac [21] firstly proposed the concept of time dependent gravitational constant on the basis of his number of hypothesis. Lau [22] proposed the variable gravitational constant G and cosmological constant Λ in the context of Einstein's field equations. Several authors such as Abdussattar and Vishwakarma [23]; Khadekar et.al. [24]; Pradhan and Yadav [25]; Pradhan and Rai [26]; Bali and Tinkar [27];

Singh et.al. [28]; Singh and Kale [29] studied the cosmological models with variable gravitational constant G and cosmological constant Λ theories in higher dimensional space-time.The cosmological constant Λ is found to be a positive decreasing function of time which is supported by results from recent supernovae type-Ia observations.

Riesset al. [30, 31]; Knop et. al. [32] proposed the concept of accelerated expansion of the universe and lead to the search for a new type of matter which violates the strong energy condition ρ+3 p<0. The matter content is responsible for such a condition to be satisfied at a certain stages of cosmological evolution called as dark energy. The type of dark energy represented by a scalar field is often called quintessence. The simplest example for dark energy is the cosmological constant Λ. In particular, one can find another type of dark energy so called Chaplygin gas which obeys an equation $p = -\frac{B}{\rho}$ of state such as [33], where p and ρ are respectively pressure and energy density, and B is positive constant. According to the point of view, Chaplygin gas unifies dark matter and dark energy in only one fluid. Subsequently the above equation was generalized to the form $p = -\frac{B}{\rho^m}$ with $0 \le m \le 1$. The present work is on modified Chaplygin gas obeying equation of state (see [34] – [40]) as follows:

$$p = A\rho - \frac{B}{\rho^m} \text{ for } A > 0.$$

In this paper, we have characterized the cosmological universe representing a modified Chaplygin gas equation of state (given in eq.1) with variable gravitational constant G and cosmological constant Λ terms in the context of higher dimensional space-time. When constructing this model, we assume that matter is treated as a perfect fluid determining the dynamics of the model.

2. Metric and Einstein's Field Equations

We consider the (n+2)dimensional homogeneous and isotropic model of the universe represented by the space-time metric in the form

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\Phi^2\right], \tag{2}$$

wherea(t)is the cosmic scale factor, and

$$d\Phi^2 = d\theta_1^2 + \sin^2\theta_1 d\theta_2^2 + \cdots + \sin^2\theta_1 \sin^2\theta_2 \ldots\ldots \sin^2\theta_{n-1} d\theta_n^2.$$

Without loss of generality the constant ktakes on only three values0 and ± 1. The constant k is related to the curvature parameters.The higher dimensional space-time is the standard cosmological model and are consistent with the observational results.

For perfect fluid distribution Einstein's field equations with the variable gravitational constant G(t) and cosmological constant Λ(t)may be written as

$$R_{ij} - \frac{1}{2}Rg_{ij} = -8\pi G(t)T_{ij} - \Lambda(t)g_{ij}, \tag{3}$$

where the symbols have their usual meaning.

The energy momentum tensor $T_{ij}$for perfect fluid is defined by

$$T_{ij} = (\rho + p)u_i u_j - pg_{ij}, \tag{4}$$

whereρ is the energy density of the matter, pstands for the isotropic pressure of the cosmic fluid and g_ij is the metric tensor and u_i is the four velocity fluid vector such that u^I u_i=1.

The Einstein's field equations (3) with energy momentum tensor (5) for the space-time metric (2) lead to the following set of field equations

whereρ is the energy density of the matter, pstands for the isotropic pressure of the cosmic fluid and g_ij is the metric tensor and u_i is the four velocity fluid vector such that u^I u_i=1.

The Einstein's field equations (3) with energy momentum tensor (5) for the space-time metric (2) lead to the following set of field equations

$$\frac{n(n+1)}{2}\left[\left(\frac{\dot a}{a}\right)^2 + \frac{k}{a^2}\right] = 8\pi G(t)\rho + \Lambda(t), \tag{5}$$

$$n\,\frac{\ddot a}{a} + \frac{n(n-1)}{2}\left[\left(\frac{\dot a}{a}\right)^2 + \frac{k}{a^2}\right] = -8\pi G(t)p + \Lambda(t), \tag{6}$$

where the dots denotes the differentiation with respect to cosmic time t.

3. Solution of the field equations

The field equations contain two independent equations in five unknown parameters name lya, p, ρ, G and Λ. There fore to obtain an exact solutions of the field equations, we need three more conditions relating these unknown parameters. For which we have used The following an satz for the scale factor of the universe [41]:

I) The following ansatz for the scale factor of the universe [41]:

$$a(t) = Ct^{\alpha} e^{\beta t}, \quad (7)$$

whereC>0,$\alpha \geq 0$ and $\beta \geq 0$ are constants. This generalized form of the scale factor is called hybrid expansion law which is a mixture of power law and exponential law. These power law and exponential law is obtain as a special case of hybrid expansion law, when α=0 and β=0 in (7) respectively. The assumption α>0 and β>0 lead to a new cosmological model. We have used the relation of energy density ρ with hybrid expansion law for scale factor to obtain the solution of field equations. Here we have also discussed some physical behavior of the model.

(ii)The modified Chaplygin gas obeying equation of state(1) and ansatz (7) for the scale factor a(t). Without loss of generality we consider the relation of energy density ρ with hybrid exponential law for scale factor in the form

$$\rho = \frac{1}{a^{\gamma}}, \quad (8)$$

whereγ>0 is a constant.

4. Physical parameters

The equations (1), (7) and (8) lead to the following expressions of isotropic pressure and energy density respectively

$$p = AC^{-\gamma} t^{-\alpha\gamma} e^{-\beta\gamma t} - BC^{m\gamma} t^{m\alpha\gamma} e^{m\beta\gamma t}, \quad (9)$$

$$\rho = C^{-\gamma} t^{-\alpha\gamma} e^{-\beta\gamma t}. \quad (10)$$

We are concerned with only physically realistic model, which is obtained for C>0. The behavior of the energy density for an appropriate choice of constants is shown in Fig. i. The energy density is positive and decreasing function of time t and with theexpansion of the Universe it converges to small positive constant value near to zero whichis in accordance to the constraints of cosmic microwave background radiation data [42].

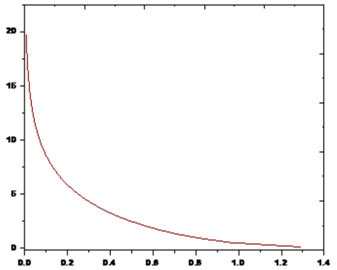

Fig.-I: Energy density of the model versus cosmic time for the appropriate choice of constants.

Variable gravitational constant

$$G(t) = \frac{n}{8\pi} \frac{[kC^{-2} t^{-2\alpha} e^{-2\beta t} + \alpha t^{-2}] C^{\gamma} t^{\alpha\gamma} e^{\beta\gamma t}}{[1 + A - BC^{(m+1)\gamma} t^{(m+1)\alpha\gamma} e^{(m+1)\beta\gamma t}]}. \quad (11)$$

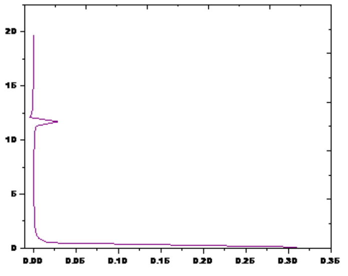

Fig.-ii: Variable gravitational constant of the model versus cosmic time for the appropriate choice of constants.

Variable cosmological constant

$$\Lambda(t) = \frac{n(n+1)}{2}\left[\left(\frac{\alpha+\beta t}{t}\right)^2 + \frac{k}{C^2 t^{2\alpha} e^{2\beta t}}\right] - \frac{n\,[kC^{-2} t^{-2\alpha} e^{-2\beta t} + \alpha t^{-2}]}{[1 + A - BC^{(m+1)\gamma} t^{(m+1)\alpha\gamma} e^{(m+1)\beta\gamma t}]}. \quad (12)$$

As from the fig. iii, it is observed that the variable cosmological constants initially negative then with expansion it becomes positive and approaches to small positive constant which is near to zero. After omitting the constants we observed the behavior of the form $G \propto \frac{1}{a}$

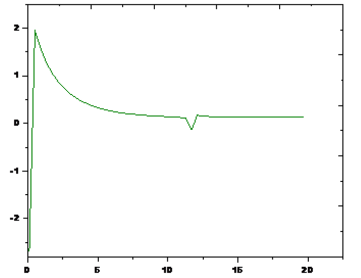

Fig.-iii: Variable cosmological constant of the model versus cosmic time for the appropriate choice of constants.

During the expansion of the model, the overall density parameter is given by

$$\Omega = \frac{16\pi G(t)\rho}{n(n+1)H^2} = \frac{2(\alpha C^{\gamma} t^{\alpha\gamma} e^{\beta\gamma t} + kt^2)}{(n+1)(\alpha+\beta t^2)(1 + A - BC^{(m+1)\gamma} t^{(m+1)\alpha\gamma} e^{(m+1)\beta\gamma t})} \quad (13)$$

The overall density parameter of the model is a function of time and increases with expansion. The behavior of overall density parameter of the model versus cosmic time for the appropriate choice of constants is shown in fig-iv.

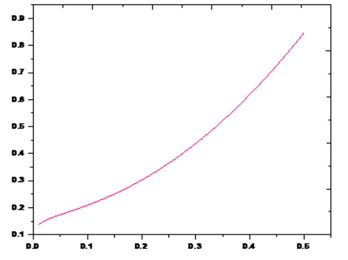

Fig.-iv: Overall density parameter of the model versus cosmic time for the appropriate choice of constants.

5. Kinematical parameters

The Spatial volume of the model is given by

$$V = C^3 t^{3\alpha} e^{3\beta t}, \quad (14)$$

In this model, we observed that the spatial volume of the Universe starts with big bang at t=0 and with the increase of time it always expands and when t=∞, then the spatial volumev$\rightarrow\infty$. Thus, inflation is possible in this model which shows that the Universe starts evolving with zero volume and expands with time t. The graphical behavior of spatial volume of the universe is shown in Fig. v.

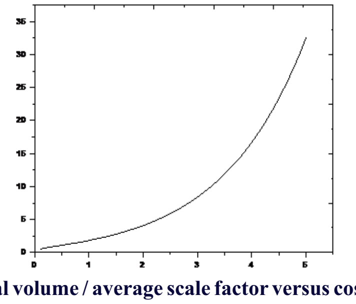

Fig.-v: Spatial volume / average scale factor versus cosmic time for the appropriate choice of constants.

The Hubble parameterof the model is given by

$$H = \left(\frac{\alpha+\beta t}{t}\right). \quad (15)$$

The Expansion scalar of the model is given by

$$\theta = 3H = 3\left(\frac{\alpha+\beta t}{t}\right). \quad (16)$$

Initially, at t=0 mean Hubble parameter, expansion scalar and the magnitude of shear scalar are all infinitely large but with the expansion of the Universe these parameters decreases. This shows that the evolution of the Universe starts with an infinite rate and with the expansion it declines. The behavior of Hubble parameter of the model verses time t is shown in Fig. vi.

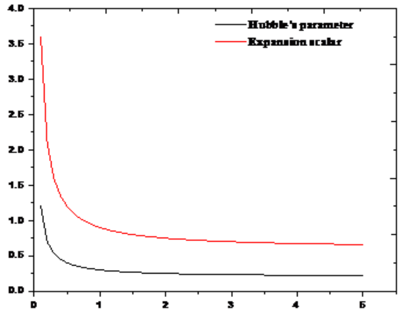

Fig.-vi: Hubble's parameter and expansion scalar versus cosmic time for the appropriate choice of constants.

The deceleration parameter of the model is given by

$$q = -1 + \frac{\alpha}{(\alpha+\beta t)^2}. \quad (17)$$

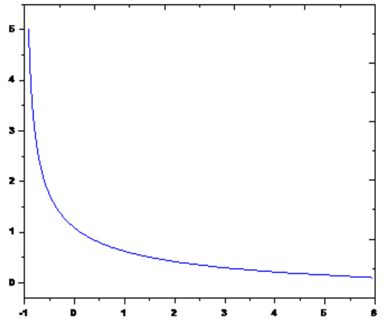

Fig.-vii: Hubble's parameter and expansion scalar versus cosmic time for the appropriate choice of constants.

The sign of deceleration parameter indicates whether the model inflates or not. The negative sign of deceleration parameter indicates inflation. Also, recent observations of type-Ia show that the present universe is accelerating and that the value of deceleration parameter lies in some place in the range $-1 \le q \le 0$. The deceleration parameter decreases rapidly and approaches -1 asymptotically, which shows de-Sitter like expansion at late time. For this model, the deceleration parameter gives a transition from a decelerating expansion phase to the present accelerating phase. These observations follow it our derived model.

6.CONCLUSION

We have studied the effect of varying gravitational constant G and cosmological constant Λ terms on cosmological universe representing a modified Chaplygin gas equation of state with energy-momentum tensor containing perfect fluid in the context of higher dimensional space-time. We have considered the scale factor in the form of hybrid expansion law which is a mixture of power law and exponential law. We observed that hybrid expansion law model of the universe evolves with a variable deceleration parameter qhaving the transition from deceleration to acceleration phase which is take place atpointt$=(\sqrt{\alpha}-\alpha)/\beta$, where $0<\alpha<1$, which is an important feature of the early stages of the evolution of the universal cosmological model. We, also observed that the deceleration parameter q approaches negative value with increase of time i.e. $q \to -1$as $\to \infty$, which shows that the universe is in the accelerated expansion phase. We observed that energy density ρdecreases with increase in time. We observed that the pressure decreases from positive to negative with increase in time. It is interesting to note that the behavior of the gravitational constant and cosmological constant are same for the all open, closed and curved Universe and approaches towards zero with increase in time.

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