Original Research Paper

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Physics

STUDYING THE MOVEMENTS OF THE RESONANT ELASTIC PENDULUM

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KEYWORDS :

1. INTRODUCTION

An elastic gravitational pendulum is an example of nonlinearly coupled oscillating systems. Resonant oscillations of this mechanical system are the subject of modern scientific research because of their mathematical similarity to other complex oscillations that are often encountered in physics. For example, the Fermi resonance in the IR spectrum of CO2 molecules is due to the nonlinear coupling of longitudinal and transverse oscillations of atoms [1]. The electrical oscillations in electromagnetic circuits with nonlinear coupling and the motion of artificial and natural satellites under the periodic influence of the third bodies are similar to mechanical oscillations of nonlinearly coupled pendulums. The solutions obtained in this paper can be used in the study of the phenomena of deterministic chaos and other physical processes.

2. Lagrangian and equations of motion

Consider a pendulum consisting of a mass m that is hanged on a spring (rubber thread). The length of an unstretched spring is l0, and k is its rigidity. The mass m satisfies the ratio m 4 kl / g that provides the resonance ratio of the frequencies of the gravitational and spring pendulums. The length of the spring r(t) and the angle of deviation of the pendulum from the equilibrium position φ(t) are chosen as generalized coordinates. The Lagrange function of the elastic pendulum is

$$L=\frac{m\dot r^2}{2}+\frac{m r^2\dot\varphi^2}{2}-\frac{k(r-l_0)^2}{2}+m g r\cos\varphi\,, \quad (1)$$

where the first and second terms are the kinetic energy of the translational and rotational motions of the mass m, the third and fourth terms are the spring potential energy and the gravitational potential energy. The system of Lagrange equations is the following:

$$\begin{cases} m\ddot r-m r\dot\varphi^2+k(r-l_0)-mg\cos\varphi=0 \\ r\ddot\varphi+2\dot r\dot\varphi+g\sin\varphi=0 \end{cases} \quad (2)$$

From equations (2) one can see that gravitational and spring oscillations are coupled in a complicated way. Analytical solutions of the differential equations system (2) are unknown; therefore, most of the known studies at present are performed in the linear approximation (small amplitude of oscillations) [1], numerical method [2] or experimentally [3].

3. Results of computer modeling

In this paper, for the study of oscillations of the resonant elastic pendulum, a computer model was created in the Wolfram Mathematica language and published on the Wolfram Demonstrations Project website (Fig. 1) [4].

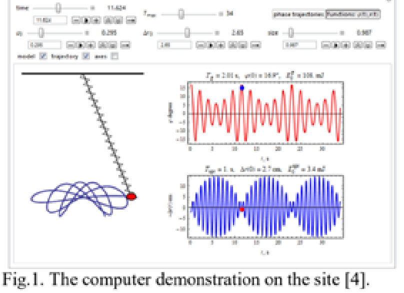

Fig.1. The computer demonstration on the site [4].

The developed program allows changing the initial conditions in a wide range and studying oscillations of the pendulum. Fig.1 demonstrates that the amplitude decrease in the gravitational pendulum oscillations is accompanied by the amplitude increase in the spring pendulum oscillations. Thus, at the friction absence, the total energy of the system remains unchanged. As a result, the graphs φ(t) and Δr(t) have the form of modulated functions that are similar to the effect of beating frequencies and oscillations of linearly coupled systems. Though, for two linear coupled oscillatory systems, the frequency of energy exchange between subsystems is determined by the system parameters and does not depend on the initial conditions. In the case of an elastic gravitational pendulum, the period of energy exchange can be changed in a wide range by the initial conditions. For example, a decrease in the initial amplitudes of oscillations leads to an increase in the period of energy transfer (Fig. 1-2).

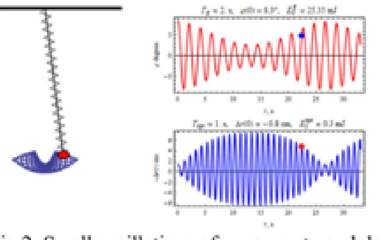

Fig 2. Small oscillations of a resonant pendulum.

Studies have shown that the value of the transferred energy between the coupled subsystems at the small initial deviations is determined by the equation

$$\Delta E=\left|E_0^g-2E_0^{spr}\right|, \quad (3)$$

where $E_0^g=mg[1-\cos\varphi(0)]r(0)$, $E_0^{spr}=k[\Delta r(0)]^2/2$

the energies of the gravitational and spring pendulums at the initial moment of time. Therefore, as follows from (3), there are such initial conditions in which there is no exchange of energies between subsystems and oscillations of pendulums that occur independently. Since the spring pendulum initial energy can be provided both by tension (r(0) 0) and compression (r(0) 0) of the spring, two types of such oscillations are possible (A and B, Fig. 3).

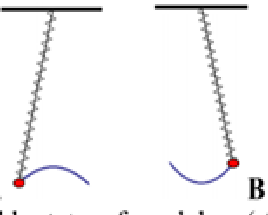

Fig.3. Stable states of pendulum ($\Delta E=0$).

When changing the initial deviations of the pendulum, there were various stable periodic trajectories of the pendulum oscillations, which are called modes. The existence of stable modes is explained by the fact that in the case when the period of energy exchange becomes a multiple of the period of the gravitational pendulum oscillations (T=N·Tg.), the system returns to the same state that was in the beginning, and hence the trajectory will repeat. Each of these modes can exist at different values of the initial conditions. In Fig. 4 solid lines show the initial conditions of several modes of oscillation. On the inserts of Fig.4 one can see trajectories that are typical for these modes.

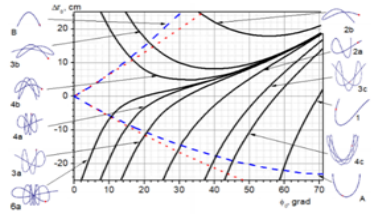

Fig.4. Distribution of oscillating modes on the plane of the pendulum initial deviations - solid lines. States **A** and **B** – dashed lines. $E_0^g = 2E_0^{spr}$ - dotted lines.

The simplest oscillation modes are observed at high energies, since then the energy transfer period is small and therefore contains a small number of the gravitational pendulum oscillations (N). For convenience, number N will denote the oscillation modes, and the Latin letters - their modifications. For example 2a, 2b,

3a, 3b, 3c .... Mode "1" has only one modification. Each of the oscillation modes when changing the initial conditions deforms, maintaining its qualitative form. The dashed lines in Fig. 4 show stable periodic oscillating states of the pendulum A and B. The curves of all modes cross one of the dashed lines A or B, so each oscillation mode can be in one of these states. Figure 5 shows the form of the mode 4a for various energy exchanges including E 0 (state A).

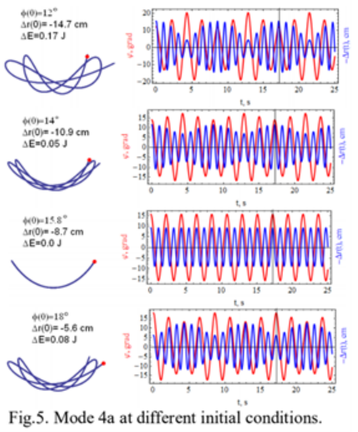

Fig.5. Mode 4a at different initial conditions.

Figure 4 shows the ratio / 2 0 0 spr g E E is performed for states A and B only at a small amplitude of the gravitational pendulum (φ <15o). This is due to the conditional division of the entire system energy into the energies of the subsystems. There is a part of the energy that belongs to both subsystems, which could be called the interaction energy [3]. The solid lines of different modes (Fig.4) are lines of equal period of energy exchange. The lines of different modes do not cross each other. There is a region of initial deviations where oscillations are unstable and small change of pendulum initial deviations produces great change of exchange energy period and pendulum trajectory.

4. CONCLUSIONS

As the result of the motion equation numerical solution, some peculiarities of the elastic gravitational pendulum oscillations were found. Different oscillation modes are systematized and their coupling with the initial conditions is established. A map of initial oscillation conditions has been created, by which one can estimate the period of energy exchange and its magnitude. The computer model of the elastic pendulum will be useful for educational and research work

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