Original Research Paper

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Mathematics

THE CHAIN FOUNTAIN

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ABSTRACT If a beaker is fully filled with a chain and an initial impulse is given on one end of the chain, then the chain inside the beaker starts coming out of the beaker and grows in height passes. After sometime, the velocity with which the chain comes out of the beaker becomes constant and depends only upon the height of the beaker and the height to which the chain rises.

KEYWORDS : Impulse, Beaker, Velocity, Constant.

INTRODUCTION

The Chain Fountain aka the Mould effect was observed by Steve Mould by accident while working on a physical model for a polymer. When a glass container is filled with chain beads and the chain is pulled from one of its ends, it is observed that the chain starts moving in a "gravity-defying motion" i.e. the height to which the chain rises increases over time. Here I try to prove that the velocity with which the chain comes out of the beaker becomes mass-independent after a long period of time and the value depends only upon the height of beaker and the height to which the chain rises.

METHODOLOGY

When an impulse is given to chain at one end, it starts coming out of the beaker and forms a U-shape. I analyzed the forces on three parts of the U-shape –

1. The part of chain moving up
2. The part of chain moving down
3. The top part

For analyzing I assumed the mass per unit length of chain as 'lambda'. An expression in terms of acceleration and velocity was obtained which was further simplified to get velocity in terms of time using integration. Then, I made the time to approach infinity which gave the expression for velocity which was mass independent and depended only on the height of beaker and the height to which the chain rose.

RESULTS AND DISCUSSION

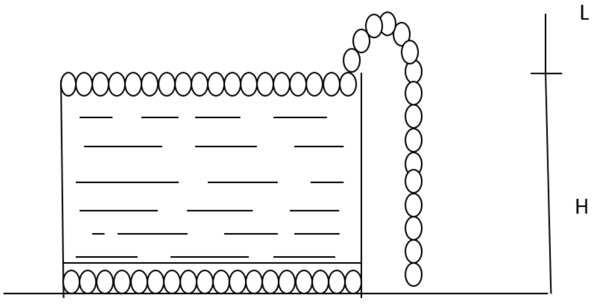


Figure 1: Schematic Diagram Of A Chain Fountain

- Let 'H' be the Height of the container
- Let 'L' be the Height to which the chain rises above the container
- Let the Force we apply on the chain to fall down act for time 'dt' cause an acceleration 'a'
- Let 'λ' be Mass per Unit Length

For a small part of chain of Length dx

Mass of that part = λdx

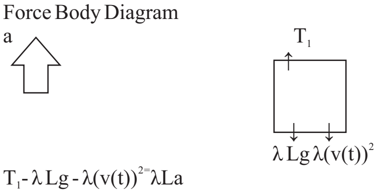
$P_i = 0$, $P_f = \lambda dx v(t)$

Note: [v is a function of t]

$$F \text{ reqd.} = \frac{dp}{dt} = \frac{\lambda dx v(t) - 0}{dt}$$
$$= \lambda v(t)\frac{dx}{dt}$$
$$= \lambda(v(t))^2 \left[\frac{dx}{dt} = v(t)\right]$$

For part of chain moving up –
Length = L, Mass = λ L

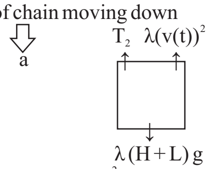
Force Body Diagram



$T_1 - \lambda Lg - \lambda(v(t))^2 = \lambda La$

$T_1 = \lambda L(g+a) + \lambda(v(t))^2$ ------1-------

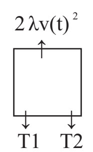
For part of chain moving down



$\lambda(H+L)g - T_2 - \lambda(v(t))^2 = \lambda(L+H)a$

$T_2 = \lambda(L+H)(g-a) - \lambda(v(t))^2$ ------ 2------

For the top part of chain [For ease of calculation neglecting force due to Gravity]



For Top part

$P_i = \lambda dx\, v(t)$

$P_f = -\lambda dx\, v(t)$

$$F = \frac{dp}{dt} = \left(\frac{-2\lambda dx v(t)}{dt}\right) = -2\lambda(v(t))^2$$

$T_1 + T_2 = 2\lambda(v(t))^2$ ------ 3-------

Adding eq. (1) and (2)

$T_1 + T_2 = \lambda Lg + \lambda La + \lambda(v(t))^2 + \lambda Lg - \lambda La + \lambda Hg - \lambda Ha - \lambda(v(t))^2$

$T_1 + T_2 = 2\lambda Lg + \lambda H(g-a)$ -------4-------

From eq. (3) & (4)

$2\lambda(v(t))^2 = 2\lambda Lg + \lambda H(g-a)$

- $2(v(t))^2 = 2Lg + Hg - Ha$
  [v is a function of t, 'a' will also be a function of t]
- $2(v(t))^2 = 2Lg + Hg - Ha(t)$
- $2(v(t))^2 = 2Lg + Hg - Hv'(t)$ [:- $a(t) = v'(t)$]
- $Hv'(t) + 2(v(t))2 - 2Lg - Hg = 0$
- $\dfrac{Hv'(t)}{-2(v(t))^2 + g(2L+H)} = 1$
- $\dfrac{Hv'(t)}{2(v(t))^2 - g(2L+H)} = -1$

Integrating with respect to time on both sides

$$\int \frac{Hv'(t)dt}{2(v(t))^2 - g(2L+H)} = \int -1\, dt$$

Let v(t) = u

$\frac{du}{dt}$= v'(t)

$dt = \frac{du}{v'(t)}$

- $\int \frac{Hdu}{2u^2-(2L+H)g}$ = - t + c
- $H\int \frac{du}{(\sqrt{2}u-\sqrt{(2L+H)g})(\sqrt{2}u+\sqrt{(2L+H)g})}$ = -t +c
- $\frac{H}{2\sqrt{(2L+H)g}}\int (\frac{1}{\sqrt{2}u-\sqrt{(2L+H)g}} - \frac{1}{\sqrt{2}u+\sqrt{(2L+H)g}})du$= - t + c
- $\frac{H}{2\sqrt{(2L+H)g}}\ln (\frac{\sqrt{2}u-\sqrt{(2L+H)g}}{\sqrt{2}u-\sqrt{(2L+H)g}})$= - t + c
- $\ln\left(\frac{\sqrt{2}u-\sqrt{(2L+H)g}}{\sqrt{2}u-\sqrt{(2L+H)g}}\right)=\frac{2\sqrt{(2L+H)g}(-t+c)}{H}$
- $\frac{\sqrt{2}u-\sqrt{(2L+H)g}}{\sqrt{2}u-\sqrt{(2L+H)g}} = e^{\frac{2\sqrt{(2L+H)g}(-t+c)}{H}}$

Where 'c' is the constant of integration.

$$\text{Let} \lim_{t\to\infty} v(t) = v$$

$$\lim_{t\to\infty} e^{\frac{2\sqrt{(2L+H)g}(-t+c)}{H}} = e^{-\infty} = 0$$

$$\frac{\sqrt{2}v-\sqrt{(2L+H)g}}{\sqrt{2}v-\sqrt{(2L+H)g}} = 0$$

$$\sqrt{2}v - \sqrt{(2L+H)g} = 0$$

$$v = \sqrt{\frac{(2L+H)g}{2}}$$

CONCLUSION

As we can see the velocity, as time approaches infinity, becomes constant and is equal to. $\sqrt{\frac{(2L+H)g}{2}}$

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REFERENCES

[1] https://royalsocietypublishing.org/doi/10.1098/rspa.2013.0689
[2] https://www.youtube.com/watch?v=_dQJBBklpQQ.