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Statistics

MARKOV SWITCHING AUTOREGRESSIVE MODEL FOR SUGAR PRODUCTION

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ABSTRACT We propose a Seasonal Markov Switching Autoregressive model and apply it to analyze sugar production in India. The dynamics is governed by two regimes, along which both the autoregressive coefficients and the innovation distributions are altering moreover; the hidden regime indicator process is allowed to be non-Markovian. After examining stationarity and basic properties of the model, we turn to its estimation by Markov Chain Monte Carlo (MCMC) methods and propose two algorithms. The values of the latent process serve as auxiliary parameters in the first one, while the change points of the regimes do the same in the second one in a reversible jump MCMC setting. After comparing the mixing performance of the two methods, the model is fitted to the Sugar production data.

KEYWORDS : Markov-switching autoregressive model, Seasonal time series, Transition probabilities, Markov switching autoregressive unit root test and R-Package.

INTRODUCTION

Markov-switching models have achieved a great expansion in non-linear time series modeling because of their great descriptive properties. The idea is that parameters of the model can acquire different values. This depends on the "regime" or "state" the model is in. The parameter switching follows the dynamic behavior of economic and financial time series quite well. Hamilton (1992) described a procedure which should be kept in case of a nonlinear modeling. In general, any principle is accepted from the specific to the more common. So any researcher may start with a simpler linear model, after maintaining given conditions, pass on to more complex non-linear models.

Markov Switching Autoregressive Model

Let S_t denotes an unobservable state variable assuming the value one or zero. A simple Switching model for the variable z_t involves two AR specifications

$$z_t = \begin{cases} \alpha_0 + \beta Z_{t-1} + \varepsilon_t & s_t = 1 \\ \alpha_0 + \alpha_1 + \beta Z_{t-1} + \varepsilon_t & , s_t = 2 \end{cases} \quad ... \quad (1)$$

where $|\beta| < 1$ and ε_t are i.i.d. random variables with mean zero and variance σ_e^2. This is a stationary AR(1) process with mean $\alpha_0/(1-\beta)$ when $s_t = 1$, and it switches to another stationary AR(1) process with mean $(\alpha_0 + \alpha_1)/(1-\beta)$ when S_t changes from 1 to 2. In this case, Z_t are governed by two distributions with distinct means, and S_t determines the switching between these two distributions (regimes or states). When $S_t = 1$ for $t = 1,2 \quad , ... T_0$ and $S_t = 1$ model with a single structural change in which the model parameter experiences one (and only one) abrupt change after t = 0. When S_t are independent Bernoulli random variables, it is the random switching model of Quandt (1972). In the random switching model, the realization of S_t is independent of the previous and future states so that Z_t may be "jumpy" (switching back and forth between different states). If S_t is postulated as probabilities satisfy $p_{i1} + p_2 = 1$. The transition matrix governs the random behavior of the state variable, and it contains only two parameters (p11 and p22). The model (2) with the Markovian state variable is known as a Markov switching model. The Markovian switching mechanism has been first considered by Goldfield and Quandt (1973). Hamilton (1989) has presented a thorough analysis of the Markov switching model and its estimation method; see also Hamilton (1994) and Kim and Nelson (1999).In the Markov switching model, the properties of Z_t are jointly determined by the random characteristics of the driving innovations ε_t and the state variable S_t.

$$Z_t = \alpha_0 + \alpha_1 s_t + \beta_1 Z_{t-1} + ... + \beta_k Z_{t-k} + \varepsilon_t \quad ... \quad (4)$$

where $s_t = 0,1$ are the Markovian state variables with the transition matrix and ε_t are independent and identically distributed random variables with mean zero and variance σ_e^2. This is a model with a general AR (k) dynamic structure and switching intercepts. For the d-dimensional time series $\{z_t\}$, the equation (4) in written as

$$Z_t = \alpha_0 + \alpha_1 s_t + B_1 Z_{t-1} + ... + B_k Z_{t-k} + \varepsilon_t \quad ... \quad (5)$$

Where $s_t = 1,2$ are still the Markovian state variables with the transition matrix (3), $B_i (i = 1 ... k)$ are $d * d$ matrices of parameters, and ε_t are independent and identically distributed random vectors with mean zero and the variance-covariance matrix $\sum_0$. Until now we have discussed so far are the 2-state Markov switching model because the state variable is binary. Further generalizations of these models are possible. Also

$$\bar{z}_t = \beta_1 \bar{z}_{t-1} + ... + \beta_k \bar{z}_{t-k} - \varepsilon_t \quad ... \quad (6)$$

Then, $\bar{Z}_t$ (and hence Z_t) depends not only on S_t but also on $S_{t-1} ... S_{t-k}$. As there are 2^{k+1} possible values of the collection $S_t; S_{t-1} ... S_{t-k}$ the model (5) can be viewed as (3) with 2^{k+1} states. Another generalization is to allow for time-varying transition the indicator variable $1_{\{\lambda_{\leq ct}\}}$ such that $s_t = 1$ or 2 depending on whether the value of λ_t is greater than the cut-of (threshold) value c, equation (1) becomes a threshold model.

In particular, suppose that S_t follows a first order Markov chain with the following transition matrix:

$\lceil IP(s \; =1|s \; . =1) \; IP(s \; =1|s \; . =1) \rceil$

$$p=\begin{bmatrix} IP(s_t=1|s_{t-1}=1) & IP(s_t=1|s_{t-1}=1) \\ IP(s_t=1|s_{t-1}=2) & IP(s_t=1|s_{t-1}=2) \end{bmatrix} \quad . \; . \; . \quad (2)$$

$$p=\begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \quad . \; . \; . \quad (3)$$

where $p_{i,j}(i; j=1,2)$ denote the transition probabilities of $s_t = j$ given that $s_{t=1} = i$. Clearly, the transition

as a ratio of the estimated parameter and its standard deviation which can be obtained from the negative of the Hessian of the log-likelihood function evaluated at the optimum. However, its distribution under the null hypothesis is non standard. Hall et al. (1999) have calculated the empirical *p*-values of this test by simulating the model under the null hypothesis.

Application to Sugar production Data

The monthly Sugar production shows an upward trend during the period 1995: 01 to 2018:12 Figure 1. Apart from the sharp increase in production, fluctuations in the production of sugar within the year have also been seen. These seasonal fluctuations in production may be due to the seasonality in the production of the Sugar. In this paper, the univariate time series analysis of monthly Sugar production in India is taken into account. The data consists of 288 monthly observations from January 1995 to December 2018.

Table 1: Estimation of the parameter of SMS-AR Model

Coefficient	AR (1)	AR(2)	AR(3)	AR(4)	SAR(1)	SAR(2)	SAR(3)	Intercept
Estimate	0.7099	-.0638	-.6303	-0.3084	0 .3975	0.2473	0.1315	52597.226
S.Errors	0.0610	.0716	0.0722	0.0595	0.0613	0.0632	0.0620	5419.528
Sigma squared estimated as 71877017 Log-likelihood=--3018.57, AIC=6055.15, AICC=6055.79, BIC=6088.11								

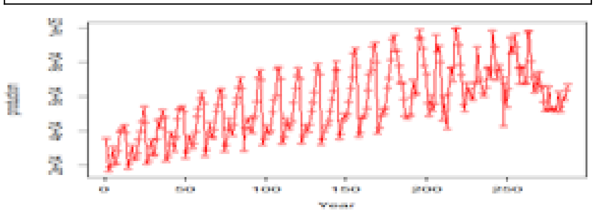

Figure 1: Actual data for Sugar Production in India (1995-2018)

Markov Switching Unit Root Test

The Markov-switching unit root test used in this chapter can be obtained by running the regression of the ADF test where the constant term is driven by an unobservable state variable S_t

$$Y_t = \rho Y_{t-1} + c_{st} + b_t + \sum_{j=1}^{k} \gamma_j \Delta Y_{t-j} + \varepsilon_t \quad . \; . \; . \quad (7)$$

where $\varepsilon_t \sim N(0,\sigma)$. The state variable is assumed to

where $\varepsilon_t \sim N(0,\sigma)$. The state variable is assumed to evolve according to an irreducible 2-state Markov chain whose transition probabilities are defined by

$$p(s_t = j / s_{t-1} = i, s_{t-2} = h \ldots \Omega_{t-1}) = p(s_t = j / s_{t-1} = i) = p_{ij}$$

where $i, j = 1,2$, and Ω_t refer to the information set up to period t^3. In short, this model endogenously permits the constant term of the time series to switch as the date and regime changes. The unit root tests are based on the *t*-statistic t_ρ, associated with $\rho = 0$. The *t*-statistic can be easily computed

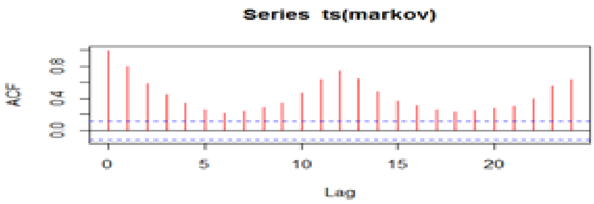

Figure 2: Sample ACF of Seasonal markov switching autoregressive models

Seasonal Markov Switching Autoregressive Model

Model Selection

Model choice will be performed by means of Bayes factors in which the marginal likelihoods are computed according to Chib (1995) and Chib and Jeliazkov (2001), through the relabeling of the hidden states by means of constrained permutation sampling. From the values of the marginal likelihoods, choose the SMSAR (2;4) as the best among all the competing models Table 2,Table 3 and Table 4.

Coefficient	Intercept	MSAR 1	MSAR 2	MSAR 3	MSAR 4	Std Dev
Low production	11778.94	0.9398227	-0.235667	0.09158303	-0.005913	11244.14
High production	12111.68	0.9398227	-0.235667	0.09158303	-0.005913	11244.14

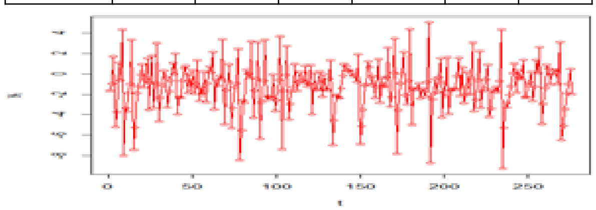

Figure 3: Seasonal markov switching model for sugar production

Table 3: The Estimated parameter of SMS-AR Model with state 1 and state 2

Model: Markov Switching SAR(2;4,0,0)(3,0,0)$_{12}$ State: 2 AIC: 6116.096, BIC: 6171.884, Loglik : -3052.048									
Low Production (State 1)					High Production (State 2)				
Coef.	Estima	Std.Er	t-stats	p-value	Estimat	Std.Er	t-stats	p-value	
Constant	12269.2	12435.8	0.9866	0.3238	11486.3	12435.8	0.9237	0.3556	
MSAR 1	0.9398	0.0797	11.791	<2.2e-1***	0.9398	0.0797	11.7917	<2.2e-1***	
MSAR 2	-0.2	0.06	-3.5	0.00041***	-0.23	0.06	-3.5	0.0004 ***	
MSAR 3	0.0916	0.0365	2.5096	0.01208***	0.0916	0.0365	2.5096	0.0120***	
MSAR 4	-0.60	8238.41	0.000	1.00000	-0.00	8238.41	0.00000	1.0000	
Residual Std.Err 11238.82					Residual Std.Err 11238.82				
Multiple R^2 = 0.6618					Multiple R^2 = 0.6542				
Standard Residuals					Standard Residuals				
Min	Q1	Med	Q3	Max	Min	Q1	Me	Q3	Max
-27232	-339	-1.90	382	3.983.	-243	-23	4	35	22

Parameter estimation

Using the criteria values of AIC value of the log likelihood function, estimated transition probability matrix that showed the probability of switching between regimes, estimated variance and considering the significant of p-value of estimated coefficients, and conclude the Seasonal Markov Switching autoregressive model. Using this selection strategy the best performance is obtained for the SMS-AR model with two regimes with four lag autoregressive components. The detail of the model fitted for SMS-AR is presented in Table 3. All estimated coefficients are statistically significant at conventional significance levels. The transition probabilities $(P(s_t = 1 | s_{t-1} = 1) = 0.592675,$

$(P(s_t = 2 | s_{t-1} = 2) = 0.387722)$ **Forecasting**

First, select the model which is fitted by the series of natural sugar production in india. Consider the problem of predicting future values from a SMS-AR $(2;4)(3,0,0,)_{12}$ process. Prediction is based on estimated SMS-AR model. In Table 5 and Figure 4, predict for 24 months (from January 2015 to December 2016), from the seasonal markov switching autoregressive model, calculate the error and do a comparison in Table 5.

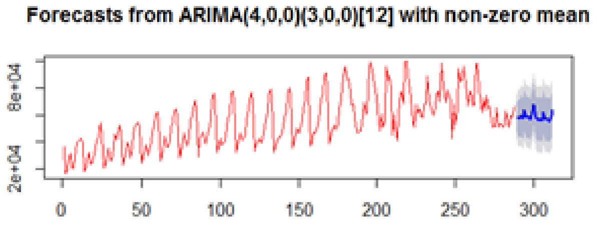

Figure 4: Forecasting from SMS-AR Model (2;4)*(3,0,0)12

Table 4: Transition Probabilities for Seasonal Markov Switching Models

S_t \ S_{t-1}	Low production	High production
Low production	0.5926750	0.4073250
High production	0.6122774	0.3877226

Table 5: Seasonal Markov Switching Model for error value

ME	RMSE	MAE	MPE	MAPE	MASE
623.745	8478.031	5838.552	-1.675	11.128	0.6771

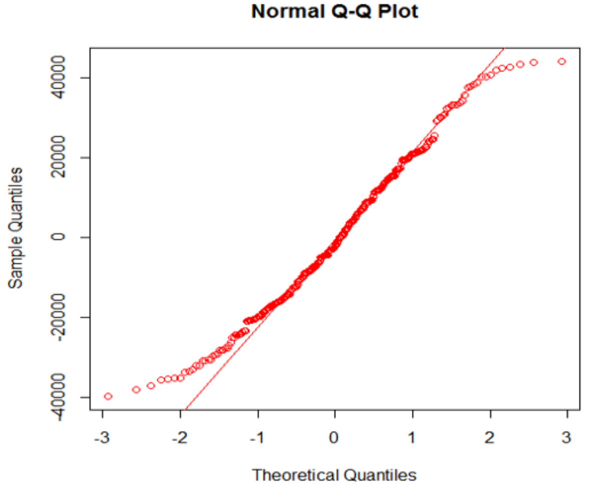

Figure 5: QQ-plot of residuals in Seasonal markov switching Models

RESULT AND DISCUSSION

Seasonal Markov Switching Autoregressive Model (SMS-AR) is selected with the order p of the autoregressive process. The AR order is p=4, therefore, there are two regime and the state 2. Consider the Markov switching autoregressive (MS-AR) model; the transitions are driven by a hidden two-state Markov chain. After that we will fit the model for using the state 2 and the AIC value 6116.096 and BIC: 6171.884.The parameters are estimated for the models it is given in the Table 3. The identified SMS-AR (2:4,0,0)(3,0,0)12 is as follows.

$$z_t = \begin{cases} 12269 + 0.93Z_{t-1} - 0.23Z_{t-2} + 0.09Z_{t-3} - 0.60Z_{t-4} + \varepsilon_t & s_t = 1 \\ 11486 + 0.93Z_{t-1} - 0.23Z_{t-2} + 0.09Z_{t-3} - 0.00Z_{t-4} + \varepsilon_t & s_t = 2 \end{cases}$$

Seasonal Markov switching autoregressive model is selected in the order (2;4,0,0)(3,0,0) the estimated values are given in the Table 3.

CONCLUSIONS

In this paper, introduce a seasonal Markov Switching Autoregressive Model has been applied in the sugar production in India. The time series data and relevant models are considered. They are able to describe the marginal distribution of the time series and thus pre-processing the data. The SMS-AR$(2;4,0,0)(3,0,0)_{12}$ model is identified. The probability of transition from low production (state1) to low production (state1) is 0.592675 and 0.6122774 denotes the probability of transition from low production (state 1) to high production (state 2) and so on. Since the single step transition probability matrix the stationary transition probability matrix is attained in 5^{th} step. At last, the values are forecasted for the 24 months ahead. The results show that the SMS-AR model is more accurate.

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