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Engineering

A MATHEMATICAL MODEL FOR OPTIMIZATION OF INSPECTION FREQUENCY FOR REPAIRABLE SYSTEMS

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ABSTRACT Inspection frequency is a denoting factor for maintenance and replacement decision making of repairable systems. This model presents an optimization model to assess inspection frequency by using the concept of Weibull distribution and Bathtub curve. Here we modified the classical Bathtub curve based on varying values of shape parameter and shape factor for different regions of bathtub curve. The objective of this study is to develop the deliberate strategy of inspection and maintenance for maximizing availability and minimizing maintenance cost by appropriate forecasting of inspection frequency.

KEYWORDS : Bathtub curve, Weibull distribution, Gamma function, inspection interval, maintenance management.

INTRODUCTION

Inspection is a significant tool of maintenance management, which is carried out to identify whether a system is well functioning or not, so that required maintenance actions can be performed in right time before failure occurs. Maintenance decisions are made on the basis of information obtained from inspections. Suitable maintenance actions maximize life of equipment, system availability, improve system safety and product quality and minimize the operating cost, failure cost, production loss due to interruptions. There are two types of items in industries repairable items and non-repairable items. After occurrence of a failure, when a device or component of equipment is repaired and put back in a well functioning condition without replacement of whole system, then the device is called repairable system. Non-repairable items are discarded and replaced with new one when they fail [1, 2]. This model deals with repairable systems. Determination of inspection frequency and inspection policy has a wide range of applications in different areas such as nuclear power plants, production machines, defense, medicine, aircraft maintenance. Inspection processes, of these heavy and critical machines, are often costly and complicated however inspection frequency should be as optimum as possible. It is necessary to determine optimum inspection frequency and make inspection policy for the purpose of forecasting the maintenance schedules. Many researchers studied and developed models for determining inspection frequency and inspection policy by considering various criterion functions and using different methodologies, various techniques and algorithms. Truong-Ba et al. [3] dilated the policy for condition-based inspection of heat exchangers by applying Markovian decision theory. The optimization has been conducted to elucidate future maintenance decisions in regard to losses due to failure and costs associated to maintenance. Xie et al. [4-5] used bathtub shaped failure rate function in reliability analysis and cost analysis, presented a modified Weibull distribution for modelling complete life time of systems. Pulcini G. [6] formulated a model to describe the failure processes of repairable equipment whose failure intensity shows a bathtub type dynamic behavior by using non-homogeneous poison process and maximum likelihood estimation.

Mathew S. [7] has determined the optimal inspection frequency of a system for all the three stages (DFR, CFR, IFR) of bathtub curve. A basic inspection model is suggested by Christer and Waller [8] by using delay time concept. Basic concept of bathtub curve is explained by Md. Modarres et al. [9] and they stated that all the three regions of bathtub curve can be represented by the Weibull distribution and it describes hazard rates conveniently. Ebeling [10] developed the basic reliability models and formulated the relationship among mean time between failures, gamma function, shape parameter and shape factor. Leung [11] proposed four optimum inspection policies to minimize the total costs associated with inspection, replacement and non detection of a failed system. Ito & Nakagawa [12] proposed a periodic inspection policy for a storage system which has to hold a higher reliability. An inspection model based on a three stage failure process, proposed by Wang [13] is an extension of the delay time concept [7]. Murthy and Naikan [14] presented a continuous monitoring strategy and monitoring interval for deferent life period of machine using the concept of bathtub curve and probability distributions. Baohe [15] addressed an optimal inspection policy for randomly failing multi-mode system by applying the SVT (supplementary variables technique). He obtained optimal critical value and optimal inspection period. Huynh et al. [16] aimed at to provide a periodic inspection model to assess the value of condition monitoring information for the maintenance decision making by using Gamma process, Non-homogeneous Poisson's process. Jardine and Hassounah [17] estimated the relation between the mean arrivals rate of breakdowns and the inspection frequency of equipment in a real-life situation. A mathematical model for determining a periodic inspection schedule for a single machine subject to random failure is developed by Hariga [18], in this model he showed that under certain conditions on the probability density function of failure, a unique optimal inspection interval can be obtained. Golmakani & Fattahipour [19] described optimal replacement policy and inspection interval using the modified transition probability matrixes and Proportional hazard model to estimate system failure rate. For avoiding huge losses due to equipment failures of degraded parts, a mathematical approach is formulated by Guo et al. [20] and an optimal inspection schedule has been set up for processing industries. The results obtained from analytical model have been validated with that of simulations and obtained distinctive throughput. Angeles and Kumral [21] have focused on estimating/quantifying optimal intervals between inspections calamitous failures cause longer downtime. The process plan and schedule have also been suggested for preventive maintenance based on the degradation rate of equipment. The presented model was found to be robust and effectual for maintenance and inspections of mining equipments.

List of Notations

$R(t)$ = Reliability function of Weibull distribution,
$f(t)$ = probability density function of Weibull distribution,
$h(t) = \lambda(t)$ =hazard function, failure rate function,
$\Gamma(t)$ =Gamma function,
MTBF = mean time between failures,
C_{RF} = maintenance cost rate factor,
M_{CR} = maintenance cost rate or maintenance cost per unit time.
t = equipment age, operating time of system t = 0 at beginning of life cycle,
T_1 = time of transition from DFR stage to slowly IFR stage,
T_2 = time of transition from slowly IFR stage to rapidly IFR stage,
T_T = transition time from one region to next region,
η = the scale parameter,
β = the shape parameter,
γ = the weight of the covariate in the hazard function,
θ = shape factor.

METHODS

The modified bathtub curve

As we know that in classical bathtub curve at constant failure rate (CFR) region, shape parameter (β) is unity, i.e., β = 1 and the curve form a straight line. This concept is true only in the theoretical point of view, but it is not true practically. It means that failure rate slowly increases with respect to time in a little amount because of wear of equipment, changes in environmental conditions and minor damages. Therefore the middle region of bathtub curve does not form a straight line rather than it forms an increasing curve because of increase of rate of rupture and shape parameter. Hence the middle region is known as slowly increasing failure rate (Slowly IFR) region which replaces the constant failure rate (CFR) region.

Now in modified bathtub curve the increasing failure rate (IFR) region starts from a slightly higher point of failure rate rather than the starting point of classical bath tub curve. In this IFR region failure rate increases faster than that of slowly increasing failure rate region and therefore this region is known as rapidly increasing failure rate (Rapidly IFR) region in modified bathtub curve. There are three following different regions are formed in modified bathtub curve:

(I) Decreasing failure rate region (DFR)
(ii) Slowly increasing failure rate region (slowly IFR)
(iii) Rapidly increasing failure rate region (Rapidly IFR)

Let these regions are defined in concern with operating time of equipment and the system is of new condition when the time is equal to zero, i.e., t = 0.
(I) 0 - T_1 represents the time interval for early life region, (DFR region)
(ii) $T_1 – T_2$ represents the slowly IFR region
(iii) Above T_2 represents the rapid IFR region.
When $\beta<1$ Represents DFR stage or early life stage,
$1<\beta<3$ Represents the slowly IFR stage,
$\beta>3$ Represents the rapidly IFR stage or wearout stage.

The model description

There has been developed the optimum inspection frequency and inspection policy model under the following assumptions.
Assumptions:
1. When the system fails the repair action returns the equipment to the conditions it was in just before the failure occurrence.
2.The condition of the monitored system deteriorates in time. Such deterioration may be measured directly by inspection at periodic interval.
3. There is no constant failure rate region in bathtub curve, the failure rate firstly decreases from 0 to T_1 time unit in DFR region, but increases slowly and slightly from T_1 time unit to T_2 time unit in SIFR region, increases rapidly in RIFR region above T_2 time unit.
4. Inspection does not degrade or rejuvenate the system.
5. Inspections are perfect and the system cannot fail during inspection.
6. The system's condition is identified by inspection only.
7. The system starts at time 0 in the running state.

Formulation of the model

The same as other inspection models, we also use the basic reliability concepts and probability distributions to develop the model. The aim is to develop a model for estimating optimum inspection frequency which is best suitable for making maintenance decisions and describing lifecycle of system [22, 23]. For the development of model we have used the concept of hazard rate and classical bathtub curve in concerns with Weibull distribution [9]. We have also evaluated the parameters of Weibull distribution such as scale parameter, shape parameter with respect to operating time of system [10]. Weibull distribution is used due to its flexibility for describing hazard rates, all three regions (early life, random failure, wearout regions) of the bathtub curve can be represented by the Weibull distribution. The Weibull distribution may be formulated in either a two or a three parameter form [13, 14]. The Reliability function and probability density function of the Weibull distribution is given by

$$R(t) = e^{-(t/\eta)^{\beta}}$$

$$f(t) = -\frac{dR(t)}{dt} \qquad (1)$$

$$f(t) = \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1} \exp\left[-\left(\frac{t}{\eta}\right)^{\beta}\right] \qquad (2)$$

When $\eta, \beta > 0, t > 0$
The failure rate or hazard rate is given by

$$\lambda(t) = \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1} \qquad (3)$$

When $\eta, \beta > 0, t > 0$

Mean time between Failures [10] of the Weibull Distribution can be derived as:

$$MTBF = \int_0^{\infty} t f(t) dt$$

$$MTBF = \int_0^{\infty} \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1} e^{-(t/\eta)^{\beta}} t dt \qquad (4)$$

$$y = \left(\frac{t}{\eta}\right)^{\beta} \Rightarrow t = \eta y^{\frac{1}{\beta}}$$

$$dy = \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1} dt$$

$$MTBF = \int t e^{-y} dy$$

By substituting the value of t we get,

$$MTBF = \int_0^{\infty} \eta y^{\left(\frac{1}{\beta}\right)} e^{-y} dy$$

We know that Gamma function

$$\Gamma(x) = \int_0^{\infty} y^{x-1} e^{-y} dy$$

However,

$$MTBF = \eta \Gamma\left(1 + \frac{1}{\beta}\right) \qquad (5)$$

By using equation (3) the hazard rate function of the Weibull distribution is given by

$$\lambda(t) = \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1} \qquad \eta, \beta > 0, t > 0$$

Determination of inspection frequency

As stated previously lifecycle of any repairable system follows the modified bath tub curve and goes from one stage to next stage of its life after certain time period, but time period or interval varies with type of system (electrical or mechanical system) [3]. A model to determine inspection frequency is described by considering various criterion function of reliability theory, this model is given as:

$$n(t) = N.\left(\frac{t}{T_T}\right)^{\theta} \qquad (6)$$

$$N = \frac{1}{MTBF} \qquad (7)$$

M_{CR} for the system may be defined by using C_{RF} as follows:

$$M_{CR} = n(t).C_{RF} \qquad (8)$$

There are following various criteria for different regions which should be satisfied for modified bathtub curve. These criteria are created as stated in various books and journals.

Lifecycle criterion of the system for DFR region

$$0 < t \le T_1 ; \theta < 0 ; T_T = T_1 ; \beta < 1$$

Criteria for Slowly IFR region,

$$T_1 < t < T_2; \theta > 0; T_T = T_1; 1 < \beta < 3$$

Criteria for rapidly IFR region,

$$T_2 \le t; \theta > 1; T_T = T_2; \quad \beta \ge 3$$

RESULTS AND DISCUSSION

Numerical application

Here we consider data modeling and evaluation of parameters by considering following data for a repairable system:

- Early life period starts at 0 and ends at 10 time units,
- Slowly IFR period starts at 10 time units and ends at 100 time units,
- Rapidly IFR or wear out period is above 100 time units.

Scale parameter of Weibull distribution is taken as 10 for all stages of bathtub curve, i.e., $\eta = 10$

Data evaluation of DFR region

$$0 < t \le T_1; \theta < 0; \beta < 1; T_T = T_1;$$

$$T_T = T_1 = 10$$

Let $\beta = 0.5$ for first point of operating time t = 1,
Using equation (5) we get,

$$MTBF = 10.\Gamma\left(1+\frac{1}{0.5}\right) = 10.\Gamma(3) = (10)(2)$$

$MTBF$=20
Now using equation (7),

$$N = \frac{1}{20} \quad N = 0.05$$

For calculating inspection frequency for t = 1 we use equation (6),

$$n(t) = 0.05\left(\frac{1}{10}\right)^{-1}, \quad n(t) = 0.5$$

Maintenance cost rate can be calculated for t = 20 units of time, Cost rate factor,
(C_{RF}) = 10000 per optimal inspection by using equation (8)

$$M_{CR} = 0.5 \times 10000 \quad M_{CR} = 5000$$

Similarly, inspection frequency can be calculated for other value of operating time t and β, in DFR region. This calculation for other values is shown in the table (1) and the graph for the calculated data is shown in the fig. (1).

Data evaluation of slowly increasing failure rate (slowly IFR) region

$$T_1 < t < T_2; \theta > 0; T_T = T_1; \quad 1 < \beta < 3;$$

$$T_T = T_2 = 100;$$

Let 2.1=β for first point of operating time ;20=t ;1.0=θ Using equation (5) we get,

$$MTBF = 10.\Gamma\left(1+\frac{1}{1.2}\right)$$

$MTBF$=9.3969
Now using equation (7),

$$N = \frac{1}{9.3969} \quad N = 0.106418$$

For calculating inspection frequency for t = 20, we use equation (6),

$$n(t) = 0.106418\left(\frac{20}{100}\right)^{-0.1}$$

$$n(t) = 0.090598$$

Maintenance cost rate can be calculated for t = 20 units of time, Cost rate factor,
(C_{RF}) = 10000 per optimal inspection by using equation (8)

$$M_{CR} = 905.98$$

Similarly, inspection frequency can be calculated for other value of operating time t and β, in slowly IFR region. This calculation for other values is shown in the table (2) and the graph for the calculated data is shown in the fig. (2).

Data evaluation for rapidly increasing failure rate (Rapidly IFR) region

$$T_2 \le t; \theta > 0; T_T = T_2;$$

$$\beta \ge 3;$$

$$T_T = T_2 = 100;$$

Let β=3.2 for first point of operating time $t = 120$;θ=3.3;

Table- 1 Inspection Frequency N(t) For Different Value Of Life Time (t) On Rapidly Dfr Region

t	H	T.T	β	1+1/β	gamma (G)	MTBF	N	θ	(t/T.T)	n(t)	Crf	Mcr
1	10	10	0.5	3	2	20	0.05	-1	10	0.5	10000	5000
2	10	10	0.55	2.8181	1.7051	17.051	0.0587	-0.9	4.2567	0.24965	10000	2496.51
3	10	10	0.6	2.6666	1.5085	15.085	0.0663	-0.8	2.6200	0.17368	10000	1736.82
4	10	10	0.65	2.5384	1.3678	13.678	0.0731	-0.7	1.8991	0.13885	10000	1388.48
5	10	10	0.7	2.4285	1.2670	12.670	0.0789	-0.6	1.5157	0.11963	10000	1196.27
6	10	10	0.75	2.3333	1.1882	11.882	0.0842	-0.5	1.2910	0.10865	10000	1086.52
7	10	10	0.8	2.25	1.133	11.33	0.0883	-0.4	1.1534	0.10180	10000	1017.96
8	10	10	0.85	2.1764	1.09	10.9	0.0917	-0.3	1.0692	0.09809	10000	980.949
9	10	10	0.9	2.1111	1.0516	10.516	0.0951	-0.2	1.0213	0.09712	10000	971.173
10	10	10	0.95	2.0526	1.0222	10.222	0.0978	-0.1	1	0.09783	10000	978.301

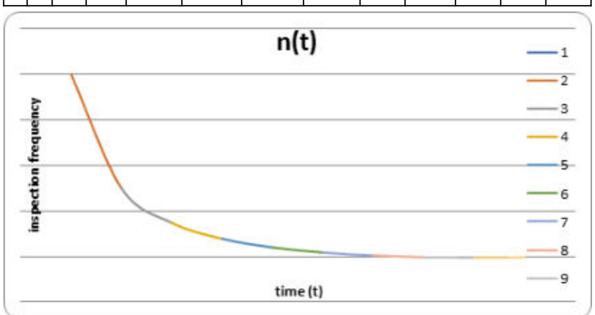

Figure 1: relationship between inspection frequency n(t) and time (t) in rapidly DFR region.

Table- 2 Inspection Frequency N(t) For Different Value Of Life Time (t) On Slow Ifr Region

t	η	(T.T)	β	(1+1/β)	gamma (G)	MTBF	N	θ	(t/T.T)	n(t)	Crf	Mcr
20	10	100	1.2	1.8333	0.93969	9.3969	0.1064	0.1	0.8513	0.0906	10000	905.979
30	10	100	1.4	1.7143	0.91057	9.1057	0.1098	0.2	0.7860	0.0863	10000	863.199
40	10	100	1.6	1.625	0.89724	8.9724	0.1114	0.3	0.7597	0.0847	10000	846.661
50	10	100	1.8	1.5556	0.88964	8.8964	0.1124	0.4	0.7579	0.0852	10000	851.871

60	10	100	2	1.5	0.88623	8.8623	0.1128	0.5	0.7746	0.0874	10000	874.035
70	10	100	2.2	1.4545	0.88566	8.8566	0.1129	0.6	0.8073	0.0912	10000	911.574
80	10	100	2.4	1.4167	0.88636	8.8636	0.1128	0.7	0.8554	0.0965	10000	965.057
90	10	100	2.6	1.3846	0.88785	8.8785	0.1126	0.8	0.9196	0.1035	10000	1035.27
100	10	100	2.8	1.3571	0.89018	8.9018	0.1123	0.9	1	0.1123	10000	1123.37
110	10	100	3	1.3333	0.89338	8.9338	0.1119	1	1.1	0.1231	10000	1231.28

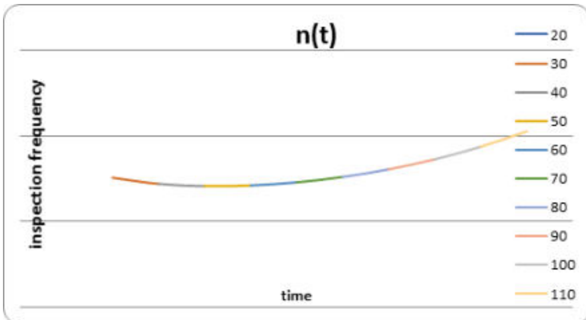

Figure 2: relationship between inspection frequency n(t) and time (t) in slow IFR region.

Table- 3 Inspection Frequency N(t) For Different Value Of Life Time (t) In Rapidly Ifr Region

T	η	T.T	β	(1+1/β)	G	MTBF	N	θ	(t/T.T)	n(t)	Crf	Mcr
120	10	100	3.2	1.313	0.896	8.960	0.1116	3.3	1.825	0.204	10000	2037.0
130	10	100	3.4	1.294	0.899	8.990	0.1112	3.6	2.572	0.286	10000	2860.3
140	10	100	3.6	1.278	0.901	9.007	0.1110	3.9	3.714	0.412	10000	4123.9
150	10	100	3.8	1.263	0.904	9.044	0.1106	4.2	5.490	0.607	10000	6070.5
160	10	100	4	1.250	0.906	9.064	0.1103	4.5	8.290	0.915	10000	9145.8
170	10	100	4.2	1.238	0.909	9.085	0.1101	4.8	12.769	1.405	10000	14054.7
180	10	100	4.4	1.227	0.911	9.108	0.1098	5.1	20.040	2.200	10000	22003.4
190	10	100	4.6	1.217	0.913	9.131	0.1095	5.4	32.009	3.505	10000	35054.7
200	10	100	4.8	1.208	0.916	9.156	0.1092	5.7	51.984	5.678	10000	56777.3

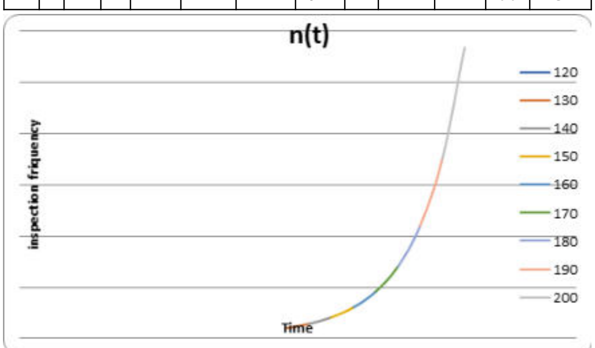

Figure 3: relationship between inspection frequency n(t) and time (t) in slow IFR region.

Using equation (5) we get,

$$MTBF = 10.\Gamma\left(1+\frac{1}{3.2}\right), \quad MTBF = 8.96$$

Now using equation (7),

$$N = \frac{1}{8.96}, \quad N = 0.11161$$

For calculating inspection frequency for t =120 units of time, we use equation (6),

$$n(t) = 0.11161\left(\frac{120}{100}\right)^{3.3}$$

$$n(t) = 0.2037$$

Maintenance cost rate can be calculated for t = 120 units of time, Cost rate factor,

(C_{RF}) = 10000 per optimal inspection by using equation (8)

$M_{CR} = 2037$ per unit time,

Similarly, inspection frequency and maintenance cost rate can be calculated for other value of operating time t and β, in rapidly IFR region. This calculation for other values is shown in the table (3) and the graph for the calculated data is shown in the fig. (3).

CONCLUSIONS

In the proposed approach an optimum inspection frequency and cost effective maintenance policy is developed. In this model shape parameter (β), shape factor (θ) and MTBF are considered as criterion function. As stated in modified bathtub curve we increased the value of both shape parameter (β<1) from 0.5 to 0.95 and shape factor (θ) from -0.1 to -0.9 as time increased from 1 to 10 units of time in DFR region as shown in table (1) and we found that values of both inspection frequency n(t) and maintenance cost rate decreases, and the related graph is shown in figure (1). Similarly in slowly increasing failure rate region (SIFR) when we increased the value of both shape parameter (β>1) and shape factor as time increased, we found that value of both inspection frequency and maintenance cost rate are increased slightly and slowly, i.e., inspection frequency and failure rate are not constant in this region as shown in table (2) and figure (2). Same as the previous two regions when we increased the value of both shape parameter (β>3) and shape factor (θ>3) as time increased, we found that value of both inspection frequency and maintenance cost rate are increased in large amount and rapidly as shown in table (3) and figure (3). From the above evaluated data we concluded that failure rate varies with time period and number of inspections in slowly IFR region is less than that of DFR region and rapidly IFR region. However this model assists us to develop maintenance policy and to make decisions of system replacement.

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