



## A STUDY ON DOMINOES: HOW THE SPACING BETWEEN SUBSEQUENT PAIRS AFFECTS THE ANGULAR VELOCITY, IN NEGLIGIBLE CONDITIONS

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**ABSTRACT** There has been much interest and studies in the dynamics of dominoes with an aim to investigate the variables acting upon overall domino propagation. Of these studies, little work is done at the mechanics of a singular domino pair. This research paper investigates the degree to which spacing affects angular velocity using scenario analysis. It further led to formulate a new algorithm that is depicted through graphical representation and substantiates an earlier study specifically on the vertical propagation of domino velocity.

### KEYWORDS :

#### INTRODUCTION

The dynamics of dominoes has been of much interest in recent years, as studies aimed to investigate the essential components and variables acting upon domino propagation has piled up. The degree to which spacing has affected angular velocity has been looked at seldom, however can be investigated using scenario analysis. However, prior knowledge can be useful in determining what the data and general trend in the relationship should be. Research done on the general velocity of propagation of dominos, and its relation to domino spacing, has been researched before in the early 1980's by where McLachlan and a group of scientists proposed the base equation by giving it the form:

$$v_{\text{McLachlan}} = \sqrt{gh} f(\lambda/h)$$

Here,  $g$  is the gravitational acceleration,  $h$  is the height of the domino and  $f(\lambda/h)$  was, at the time, an unknown function that related a ratio of the domino spacing to the height of the domino. It was unsure what exactly factored the relationship between these quantities, specifically for dominoes in linear motion. This was then figured out by Sun in 2020, who managed to fit his experimental data to a constant and a known trend. He then came up with:

$$v_{\text{Sun}} = C \lambda^{0.5} \sqrt{(\delta g/h)}$$

#### Research Question

How does the spacing between subsequent dominoes affect the angular velocity of a particular collision pair?

#### Experimental Design

1. From whichever domino set being used, pick out 6 dominoes, in particular, ones which are very similar in dimensions from a visual evaluation.
2. Once these dominoes are chosen, measure out the length, height and breadth of each domino using a ruler. This should be done by placing them flat on a table and start the measurement from the edges of the dominoes. Small variances of  $\pm 0.5$  mm should be fine.
3. After they have been picked out, get the tape and the ruler ready. Pick out a platform, preferably one that is rough in nature, and then use an inclinometer to see that there is no slant on it.
4. Using the tape, attach the ends of the ruler onto the surface, but make sure not to cover the end readings of the ruler so they can be read properly. So, tape the ends with a small strip horizontally.
5. Now, place the dominoes right next to the ruler to get the correct spacings between the dominoes.
6. Start with the first domino's front edge at the zero cm reading on the ruler.
7. Place the back edge of the domino exactly 10 mm after the first one.
8. Then, read where the front edge of the second end of the domino ends and add the breadth of the domino to get where the second domino will end.
9. From the second back edge, add the next 10 mm and then repeat steps 6-8 again until 5 of the 6 dominoes are placed
10. On the last collision pair of the 5 dominoes placed, i.e., between the fourth and fifth domino, place the tripod right in front of the pair.
11. Then adjust the camera such that it is securely on the tripod.
12. Switch on the camera and adjust the angle and view so that the complete vision of the fourth and fifth domino can be seen.
13. Once that has been done, start the recording on the camera and keep it as such so that it can capture everything. This will be used to analyse

the footage and get the time for fall to occur.

14. Using a protractor, use the sixth domino and place it approximately 40 mm from the first domino and hold the domino at an incline of 70 degrees

15. Once everything is in place, leave the domino at said incline and keep the camera recording until all the dominoes have fallen.

16. Repeat steps 5-15 to get 5 trials and create a folder on the camera that is titled "10 mm trials."

17. Repeat steps 5-16 but this time, using different spacings between dominoes in the given increments, 20, 30, 40, 50 mm (renaming the file of the camera associated with the spacing).

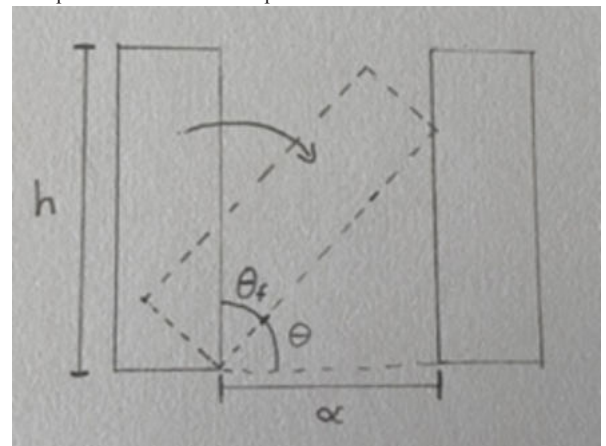
18. To get the time taken, upload all the recordings onto a device with LoggerPro downloaded.

19. Upload the first video by selecting the inset option on the tool bar and picking the "Movie..." option.

20. This will bring up the video analysis tab. On the window with the video opened, select the three red dots, and select the second option that is called add point

21. Forward through the frames of the video and select the exact frame where the fourth domino gets hit and then place the next point where the fourth one hits the fifth one

22. LoggerPro will automatically place the time for the frames in the table provided and do these steps for all the videos.



**Figure 1.3 – Domino Diagram**

Formula generated for data collection based on diagram above:  
 $\theta_f = \pi/2 - \arccos(\alpha/h)$

$$\omega = \theta_f/t$$

#### RESULTS

Domino Spacing/L/mm/ $\pm$ 0.5mm	Average Angular Velocity/AV/ $\omega$ / rad s <sup>-1</sup>
10.00	5.280
20.00	8.376
30.00	10.07
40.00	11.20
50.00	11.65

The bars on the data points are the associated uncertainties in the values calculated/measured.

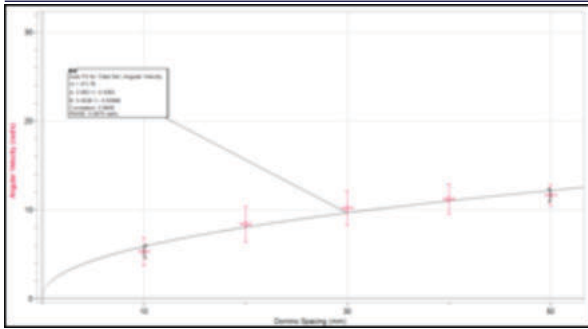


Figure 1.1 – Graph Of Observed Relationship

The results have to be linearized for analysis. And this can be done by looking at the base equation from the plot diagram above:

$$y = ax^b$$

$$y = 2.063x^{0.4538}$$

$$^{0.4538}\sqrt{y} = ^{0.4538}\sqrt{2.063} x$$

$$^{0.4538}\sqrt{y} \propto x$$

Domino Spacing/L/mm (±0.05 mm)	0.4538 <sup>th</sup> Root of Angular Velocity/ω <sup>2.02</sup> /rad <sup>2</sup> s <sup>-2</sup>
10	39.12
20	108.1
30	162.3
40	205.2
50	223.8

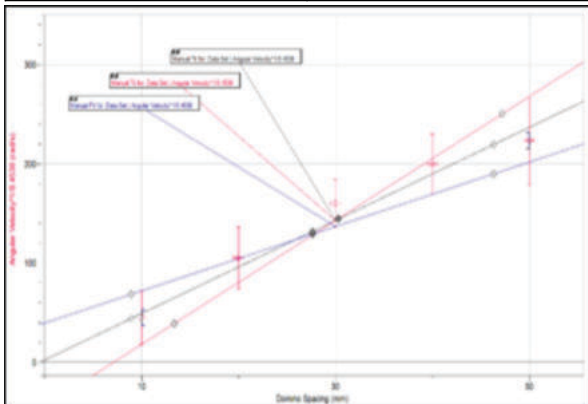


Figure 1.2 – Graph of linearized relationship

**CONCLUSION**

Based on the interesting results received from the investigation, quite a few good graphs and relationships were observed through their linearized and non-linearized variants. In the start, the hypothesis given suggested that the relationship between domino spacing and angular velocity would be a square root relationship, wherein, the angular velocity would increase in response to the domino spacing increasing. This was hypothesized under the pretense of previous research that concluded the relationship between domino spacing and linear propagation. The results received are in fair agreement with this hypothesis, as, the graph plotted, based on a best fit approach, showed that the best comparison of the 2 variables can be gotten from using a nth power graph, with it being 0.4538, while the graph depicted a relationship where, as one variable increased, the other increases at a decreasing rate.

This can be further proved by some mathematical proof, by finding the instantaneous rate of change at any point on the graph by taking the derivative of the found equation.

$$y = 2.063x^{0.4538}$$

By differentiating both sides with respect to x, and the use of power rule:

$$dy/dx = (2.063 \times 0.4538) (x)^{0.4538-1}$$

$$dy/dx = 0.936/x^{0.545}$$

From the deduced derivative, it is seen that as x approaches infinity, the expression evaluates to 0. Hence, showing that the more the value in the denominator increases, the more the rate of change, at that point, will decrease. Although the rate itself does not decrease at a very extreme rate, it is almost certain that the proposed equation is met.

**Literature Value Evaluation:**

Because the actual graph from Sun was shown to be a square root relationship, it would make sense to say that as velocity and angular velocity are directly proportional to each other, they would be raised to the same powers. The literature value can be taken as 0.5 and the experimentally determined value as 0.4538.

$$(|Lit. Value - Exp. Value| / Lit. Value) \times 100$$

$$(|0.4538 - 0.5| / 0.5) \times 100$$

$$9.2\%$$

The final evaluation of whether the real value could be in the given region of error would have to be evaluated from the uncertainty in the gradient. This was found to be:  $2/4.7 \times 100 = 42\%$ . Hence, because of the massive systematic uncertainty from the materials used, it is very much possible that the real value of the power could have been within this range, due to percentage being smaller than the percentage uncertainty. And so, the equation for finding a domino's angular velocity, within its given height, is given by:

$$\omega = (\pi/2 - \arccos(a/h))/t$$

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