Original Research Paper

Volume - 12 | Issue - 06 | June - 2022 | PRINT ISSN No. 2249 - 555X | DOI : 10.36106/ijar

Statistics

EVALUATION OF COHERENT FUNCTIONAL MODEL IN THE PROJECTIONS OF MORTALITY RATE

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ABSTRACT The present work proposed a new method for coherent mortality forecasting that incorporates forecasting interpretable product and ratio functions of rates using the functional data paradigm introduced in Hyndman and Ullah. The product-ratio functional forecasting method can be applied to two or more sub-populations incorporates convenient calculation of prediction intervals as well as point forecasts framework such as Hyndman and Booth. The main aim of study is to propose the use of Coherent functional demographic model as new approach for forecasting mortality. For analysis, two datasets in age-period format required for males and females separately: central death rates (mx) = number of deaths/mid-year population, exposures to the risk of death. We compare our results to longer fitting periods by making use of the extrapolated data. Our analysis shows, the estimated average gender-specific mortality (ax), the male and female mortality are almost identical till 20–24; but for subsequent age-groups, the average female mortalities are consistently lower than the male. The functional coherent model predicts a substantially sharper fall in female mortality as compared to the male counterparts. The gradual bulge towards the top is reflective of reduced mortality across ages. In conclusion, the product-ratio functional method may produce more accurate forecasts than other methods. The evaluations show that the functional data model of this study produces more accurate forecasts of death rates than the Lee Carter method and its variants in 13 out of 20 populations.

KEYWORDS : Coherent functional demographic model, morality, sex, age

Introduction:

In recent years, however, interest in the development of new models and strategies for modelling and forecasting mortality has slightly decreased in actuarial science and demography.[1]

In many countries all over the world, mortality forecasts are used to create and modify old-age or disability insurance systems and other social security programmes. Some authors have made important contributions to mortality in national forecasts. Their results build on the works on general population forecasting published by Lee (1974), Lee and Carter (1992) and many others.[2]

It is mostly urgent and required that the mortality forecast rates do not become divergent over time. The non-divergent forecasts for sub-populations within a larger population have been labelled "coherent". Such coherent mortality rate forecasting seeks to ensure in maintaining certain structural relationships based on extensive historic observation. Study carried out by Booth (2006) reports a general framework for forecasting life expectancy as the sum of a common trend and the population-specific rate of convergence towards that trend.[3]

Coherent models have recently been created to alleviate the constraints of independent mortality predictions. The main benefit of the coherent functional demographic model is its ability to ensure that male and female death rates do not diverge over time. The method involves forecasting product and ratio functions of rates using the functional data paradigm.

In the present research work, using a new method for coherent mortality forecasting which involves forecasting interpretable product and ratio functions of rates using the functional data paradigm introduced in Hyndman and Ullah (2007).[4] The new method is simple to apply, flexible in its dynamics, and produces forecasts that are at least as accurate in overall terms as the comparable independent method.

Data:

The data required for analysis purpose, we mostly rely on secondary data which will be obtained from census or SRS source. For analysis, two datasets in age-period format was required for males and females separately:

central death rates (mx) = number of deaths/mid-year population (with a more complicated calculation for mx at age 0 reflecting the skewed distribution of deaths in the first year of life)

exposures to the risk of death (ie mid-year population). To deal with this efficiently in the analysis, we will try to simply shift the year in the population datasets, so that the year for the population is the same as the year for the mx value it relates to. For initial analysis we will use actual data only by restricting our fitting period to the years 2001 to 2011.

Coherent Functional Method

The current study initially framed the problem in terms of forecasting male and female age-specific death rates because the two-sex application is the most common and the best understood.

Let $m_{t,F}(x)$ denote the female death rate for age x and year t, $t = 1, \dots n$. We will model the log death rate, $y_{t,f}(x) = \log[m_{t,F}(x)]$. Similar notation applies for males, $m_{t,M}(x)$ denote the male death rate for age x and year t, $t = 1, \dots n$. We will model the log death rate, $y_{t,m}(x) = \log[m_{t,M}(x)]$.

Functional Data Models

In the functional data paradigm, we assume that there is an underlying smooth function $f_{t,F}(x)$ that we are observing with error. Thus,

$$y_{t,F}(x_i) = \log\left[f_{t,F}(x_i)\right] + \sigma_{t,F}(x_i)\,\varepsilon_{t,F,i}, \qquad (1)$$

where x_i is the center of age group i $(i\ 0\ 1, \dots p)$, $E_{t,F,i}$ is an independent and identically distributed standard normal random variable, and $\sigma_{t,F}(x_i)$ allows the amount of noise to vary with age x. Analogous notation is used for males. For smoothing, we use weighted penalized regression splines (Wood 1994)[5] constrained so that each curve is monotonically increasing above age $x = 65$ (Hyndman and Ullah 2007)[4]. The weights are to take care of the heterogeneity in death rates across ages. The observational variance $\sigma_{t,F}(x)$ is estimated by using a separate penalized regression spline of $\{yt(xi) - \log[f_{t,F}(x_i)]\}^2$ against x_i, for each t.

Product-Ratio Method for Males and Females

We define the square roots of the products and ratios of the smoothed rates for each sex:

$$p_t(x) = \sqrt{f_{t,M}(x) f_{t,F}(x)} \text{ and } r_t(x) = \sqrt{f_{t,M}(x)/f_{t,F}(x)}.$$

We model these quantities rather than the original sex-specific death rates. The advantage of this approach is that the product and ratio will behave roughly independently of each other, provided that the subpopulations have approximately equal variances. On the log scale, these are sums and differences that are approximately uncorrelated. We use functional time series models (Hyndman and Ullah 2007)[4] for $p_t(x)$ and $r_t(x)$:

$$\log\left[p_t(x)\right] = \mu_p(x) + \sum_{k=1}^{K} \beta_{t,k}\phi_k(x) + e_t(x) \qquad (2a)$$

$$\log\left[r_t(x)\right] = \mu_r(x) + \sum_{l=1}^{L} \gamma_{t,l}\psi_l(x) + w_t(x). \qquad (2b)$$

where the functions {□$_k$(x)} and {ψ_l(x)} are the principal components obtained from decomposing {pt(x)} and {rt(x)}, respectively, and $\beta_{t,k}$ and $\gamma_{t,l}$ are the corresponding principal component scores. The function $\mu_p(x)$ is the mean of the set of curves {pt(x)}, and $\mu_r(x)$ is the mean of {$r_t(x)$}. The error terms, given by $e_t(x)$ and $w_t(x)$, have zero mean and are serially uncorrelated. The models are estimated using the weighted principal components algorithm of Hyndman and Shang [6], which places more weight on recent data and so avoids the problem of the functions {□$_k$(x)} and {ψ_l(x)} changing over time (Lee and Miller 2001)[7]. The coefficients, {βt,1,..., βt,K} and {gt,1,..., gt,L}, are forecast using time series models. To ensure that the forecasts are coherent, we require the coefficients {gt,l} to be stationary processes. The forecast coefficients are then multiplied by the basis functions, resulting in forecasts of the curves pt(x) and rt(x) for future t. If pn+h|n(x) and rn+h|n(x) are h-step forecasts of the product and ratio functions, respectively, then forecasts of the sex-specific death rates are obtained using fn+h|n,M(x) 0 pn+h|n(x)rn+h|n(x) and fn+h|n,M(x) 0 pn+h|n(x)/rn+h|n(x).

Result:

The fitted and forecasted mortality rates based on Coherent functional model were determined. The fitted mortality rates for 2001, and the forecasted ones for year 2040, 2060, 2080 and 2100 are shown in Figures 1 and 2, respectively in the logarithmic scale.

Figure 1: Observed and forecasted Male Indian mortality in log scale

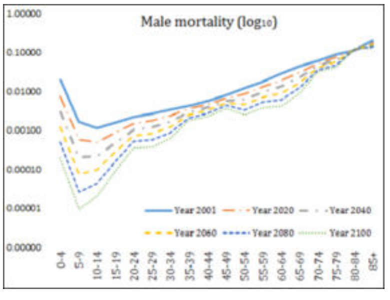
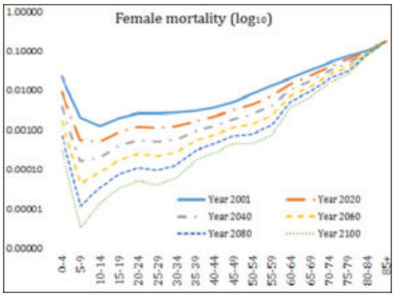


Figure 2: Observed and forecasted Female Indian mortality in log scale



In above figures 1 and 2, functional coherent model predicts a substantially sharper fall in female mortality as compared to the male counterparts. These projections not only help us to build the population pyramids as demonstrated in the next section, but also projects a realistic possibility of achieving progressively greater gender imbalance in future.

Table 1: Forecasted Male vs. Female Indian mortality for select age groups

Age groups	2001	2007	2017	2027	2037	2047	2057	2067	2077	2087	2097
0 to 4 male	76210	26656	82999	20834	50671	21145	6078	10144	26946	52851	31841
0 to 4 female	84581	31206	104099	25545	60439	25351	6865	12541	32988	61095	33406
25 to 29 male	14.6	18.9	28.6	18.3	22.7	28.4	17.5	29.4	23.4	17.5	9.4
25 to 29 female	11.0	17.1	25.4	22.6	19.3	19.9	12.9	23.6	22.4	15.6	4.9
55 to 59 male	978	935	919	989	920	861	968	892	941	965	1058
55 to 59 female	993	958	918	991	919	879	972	889	948	973	1084

Above table-1 provides similar detailed comparison between male and female mortality. The biggest gender gap is for the age-group 55–59, with the female mortality being substantially lower. On the other hand, male mortality is marginally lower for 0–4 age-group, and the mortality rates are almost identical for the age group 25–29.

Discussion:

Several studies have reported that the Coherent functional model has been based on functional forecasting of simple functions of the products and ratios of the subpopulation death rates rather than the rates themselves.8-11 Using two-sex data for India, we demonstrated the coherence of our results while also demonstrating that the coherent method is at least as accurate in overall terms as independent forecasts. The terms "non divergent" and "coherent" have previously been used interchangeably.12 Coherence, the essential feature of the product-ratio method, is ensured by the convergence of the ratio function to constant age-specific ratios.13 In the two-sex case, eventually constant ratios are in keeping with their biological basis.

Convergence to constant ratios does not imply that mortality differences between subpopulations tend to a set of constants. Only if both the forecast ratio function and forecast product mortality were constant would subpopulation differences be constant. The forecast ratio function typically increases at some ages and decreases at others; the net effect is that differences in life expectancy may be convergent or divergent. As the ratios converge to their historic means, their ability to effect change diminishes. For constant ratios, changing product death rates produce differences between subpopulations that change at the same rate as product mortality. Thus, change in product mortality governs whether and how differences converge or diverge in the long term. Convergence is constrained by the level of mortality and would occur only when death rates reach zero. Divergence and constant differences are similarly constrained, and for most populations are unlikely to be forecast given observed mortality trends.

For life expectancies at different ages, the age pattern of the mean ratios also influences forecast trajectories. This may result in widening differences in life expectancy at certain ages even when mortality is declining at all ages. Thus, while forecast death rates are constrained to tend toward a defined relative differential, forecast life expectancies are less precisely constrained.

It has been shown that the product-ratio functional method ensures non-divergence without compromising the overall (average) accuracy of the forecasts. In the two-sex case, the accuracy of the male mortality forecast was improved at the expense of accuracy in female mortality forecast; in other words, by adopting the coherent method, the accuracy of the forecasts for the two sexes was (partly) equalized. This feature of the method is useful in practical applications such as population forecasting, where it is preferable to maintain a balanced margin of error and hence a more balanced forecast population structure than might occur in the independent case. Coherent forecasting incorporates additional information into the forecast for a single subpopulation. The additional information acts as a frame of reference — limiting the extent to which a subpopulation forecast may continue a trend that differs from that of trends in related subpopulations. A similar approach has previously been adopted in forecasting fertility and for mortality.1

CONCLUSION:

According to the evidence, the product-ratio functional technique produces more accurate forecasts than other methods. In 13 of 20 populations, the analyses reveal that the functional data model used in this work delivers more accurate death rate projections than the Lee Carter approach and its modifications.

REFERENCES:

1. Booth H, Tickle L. (2008), "Mortality modelling and forecasting: A review of methods. " Annals of actuarial science. 3(1-2),3-43.
2. Giacometti R, Bertocchi M, Rachev ST, Fabozzi FJ. (2012), "A comparison of the Lee–Carter model and AR–ARCH model for forecasting mortality rates. " Insurance: Mathematics and Economics. 50(1),85-93.
3. Booth H, Hyndman RJ, Tickle L, De Jong P. (2006), "Lee-Carter mortality forecasting: a multi-country comparison of variants and extensions. " Demographic research. 15,289-310.
4. Hyndman RJ, Ullah MS. (2007), "Robust forecasting of mortality and fertility rates: a functional data approach. " Computational Statistics & Data Analysis. 51(10),4942-56.
5. Wood SN. (1994), "Monotonic smoothing splines fitted by cross validation. " SIAM Journal on Scientific Computing. 15(5),1126-33.
6. Hyndman RJ, Shang HL. (2010), "Rainbow plots, bagplots, and boxplots for functional data." Journal of Computational and Graphical Statistics. 19(1),29-45.
7. Lee R, Miller T. (2001), "Evaluating the performance of the Lee-Carter method for

forecasting mortality. " Demography. 38(4),537-49.
8. Hyndman RJ, Booth H, Yasmeen F. (2013), "Coherent mortality forecasting: the product-ratio method with functional time series models. " Demography. 50(1),261-83.
9. Shair S, Purcal S, Parr N. (2017), "Evaluating extensions to coherent mortality forecasting models. " Risks. 5(1),16.
10. Li H, Lu Y. (2017), "Coherent forecasting of mortality rates: A sparse vector-autoregression approach. " ASTIN Bulletin: The Journal of the IAA. 47(2),563-600.
11. Bergeron-Boucher MP, Canudas-Romo V, Oeppen J, Vaupel JW. (2017), "Coherent forecasts of mortality with compositional data analysis. " Demographic Research. 37,527-66.
12. Shang HL. (2016), "Mortality and life expectancy forecasting for a group of populations in developed countries: a multilevel functional data method. " The Annals of Applied Statistics. 10(3),1639-72.
13. Zhu N, Bauer D. (2011), "Coherent modeling of the risk in mortality projections: A semi parametric approach. " Working Paper, Georgia State University.