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Plant Biology

MODELLING OF TREE DIAMETER GROWTH USING BETA FUNCTION

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ABSTRACT Diameter growth data over time of two common tree species of Delhi urban forest viz. Polyalthia longifolia L. (PL) and Pongamia pinnata L. (PP) were evaluated. Mean diameter growth was measured over a time period of 11 consecutive years and was eventually modelled applying a three parametric beta function. Results, suggested that PL performed slightly better than PP on the basis of model performance. RMSE (0.551) and AIC (-10.601) values for PL were found to be low than that of PP. However, adjusted R^2 for both data sets were same (0.998). Moreover, maximum growth rate at ti was higher for PP (4.49) as compared to PL (3.22). Diameter growth of PP was quick and inflection time i.e., time at which maximum growth rate is achieved hits early than that of PL. All model parameters were stable and showed statistical significance over time. The objective of this study was to use beta function to predict diameter growth and derive growth parameters of two tree species in question. The function was found to be suitable to indicate for the sigmoidal pattern of determinate growth.

KEYWORDS : Delhi urban forest, Polyalthia longifolia, Pongamia pinnata, Beta function

INTRODUCTION

Forest biomass is the total amount of plant organic matter accumulated in the ecosystem which is the source of nutrients and energy base of the whole forest ecosystem[1]. The carbon sequestration ability of a forest can be directly reflected by biomass, and the carbon sink potential of forest ecosystem is determined by the accumulation rate of carbon[2]. Tree diameter is an important variable of biomass measurement, and also the most fundamental component for growth and yield modelling research, which assists in the study of basic growth law of forest trees and the establishment of forest management systems.

Modelling tree diameter growth over time is an important step towards collecting stand data, yield estimation and damage appraisals. Though, ample diameter- growth models have been developed in the past which are mostly linear in nature for various tree species but in the recent times nonlinear sigmoidal models have gained much popularity which are widely used in forest sciences to model diameter growth of individual trees and forest stands. Such models, works on the principle that there is always an upper limit of growth because of limited resources, competition among individuals and environmental constrains. Growth under such conditions forms a typical sigmoid curve, with an upper asymptote. In this type of growth pattern, earlier growth phase is relatively slow which accelerates thereafter and decreases in the final phase, approaching the upper asymptote. The entire growth process can also be demonstrated by an early exponential phase, a middle linear phase and eventually the curve levelling off.

Growth models assist forest managers in a variety of ways. Some important use includes the ability to predict future yields, and to explore silviculture options. Model- functions provides an efficient way to construct resource forecasts, but a more predominant role is their ability to explore management options and silviculture alternatives. The process of developing a growth model also offer interesting and new insights into the field of forestry. Growth models also have a broader role in forest management and in the formulation of forest policies. The model information linked with other environmental data assists in making predictions, formulate prescriptions and ultimately guide forest policy decisions. The study thus demonstrates, the application of Beta function to simulate diameter growth in two most common tree species comprising Delhi urban forest, in order to provide scientific basis for the growth dynamics prediction of tree DBH and provide basic data for scientific and reasonable forest management strategy.

MATERIAL AND METHODS

Delhi is located at 28° 37' N 77° 14' E/ 28.61°N 77.23°E and lies in north India. The city has a humid sub- tropical climate. Temperature ranges from 5 (Jan)- 40°C (June), with an annual mean temperature of 25°C. Delhi receives an annual rainfall between 600-800 mm, three-fourth of which falls in the month of July, August and September. The soil in this part of the city is mostly fertile and alluvial in nature. Vegetation of Delhi is thorny- scrub which is peculiar to arid and semi-arid regions.

The experiment was conducted at a District Park in Mayur Vihar Phase 1 managed by Delhi Development Authority (DDA) in East Delhi area from 2011- 2022. The area of the park was around 20.25 acres The soil of the experimental site was also analyzed for various physio-chemical characteristics (all numbers represent mean values) viz. pH (7.8), Water Holding Capacity (18.83%), Conductivity (1.57 dS/m), Total Organic Matter (1.04%), Total Carbon (0.6%), Total Nitrogen (59.57 Kg/ha) and Total Phosphorus (0.44 ppm). Two tree species viz. Polyalthia longifolia L. and Pongamia pinnata L. were planted in 2011 at the district park and diameter growth data was recorded at one year interval till 2022. All tree saplings were healthy, disease free and more or less uniform in size. The experiment was laid out in Randomized Complete Block Design (RCBD) with 2 replicates. The trees were planted at a spacing of 2m X 2m and examined for 11 consecutive years.

For constructing growth curves, mean diameter of trees was considered. Initially, Circumference at Breast Height (CBH) was measured using an inch tape which was subsequently converted to corresponding DBH values. In the next step, the diameter growth data of tree species were modelled against time by applying a 3- parametric Beta function. The rationale behind the use of growth functions lies in the fact that these functions have parameters with distinct biological meaning which in turn assists in mathematical interpretation of the entire growth process. Maximum information from the data sets can be extracted using nonlinear regression models.

Here, a three- parametric Beta function[3] was used as a model- function which is expressed in linear form as: *D*

$$D(t) = \alpha(1 + te - t/te - ti) * (t/te)^{\wedge}te/te - ti \quad \text{with } 0 \le ti < te \quad ...... \text{(I)}$$

Where, *D(t)* is the value of diameter at a given time "t",α is the upper asymptote i.e., maximum stable value of D, *ti* the inflection time i.e., time at which growth rate is maximum, *te* is the time at the end of growth period and *t* is a constant resembling time of growth (an independent variable).

Maximum growth rate (μi) at ti was obtained using the function:

$$\mu i = 2(te - ti)/te(te - ti) * (ti/te)^{\wedge}ti/te - ti * \alpha \; ..... \text{(II)}$$

The model was evaluated on basis of Adjusted R^2 [4], Root Mean Square Error[5] and Akaike Information Criterion[4]. Statistical significance of model parameters was calculated using Jackknife resampling procedure[6]. Modelling techniques were all implemented in Microsoft Excel, 2021. Statistical analysis was performed using add in functions viz. "Real Statistics Resource Pack" and "Data Analysis Tools". Nonlinear curve fitting was performed in excel "solver". Model with least RMSE/ AIC and maximum Adj. R^2 was considered to perform the best.

RESULTS AND DISCUSSION

Beta function was applied to diameter- growth data of two tree species over time. Coefficient of model parameters were calculated and statically examined. Parameter values of the growth function fitted well to observed values of diameter growth data, the details of which is provided in Table 1. The visual illustration of the fitting by beta function in figure 1 (a and b), exhibited a sigmoidal shape. Similar trends of sigmoid growth have also been observed for other tree variables as well viz. tree height[7], leaf area[8] and biomass[9,10,11]. Statistical analysis indicated that both fitting and evaluation statistics of PL was slightly better than that of PP in terms of RMSE and AIC values. However, adjusted R^2 for both data sets appeared to be similar.

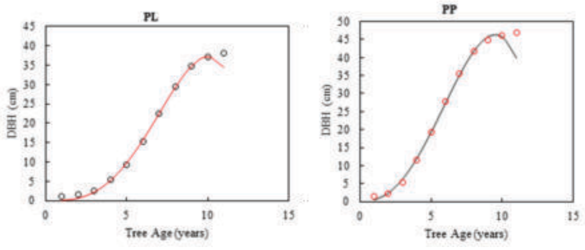

Fig 1 (a and b): Observed (circles) and fitted (line) values of diameter growth over time applying a 3- parametric beta function *PL= Polyalthia longifolia, PP= Pongamia pinnata

The inflection time (ti) i.e., time at which maximum growth rate is achieved for PP hits earlier (5.94 years) than that of PL (7.13 years). The diameter growth at ti for PP and PL were 27.22 and 22.43 cm respectively indicating that at ti, PP had a higher cumulative diameter growth than PL. Maximum growth rate at ti was higher in PP (4.49) as compared to PL (3.22) which suggested that the PP is fast growing in nature. The maximum expected diameter value at the end of growth for PP was 47.7 cm while for PL it was 38.4 cm. The length of exponential phase also varied for two species and the initial growth for PP was rather quick. At the end of exponential phase, mean diameter growth of PP was higher (11.7 cm) as compared to PL (5.1 cm). In this case, a 3-parametric growth function, with an initial diameter value of zero was applied. Here α is the maximum value of D, which is reached at time te. Equation I obey the constrains that D = 0, at the start of growth (i.e., t = 0), and D = α, when growth is terminated (i.e., t = te).

Table 1: Model fitting and evaluation statistics (± Standard Error)

(Model parameters)	P. longifolia	P. pinnata
α	38.425±0.122	47.742±0.151
te	10.428±0.036	10.033±0.051
ti	7.131±0.011	5.931±0.013
Adj. R2	0.998	0.998
RMSE	0.551	0.677
AIC	-10.601	-6.08

This equation is often applied to growth within the time span of $0 \leq t \leq$ te; otherwise, D has to be set as 0 if $t < 0$, and D if $t >$ te. Because equation I still produces an asymmetrical unimodal curve if te is exceeded, the equation with the extension that D is α, if $t >$ te is commonly referred to as the beta function of sigmoidal growth. The present study thus demonstrated that Beta function has the ability to model diameter growth of trees and the model- function can be broadly applied to evaluate growth of individual tree species in a forest.

CONCLUSIONS

To conclude, a three- parametric Beta function was applied to model diameter growth of two tree species over time, which forms an important composition of Delhi urban forest. A nonlinear and sigmoid function was used in order to mathematically interpret the entire growth process involving three model parameters which had distinct biological meaning. Results confirmed that beta function is suitable to indicate for the sigmoidal pattern of determinate growth.

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