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Mathematics

# A STUDY OF POISEUILLE FLOW BEHAVIOUR OF MICROPOLAR FLUID WITH SLIP AT THE WALL

**Santosh Kumar Singh Bhadauria** Department of Mathematics,Pt.J.N.College Banda

**Neelam Singh** Department of Mathematics, Bundelkhand College Jhansi

KEYWORDS :

The fluid mechanical and biomechanical processes which occur during the movement of an erythrocyte through the human capillary are very complex. Most of the rheological properties of blood depend on RBCs which in normal condition constitute about 95% number of total cells in the blood. Caro[4], Chien[8], Dintenfass [9], Kline[11], Woodcock [14] and many other researchers have shown that the knowledge of fluid mechanical and rheological properties of the flow of blood, like velocity profiles, velocity gradients and shear stress at the wall. Apparent viscosity of blood, deformability of red cells and spin boundary conditions could play a vital role in the diagnosis and treatment of many diseases.

The viscosity of blood varies greatly in different areas of circulation and depends on such factors as internal viscosity of red cells as well as on the lumen of the blood vessels, its diameter and on the flow velocity. When the diameter of blood vessel is less than 500 µm the circulation is called microcirculatory. This is responsible for 80 percent pressure drop in circulatory system. In capillary bed the transfer of nutrients to and removal of wastes from the living cells of the body is a part of microcirculation.

The constitutive equations for micro polar fluid [1,2] are
τij=(-p+µ'dkk ) δij+2µdij+2µ1 ϵijk (Wk-σk ).... (1)
Mij=ασ(p,p) δij+βσ(i,j)+γσ(j,i)...........(2)

τij-Stress tensor; Mij- couple tensor, ϵijk -alternating tensor, k- micro rotation,
Wk- vorticity, p- thermodynamic pressure, $d_{ij} = \frac{1}{2}[v_{i,j} + v_{j,i}], W_k = \frac{1}{2}\epsilon_{kij}v_{i,j}, \sigma_i$ time differential of micro polar rotation vector.

We assume RBC a rigid particle of finite dimension suspended in aqueous phase plasma. In tube of radius 20-500µmwe can well use the micro polar fluid model to explain the flow parameters characterizing the blood behavior. Several authors like Brunn [3], Chaturani [7], Chaturani and Biswas [5, 6], Oka Singh[12] and Shraokov [13]have used slip at the wall as a mechanism in explaining the blood flow.

In this paper, it is assumed that the blood flow does not obey the slip condition; the presence of red cell moving close to wall may be thought to be equivalent to a continuum which is slipping at the wall. The mutual inter actions of the particle give rise to spin at the boundary. It is assumed that shear viscosity of blood is different to that of solvent viscosity [5]. The study may be helpful in understanding the pathological conditions arising from the flow abnormality of blood.

**The Mathematical Model-**
A Poiseuille flow of incompressible micro polar fluid in a rigid circular tube is considered. It is assumed that radius of the tube is constant and blood is slipping at the vessel's wall.

**The Governing Equations-**
The equations of motion for the Poiseuille flow of micro polar fluid through a rigid tube of uniform circular cross section whose origin lies on the axis of the tube are

$$(\mu_1+\mu_2)\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v}{\partial r}\right)+2\mu_2\frac{1}{r}\frac{\partial}{\partial r}(r\omega) = \frac{\partial p}{\partial z} \quad \text{..............................(1.1)}$$

$$\mu_3\left(\frac{\partial^2\omega}{\partial r^2}+\frac{1}{r}\frac{\partial\omega}{\partial r}-\frac{1}{r^2}\omega\right)-2\mu_2\frac{\partial v}{\partial r}-4\mu_2\omega=0 \quad \text{..........................(1.2)}$$

where v,ω are axial and angular velocities of the particles, z,r are the axial and radial coordinates, p the pressure, µ1 the shear viscosity, µ2 the rotational viscosity and µ3 the angular viscosity.

**The boundary conditions-**
The appropriate boundary conditions are given by
v and ω remain finite at r=0……………………………….(1.3)
v=-m dv/dr at r=R……………………………………..(1.4)
ω=-n/2 dv/dr at r=R……………………………………..(1.5)

Where R is the radius of the tube, m the slip parameter and n is the wall effect parameter.

**The mathematical analysis-**
Integrating equation (1.1) , we get

$$(\mu_1+\mu_2)r\frac{\partial v}{\partial r}+2\mu_2 r\omega=\frac{r^2}{2}\frac{\partial p}{\partial z}+D$$

Where C is constant of integration.
Putting value of $\frac{\partial v}{\partial r}$ in equation (1.2), we get

$$\mu_3\left(\frac{\partial^2\omega}{\partial r^2}+\frac{1}{r}\frac{\partial\omega}{\partial r}-\frac{1}{r^2}\omega\right)-2\mu_2\frac{\left[\frac{r}{2}\frac{\partial p}{\partial z}+\frac{D}{r}-2\mu_2\omega\right]}{(\mu_1+\mu_2)}-4\mu_2\omega=0$$

$$\text{Or } \mu_3\left(\frac{\partial^2\omega}{\partial r^2}+\frac{1}{r}\frac{\partial\omega}{\partial r}-\frac{1}{r^2}\omega\right)-4\mu_2\omega+\frac{4\mu_2^2}{(\mu_1+\mu_2)}\omega=\frac{\mu_2}{(\mu_1+\mu_2)}r\frac{\partial p}{\partial z}+\frac{2\mu_2}{(\mu_1+\mu_2)}\frac{D}{r}$$

$$\text{Or } \frac{\partial^2\omega}{\partial r^2}+\frac{1}{r}\frac{\partial\omega}{\partial r}-\left(\frac{4\mu_1\mu_2}{\mu_3(\mu_1+\mu_2)}+\frac{1}{r^2}\right)\omega=\frac{\mu_2}{\mu_3(\mu_1+\mu_2)}r\frac{\partial p}{\partial z}+\frac{2\mu_2}{\mu_3(\mu_1+\mu_2)}\frac{D}{r}$$

$$\text{Or } \frac{\partial^2\omega}{\partial r^2}+\frac{1}{r}\frac{\partial\omega}{\partial r}-\left(h^2+\frac{1}{r^2}\right)\omega=\frac{\mu_2}{\mu_3(\mu_1+\mu_2)}r\frac{\partial p}{\partial z}+\frac{2\mu_2}{\mu_3(\mu_1+\mu_2)}\frac{D}{r}$$

$$\text{Or } \frac{\partial^2\omega}{\partial r^2}+\frac{1}{r}\frac{\partial\omega}{\partial r}-\left(h^2+\frac{1}{r^2}\right)\omega=-\frac{\mu_2}{\mu_3(\mu_1+\mu_2)}rC+\frac{2\mu_2}{\mu_3(\mu_1+\mu_2)}\frac{D}{r} \quad \text{.......................(1.7)}$$

$$\text{Where } \eta=\frac{\mu_2}{\mu_1+\mu_2},\ h^2=\frac{4\mu_1\eta}{\mu_3} \text{ and } \frac{\partial p}{\partial z}=C$$

$\because \omega$ remain finite at $r=0 \rightarrow D=0$

Solving equation (1.7) , we get

$$\omega = A\,I_1(hr)+BK_1(hr)+\frac{1}{4\mu_1}rC$$

Where I1 (hr) and K1 (hr) are solutions of modified Bessel's equation

$$\frac{\partial^2\omega}{\partial r^2}+\frac{1}{r}\frac{\partial\omega}{\partial r}-\left(h^2+\frac{1}{r^2}\right)\omega=0$$

Applying boundary condition (1.3) , B=0
Also $\frac{1}{(4\mu_1)}$ rC is particular integral of differential equation (1.7).

Solution of differential equation (1.7) is given by
$\omega = A I_1(hr)+\frac{1}{(4\mu_1)}rC$ ……………………(1.8)

Now, $\omega(R)=A\,I_1(hR)+\frac{1}{4\mu_1}RC$ & $\left(\frac{\partial v}{\partial r}\right)_{r=R}=\frac{\left[\frac{R}{2}\frac{\partial p}{\partial z}-2\mu_2\omega(R)\right]}{(\mu_1+\mu_2)}$

$$=\frac{RC}{2(\mu_1+\mu_2)}-\frac{2\mu_2}{(\mu_1+\mu_2)}\left\{A\,I_1(hR)+\frac{1}{4\mu_1}RC\right\}$$

$$\because \omega=-\frac{n}{2}\frac{dv}{dr}\text{ at } r=R$$

$$\therefore A\,I_1(hR) + \frac{1}{4\mu_1}RC$$
$$-\frac{n}{2}\left[-\frac{RC}{2(\mu_1+\mu_2)}-\frac{2\mu_2}{(\mu_1+\mu_2)}A\,I_1(hR)\quad\frac{\mu_2}{2\mu_1(\mu_1+\mu_2)}RC\right]$$

$$--\frac{n}{2}\left[-\frac{1}{2\mu_1}RC-\frac{2\mu_2}{(\mu_1+\mu_2)}A\,I_1(hR)\right]--\frac{n}{2}\left[-\frac{1}{2\mu_1}RC-2\eta A\,I_1(hR)\right]$$

Or $A\,I_1(hR)+\frac{1}{4\mu_1}RC-\frac{n}{4\mu_1}RC+n\eta A\,I_1(hR)$

Or $A\,I_1(hR)(1\quad n\eta)-\frac{(n-1)RC}{4\mu_1}$

Or $A-\frac{(n-1)RC}{4\mu_1(1-n\eta)I(hR)}$

Putting value of $A$ in equation (1.8), we get

$$\omega = A\,I_1(hr)+\frac{1}{4\mu_1}rC=\frac{(n-1)RC}{4\mu_1(1-n\eta)J_1(hR)}I(hr)+\frac{1}{4\mu_1}rC$$

$$or\ \omega=\frac{CR}{4\mu_1}\left[\frac{r}{R}-n'\frac{I_1(hr)}{I_1(hR)}\right] \quad \text{.................................(1.9)}$$

where $n'\quad\frac{1-n}{1-n\eta},(0\le n'\le 1)$

Writing $\omega_1-\frac{CR}{4\mu_1}\ \&\ \bar\omega-\frac{\omega}{\omega_1}$,

We get $\bar\omega=\left[\frac{r}{R}-n'\frac{I_1(hr)}{I_1(hR)}\right]$........................(1.10)

Now using equation (1.8) in equation (1.6) with boundary conditions , we obtain

$$\frac{\partial v}{\partial r}=\frac{\left[\frac{r}{2}\frac{\partial p}{\partial z}+\frac{D}{r}-2\mu_2\omega\right]}{(\mu_1+\mu_2)}=-\left[\frac{Cr}{2(\mu_1+\mu_2)}+2\eta\omega\right]$$

$$or\ \frac{\partial v}{\partial r}=-\left[\frac{Cr}{2(\mu_1+\mu_2)}+2\eta\frac{CR}{4\mu_1}\left\{\frac{r}{R}-n'\frac{I_1(hr)}{I_1(hR)}\right\}\right]$$

Integrating

$$v(r)=-\left[\frac{Cr^2}{4(\mu_1+\mu_2)}+2\eta\frac{CR}{4\mu_1}\left\{\frac{r^2}{2R}-n'\frac{I_0(hr)}{h\,I_1(hR)}\right\}\right]+B$$

At $r-R$

$$v(R)=-\left[\frac{CR^2}{4(\mu_1+\mu_2)}+2\eta\frac{CR}{4\mu_1}\left\{\frac{R^2}{2R}-n'\frac{I_0(hR)}{h\,I_1(hR)}\right\}\right]+B$$

And $\left(\frac{\partial v}{\partial r}\right)_{r=R}=-\left[\frac{CR}{2(\mu_1+\mu_2)}+2\eta\frac{CR}{4\mu_1}\{1-n'\}\right]$

$$\therefore v=-m\frac{dv}{dr}\ at\ r=R\Rightarrow v(R)=-m\left(\frac{\partial v}{\partial r}\right)_{r=R}$$

$$\Rightarrow-\left[\frac{CR^2}{4(\mu_1+\mu_2)}+2\eta\frac{CR}{4\mu_1}\left\{\frac{R^2}{2R}-n'\frac{I_0(hR)}{h\,I_1(hR)}\right\}\right]+B\quad m\left[\frac{CR}{2(\mu_1+\mu_2)}+2\eta\frac{CR}{4\mu_1}\{1-n'\}\right]$$

$$\therefore B\quad\frac{CR^2}{4(\mu_1+\mu_2)}+2\eta\frac{CR}{4\mu_1}\left\{\frac{R^2}{2R}-n'\frac{I_0(hR)}{h\,I_1(hR)}\right\}+m\left[\frac{CR}{2(\mu_1+\mu_2)}+2\eta\frac{CR}{4\mu_1}\{1-n'\}\right]$$

$$\therefore v(r)--\left[\frac{Cr^2}{4(\mu_1+\mu_2)}+2\eta\frac{CR}{4\mu_1}\left\{\frac{r^2}{2R}-n'\frac{I_0(hr)}{h\,I_1(hR)}\right\}\right]+\frac{CR^2}{4(\mu_1+\mu_2)}$$
$$+2\eta\frac{CR}{4\mu_1}\left\{\frac{R^2}{2R}\quad n'\frac{I_0(hR)}{h\,I_1(hR)}\right\}+m\left[\frac{CR}{2(\mu_1+\mu_2)}+2\eta\frac{CR}{4\mu_1}\{1\quad n'\}\right]$$

Or $v(r)-\frac{C(R^2-r^2)}{4(\mu_1+\mu_2)}+\frac{\eta C(R^2-r^2)}{4\mu_1}+\frac{2\eta CR\,n'}{4\mu_1}\left(\frac{I_0(hr)-I_0(hR)}{hI_1(hR)}\right)+\frac{mCR}{4\mu_1(\mu_1+\mu_2)}[2\mu_1+2\eta(1\quad n')(\mu_1+\mu_2)]$

Or $v(r)-C(R^2\quad r^2)\left[\frac{1}{4(\mu_1+\mu_2)}+\frac{\mu_2}{4\mu_1(\mu_1+\mu_2)}\right]++\frac{2\eta CR\,n'}{4\mu_1}\left(\frac{I_0(hr)-I_0(hR)}{hI_1(hR)}\right)+\frac{mCR}{4\mu_1(\mu_1+\mu_2)}[2\mu_1+2\mu_2(1-n')]$

Or $v(r)-\frac{C(R^2-r^2)}{4\mu_1}+\frac{2\eta CR\,n'}{4\mu_1}\left(\frac{I_0(hr)-I_0(hR)}{hI_1(hR)}\right)+\frac{2mCR}{4\mu_1}[1-\eta n']$

Or $v(r)-\frac{CR^2}{4\mu_1}\left[1-\frac{r^2}{R^2}-2\eta n'\left(\frac{I_0(hR)-I_0(hr)}{RhI_1(hR)}\right)+2b[1-\eta n']\right]$..................(1.1 1)

$\bar v\quad\frac{v(r)}{v_1}\quad 1-\frac{r^2}{R^2}-2\eta n'\left(\frac{I_0(hR)-I_0(hr)}{RhI_1(hR)}\right)+2b[1-\eta n']$..................(1.12)

Where $b-\frac{m}{R}$ and $v_1-\frac{CR^2}{4\mu_1}$

Velocity at the axis ($r=0$) is

$$v_0\quad\frac{CR^2}{4\mu_1}\left[1-2\eta n'\left(\frac{I_0(hR)-1}{Rh\,I_1(hR)}\right)+2b[1-\eta n']\right]$$

Or

$$\overline{v_0}-1-2\eta n'\left(\frac{I_0(hR)-1}{Rh\,I_1(hR)}\right)+2b[1-\eta n']$$

**Volume Flow Rate-**

The volume flow rate Q of the fluid flowing per unit time across the cross section of tube is given by

$$Q=\int_0^R 2\pi r\,v(r)dr$$

$$-\frac{\pi CR^2}{2\mu_1}\int_0^R\left[r-\frac{r^3}{R^2}-2\eta n' r\left(\frac{I_0(hR)-I_0(hr)}{Rh\,I_1(hR)}\right)+2br(1-\eta n')\right]dr$$

$$-\frac{\pi CR^2}{2\mu_1}\left[\frac{r^2}{2}\quad\frac{r^4}{4R^2}+2b\frac{r^2}{2}(1\quad\eta n')\right]_0^R$$
$$-\frac{\pi CR^2}{2\mu_1}2\eta n'\int_0^R\left[\frac{r\,I_0(hR)}{Rh\,I_1(hR)}-\frac{r\,I_0(hr)}{Rh\,I_1(hR)}\right]dr$$

$$=\frac{\pi CR^2}{2\mu_1}\left|\frac{R^2}{2}\quad\frac{R^4}{4R^2}+bR^2(1\quad\eta n')\right|\quad\frac{\pi CR^2}{\mu_1}\eta n'\left|\frac{I_0(hR)}{Rh\,I_1(hR)}\frac{r^2}{2}\quad\frac{r\,I_1(hr)}{Rh^2\,I_1(hR)}\right|_0^R$$

$$-\frac{\pi CR^4}{8\mu_1}[1+4b(1\quad\eta n')]\quad\frac{\pi CR^4}{8\mu_1}4\eta n'\left[\frac{I_0(hR)}{Rh\,I_1(hR)}\quad\frac{2}{(Rh)^2}\right]$$

$\therefore Q-\frac{\pi CR^4}{8\mu_1}\left[1+4b(1-\eta n')-4\eta n'\left\{\frac{I_0(hR)}{RhI_1(hR)}-\frac{2}{(Rh)^2}\right\}\right]$..................(1.13)

$Q=\frac{Q}{Q_0}=\left[1+4b(1-\eta n')-4\eta n'\left\{\frac{I_0(hR)}{RhI_1(hR)}-\frac{2}{(Rh)^2}\right\}\right]$......................(1.14)

Apparent fluidity $\phi_a$ is given by $\phi_a=\frac{8Q}{\pi CR^4}$

$$\therefore\phi_a=\frac{8Q}{\pi CR^4}=\frac{1}{\mu_1}\left[1+4b(1-\eta n')-4\eta n'\left\{\frac{I_0(hR)}{Rh\,I_1(hR)}-\frac{2}{(Rh)^2}\right\}\right] \text{............(1.15)}$$

Or $\phi_a-\frac{1}{\mu_1}[1+4b(1-\eta n')-4\eta n'\phi]$ ....................(1.14)

Where $\phi=\frac{I_0(hR)}{Rh\,I_1(hR)}-\frac{2}{(Rh)^2};0\le\phi\le 1$

**RESULTS AND DISCUSSION-**

When H=40%, , where solvent viscosity μ_s=1.2 ,μ_2=0.98 and μ3=1.2×10[-8] gm cm/sec , b=0.02 ,R=40 μm & μ1=μs+μ2

Then $\omega\quad\left[\frac{r}{R}\quad n'\frac{I_1(hr)}{I_1(hR)}\right]\quad\left[\frac{r}{R}\quad 3.19\,n'I_1(hr)\right]$

Or $\bar\omega=x-3.19\,n'I_1(0.6004\,x)$, where $x=\frac{r}{R}$ .........................(1.15)

$$\bar v=1-\frac{r^2}{R^2}-2\eta n'\left(\frac{I_0(hR)-I_0(hr)}{Rh\,I_1(hR)}\right)+2b[1-\eta n']$$

Or $\bar v-1.04-x^2-3.61n'-3.292\,n'I_0(0.6004x)$ ....................(1.16)

$$\overline{v_0}=1-2\eta n'\left(\frac{I_0(hR)-1}{Rh\,I_1(hR)}\right)+2b[1-\eta n']=1.04-0.4275224n'$$

i.e. $v_0\quad 1.04-0.428\,n'$ ..............................(1.17)

$$Q\quad 1+4b(1\quad\eta n')\quad 4\eta n'\left\{\frac{I_0(hR)}{Rh\,I_1(hR)}\quad\frac{2}{(Rh)^2}\right\}=1.08-0.56\,n'$$

i.e. $\bar Q=1.08-0.56\,n'$ .....................(1.18)

From equation (1.9) and (1.10), we see that particles rotational velocity is unaffected with slip at the wall. When we plot graph between $x\left(=\frac{r}{R}\right)$ and $\bar{\omega}$ for different values of n', we see that rotational velocity increases with respect to r for n' lies in the range 0 to 0.85. When values of n' lies between 0.95 to 1.0, the rotational velocity first increases and then decreases slowly as $\frac{r}{R}$ approaches to 1. Clearly particle rotational velocity decreases as n' increases. At $\frac{r}{R} = 1$, *i.e.* at the wall, the rotational velocity reduces to its minimum value when n'=1.

From equation (1.12), we observe that the axial velocity increase with slip at the wall. Equation (1.16) shows the variation of particle's axial velocity $\bar{v}$ with respect to radial coordinate r for different values of n', we see that axial velocity decreases as n' increases.

Equation (1.17) between $\overline{v_0}$ and n′ is linear relation, so represent line. This shows the variation of velocity at the axis $v_0$ for different values of $n'$. It is clear that velocity $v_0$ decreases as $n'$ increases. We also observe that velocity is maximum for $n' = 0$.

Also Equation (1.18) between $Q$ and n′ is linear relation, so represent line. This shows the variation of volume Flow Rate $\bar{Q}$ with respect to n′. Clearly $\bar{Q}$ decreases as n′ increases from 0 to 1. Volume Flow Rate $Q$ is maximum when n′=0.

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