RESEARCH PAPER | Engineering | Volume - 5 | Issue - 1 | Jan Special Issue - 2015 | ISSN - 2249-555X

Analytical Method for Prediction of Stability for Cutting Tool System

KEYWORDS: stability, stiffness, damping ratios, chatter.

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ABSTRACT The machine tool structure loose stability due to the dynamic forces which arise during the cutting operation. Due to this self excited vibrations occurs which is known as chatter. Chatter is a so-called self-excited oscillation because the vibration itself generates the energy that again creates the vibration. The single degree of freedom chatter theory has been considered for only those cases where rigidity of the tool and support is relatively small in one direction which may allow the tool to vibrate in that direction. Otherwise, the tool motion will not be straight and two degree of freedom theory will have to be used for analyzing the problem. An analytical model is simulated using MATLAB. The model proposed in this work is an analytical model used for the prediction of stability limit for hard turning systems during cutting process by using various modes, speeds, width of cut, damping ratios and stiffness.

I. INTRODUCTION

With the modern trend of machine tool development, accuracy and reliability are gradually becoming more prominent. To achieve higher accuracy and productivity it is not enough to design the machine tools from static considerations without considering the dynamic instability of the machine tools. If there are any relative vibratory motion present between the cutting tool and the job, it is obvious that the performance of the machine tool will not be satisfactory. During operations, machine tools are subjected to static or dynamic loads, these loads/forces may act in either of the following manners,

a) Dynamic behavior caused absolutely by the load acting during the action of the load (forced vibrations),
b) Dynamic behavior initiated by a load but persisting after load has ceased to act (free vibrations),
c) Dynamic behavior through an interaction between the structure and cutting process (Self-excited vibrations).

The parameters like mass, stiffness of tool, rake angle, damping of tool and speed are considered to implement them in MATLAB.

II. MATHEMATICAL MODELING : STABILITY OF CUTTING TOOL SYSTEM

Basic and important concepts and equations of the structural dynamics, which are used in the following sections, which are used to develop matlab program, with the discussion of a single degree of freedom system. A viscously damped single degree of freedom system model is shown in Figure 1. Assuming that, any increment of force (P) due to regeneration effect occurring in y direction continues to act in β–direction and that is the only force acting on the system.

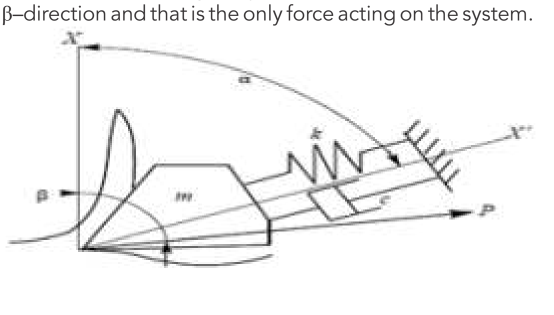

Figure 1. Single Degree of Freedom turning operation

surface, the motion along y is related to motion along x by,

$$y = x\cos\alpha \qquad y = y(t) - y(t-T) \qquad y = [x(t)] - x(t-t)]\cos\alpha \tag{1}$$

If the coupling coefficient between the force along y and z is given by r,

$$P(t) = -ry = -r[x(t)] - x(t-t)]\cos\alpha \tag{2}$$

The force component along x is given by:

$$P_x(t) = P(t)\cos(\alpha-\beta) \tag{3}$$

The equation of motion along x is:

$$m\ddot{x} + c\dot{x} + kx = P_x(t) = P(t)\cos(\alpha-\beta) \tag{4}$$

where
m = Mass
c = Damping coefficient
k = Stiffness (k = P / x)
The solution is given by:

$$x(t) = \frac{P(t)\cos(\alpha-\beta)}{k}\,\frac{1}{1-\left(\frac{\omega}{\omega_o}\right)^2 + j2\xi\left(\frac{\omega}{\omega_o}\right)} \tag{5}$$

where,

$$\xi = \text{damping ratio} = \frac{c}{c_c}$$

$$\omega_o = \text{natural frequency} = \frac{k}{m}$$

$$\omega = \text{forcing frequency}$$

$$r = \text{frequency ratio} = \frac{\omega}{\omega_o}$$

Re-arranging the equation (5)

$$P(t) = \frac{Kx(t)}{\cos(\alpha-\beta)}\left[(1-r^2) + j2\xi r\right]$$

Using the equation (2) the above equation can be re-written as

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$$x(t)\left\{\frac{K}{\cos\alpha\cos(\alpha-\beta)}\right\}\left[(1-r^2)+j2\zeta r\right]=-r[x(t)]-x(t-T)$$

Let

Coupling coefficient $u=\cos\alpha\cos(\alpha-\beta)$ (7)

Cross receptance $\phi=\frac{u}{k}\frac{1}{(1-r^2)+j2\zeta r}$ (8)

Combining equation (6) with equation (8)

$$\frac{x(t)}{\phi}=-r[x(t)]-x(t-T)]$$

Hence,

$$\frac{|x(t)|}{|x(t-T)|}=\frac{r}{\left(\frac{1}{\phi}\right)+r}=\frac{\phi}{\phi+\left(\frac{1}{r}\right)}=q \qquad (9)$$

q<1, the system is stable, i.e. the amplitude does not build up, but when q=1,the system is at the threshold of stability.

Tlusty and Polacek assumed 'r' as real and hence they solved the stability criterion with only real part of ϕ

$$\phi=\frac{u}{K}\left[\frac{(1-r^2)}{(1-r^2)^2+j4\zeta^2r^2}-j\frac{2\zeta r}{(1-r^2)^2+j4}\right] \qquad (10)$$

$$\phi=G'+jH'=u(G+jH) \qquad (11)$$

Replacing ϕ by Re (ϕ) in equation (9),

$$q=\frac{|G'|}{\left|G'+\frac{1}{r}\right|}=1 \text{ (threshold of stability)}$$

This equation is satisfied, if

$$|G'|=|G'|+\frac{1}{r} \qquad (12)$$

From which

$$r^*=-\frac{1}{2G'}$$

The limiting coupling coefficient is given by limiting value of the negative reciprocal of the real part of cross receptance for threshold criterion of stability.

$$G'=G^*\cos\alpha*\cos(\alpha-\beta)$$

When $\alpha=\beta=0$, G' = G.

The limiting coupling co-efficient is given by:

$$r_{max}^*=-\frac{1}{2G_{min}} \qquad (13)$$

The governing equations of machine chattering can be derived from the general equation of vibration and the regenerative chatter equations.

In orthogonal cutting, the cutting force P(t) is proportional to the cutting area (the product of the chip width or depth of cut ap and thickness h)

$$P(t)=k_sa_ph=k_sa_p\left[x(t-T)-x(t)\right] \qquad (14)$$

T is the time interval between the previous and current cuts. Substituting equation (14) into the general equation of vibration:

$$m\ddot{x}+c\dot{x}+kx=P(t) \qquad (15)$$

The depth of cut is found from equation (14)

$$a_p=-\frac{1}{2k_sG} \qquad (16)$$

where G is the real part of the frequency response function (FRF) and from the equations (10) and (11)

$$G=\frac{1}{K}\left[\frac{(1-r^2)}{(1-r^2)^2+4\zeta^2r^2}\right]$$

r = Ratio of chatter frequency to natural frequency (r = f / fn).
fn = Natural frequency of the machining system is also called the modal frequency.
ζ = Ratio of the damping coefficient to the critical damping coefficient (ζ = c / cc).
c_c= Critical damping coefficient (c_c = 2√km)

Solving the equations (14) and (15) together, the depth of cut ap is dependent on the frequency f of machine vibration or chatter through the frequency ratio r. For each chatter frequency generated on a machining system, there is a corresponding critical chip width (minimum depth of cut) ap. The cutting process is stable when its depth of cut is less than the critical value and unstable otherwise.

Roughness or waviness always exists on the machined surface of workpiece due to vibrations. According to regenerative chatter theory, chatter occurs whenever there is a shift of the phase angle ε between the current and previous surface waviness. Therefore, the ratio of chatter frequency f to tooth stroke frequency ft represents the number of surface waves between consecutive cutter teeth, and can be written as an integer n (also called the lobe number. n = 0, 1, 2, …) plus a fraction of ε / 2π radians

$$\frac{f}{f_t}=n+\varepsilon/2\pi \qquad (17)$$

where rt is the ratio between the tooth frequency and natural frequency
(r_t = ft / fn). The phase shift angle ε between the current and previous surface waviness may be expressed as

$$\varepsilon=\pi+2\tan^{-1}H/G \qquad (18)$$

Since the real and imaginary FRF's G and H are both negative as shown in equation (10), using the principal range of $-\pi/2<\tan^{-1}x<\pi/2$, and the phase shift angle ε is between $\pi<\varepsilon<2\pi$ Substitute equation (16) into equation (18) and then into equation (17), we obtain the equation of regenerative chatter

$$\frac{f}{f_t}=n+\frac{1}{2}+\frac{1}{\pi}*\tan^{-1}\left(\frac{2\zeta r}{1-r}\right) \qquad (19)$$

Equation (19) represents the relationships among the chatter frequency f, the tooth frequency f_t), and the lobe number n. Together with equation (16), they form the governing relationship between the depth of cut ap and spindle speed N. The spindle speed N can be related to the tooth frequency ft (ft = n_t N / 60), where n_t is the number of teeth on the cutter.

III. STABILITY ANALYSIS

1. Effect of Mode Orientation Angle on Stability

From the equation (7), the effect of mode orientation angle A on cross receptance is simulated with the help of software MATLAB program. The function u (coupling coefficient) for β = 60° is plotted in Figure

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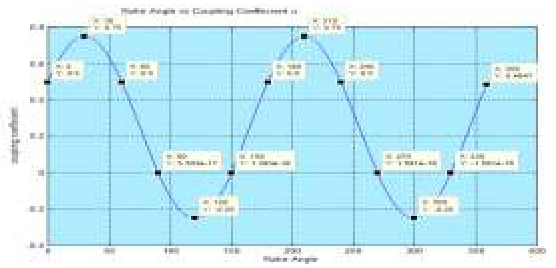

Figure 2. Effect of mode-coupling coefficient u on mode coupling angle α

From Figure 2, it is observed that for α= (β/2) and α= [(π/2) + β], the coupling coefficient, u becomes zero and an unconditional stability is achieved. This is also clearly shown in Figure 3. where a stability plot for different values of A keeping B invariant shows that the stability limits is affected by the mode orientation angle α.

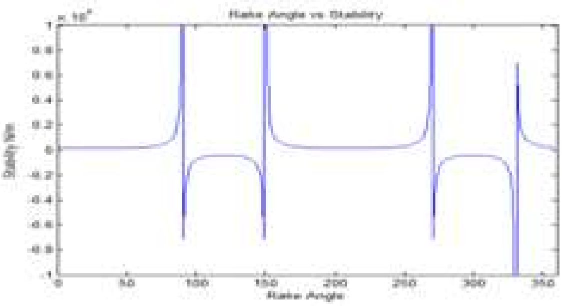

Figure 3. Variation of stability as a function of α

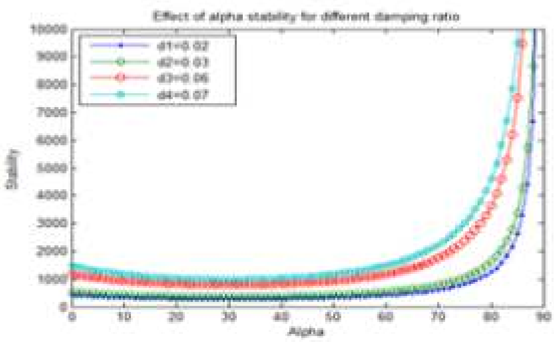

Figure 4. Effect of α on stability for different damping ratio

The limit of stability, connoted by coupling coefficient, r*, is minimum for α= (β /2). From the Figures 3 and 4, it is indicated that the permissible stability value r* is minimum at α1= 30°. Figure 4 shows the effect of mode coupling angle α on stability limit for finding various damping ratio and keeping β invariant. The higher damping ratio has then higher stability region. The stability limit is increased when the damping ratios increase.

2. Effect of Cutting Speed on Stability for Various damping ratios

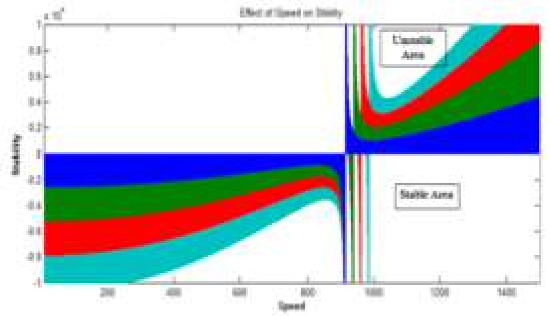

Figure 5. Effect of speed on system stability for different damping ratio

From the equation (12), it is indicated that the effect of speed on stability is simulated with the help of software MATLAB program. Figure 5 shows the stability plot of conventional characteristic equation method. The stability plot is based on the approximated values of mass and damping, the plot shows the effect of the variation of stability with the change in speed by keeping the stiffness and mass of the system constant with different values of damping ratio. The chatter frequency is greater than the resonant frequency. Hence, the stability plot should be discussed after the resonant frequency. Higher damping ratio moves the stability boundary upward and stability region become wider. From the Figure 5, it understood that the higher damping ratio has a wider stability region.

3. Effect of Speed on System stability for different stiffness

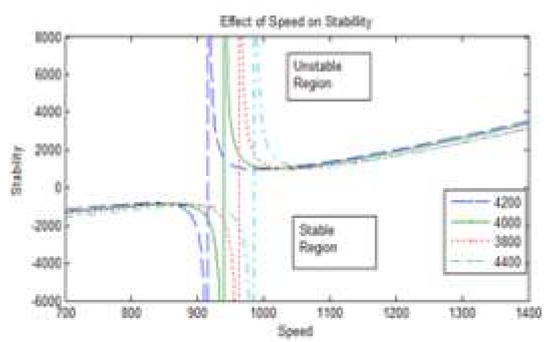

Figure 6. Effect of speed on stability for different stiffness

The variation of stability as function of speed for different stiffness of cutting tool is clearly shown in the Figure6.

4. Characteristics of Stability Lobe According to Natural Frequency and Damping Ratio

The characteristics of the stability curve with respect to the dynamic parameters, damping ratio and stiffness are combined and presented in the Figure 7

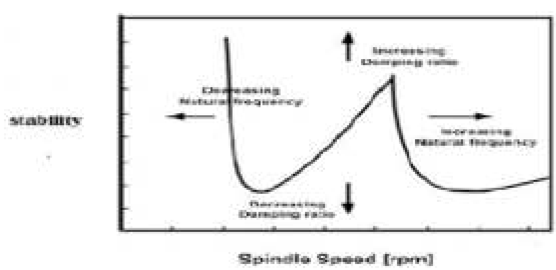

Figure 7 stability lobe behavior according to natural frequency and damping ratio

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From the Figure 7 it is seen that the stability lobe boundary moves upward and the stability region becomes wide when the damping ratio increased. However, the boundary region moves downward for lower damping ratio and the size of stable region becomes smaller and narrower. If the natural frequency increased then the boundary curve moves right. On the contrary, if the natural frequency decreases then, the curve moves left.

5. Stability Lobe for Cutting Tool System

The relationships among the chatter frequency f, and the tooth frequency ft, and the lobe number n, together with equation (12), they form the governing relationship between the depth of cut ap and spindle speed N. Using MATLAB program relationship between depth of cut ap and spindle speed N are plotted in Figure 8. Since the series of relationships curve in Figure 8 is shaped like lobes, the graph is usually called a stability lobe diagram. A stability lobe diagram shows the relationship between chip width (or depth of cut) and spindle speed, with the lobe number as a parameter. Usually, the variable on its x-axis is represented as spindle speed N, tooth frequency ft, the variable on its y-axis is represented as depth of cut ap.

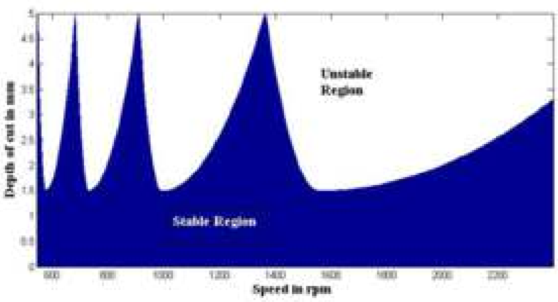

Figure 8 Stability Lobe Diagram for hard turning

There is an optimal depth of cut for every cutting speed, which gives maximum cutting speed boundary value. There is an optimal cutting speed, which gives the best margin of cutting speed.

6. Results

The above simulation revealed the following

- The stability limit of cutting tool with different dampers can be predicted using MATLAB.
- The permissible stability value r* is minimum at α= β/2 = 30. And between 60 < α <180 the self-excited vibrations due to mode - coupling would not occur.
- Higher damping ratio lifts the stability lobe upward and widens the stability region. However, the boundary region moves downward for lower damping ratio and the size of stable region becomes smaller and narrower.
- By increasing the stiffness of system, the natural frequency is increased and the boundary curve moves right. Similarly, if the stiffness is decreased then the curve moves left.
- This finding confirms that the dynamic stability can be increased without any modification in structure and the weight of the existing machine tool.

7. Conclusion

Some interesting characteristics exist in the stability lobe diagram, which may be utilized to optimize the chip width or depth of cut and obtain the maximum material rate (MRR) in machining processes In a stability lobe diagram, a series of scallop shaped lobes intersects with each other. These lobes form the limits for chattering. Locally, for each lobe, it is stable below the lobe and unstable above the lobe. Since the lobes intersect, a point located below one lobe could be above the neighboring lobe. This point must be treated as unstable. Therefore, globally we must consider the relationship between adjacent lobes in determining the stability. The upper portion of any two adjacent lobes above their intersection point should be trimmed off. The intersection point connects all the lobes into 'chained' chatter lines. All points above the chatter line are unstable, and below are stable. As spindle speed increases, the lobes become wider with larger intervening spaces between consecutive lobes, and intersection points are higher. This phenomenon creates a desirable situation for machining at both higher speed and deeper cut simultaneously and a wider speed range as well. It is very convenient to predict the stability lobe for machining by proper mathematical modeling and simulation.

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