Volume : 4 | Issue : 10 | Special Issue Oct- 2015 • ISSN No 2277 - 8179

A study of some Generalized Measures of Entropy Under Intuitionistic Fuzzy Environment

Mathematics

KEYWORDS: Intuitionistic Fuzzy Sets (IFSs), Fuzzy Information Theory, Entropy Measures.

Kumar, Vijay	Manav Rachna International University,Faridabad.
Arora, Hari	Amity University, Noida
lata, Hem	Amity University, Noida

ABSTRACT

In this paper, we used the concept of probability for defining the fuzzy entropy of intuitionistic fuzzy sets(IFSs). Here, we propose three families of entropy measures for IFSs and also construct the axiom definition and properties.

1. Introduction.

The concept of information theory was given by C. E Shannon [1948], but the real connectivity between the information theory and fuzzy sets theory was established by Weaver. According to Seising[2009], Weaver in sighed the non-technical and the philosophical aspects of information theory and developed the concept of fuzzy information theory. After that many generalized versions of the said theory were took place and having applications in various domains. One such generalization is the Intuitionistic fuzzy set(IFS) theory as given by Atanassov [1986]. An IF set presents the degrees of membership, non-membership and hesitancy. Therefore, the notion of entropy constituted a fundamental aspect from the beginning of the development of fuzzy sets theory. In the present work, we will study some generalized measures of intuitionistic fuzzy entropy.

1.1 Intuitionistic Fuzzy Sets(IFSs).

Atanassov [1986], For a fixed set X, an IFS of A is defined as : $A=\{<x, \mu_A(x), \nu_A(x)>|x\in X\}$, where $\mu_A(x): X\to[1,0]$ and $\nu_A(x): X\to[0,1]$ define the degree of membership and degree of non-membership of the element $x\in X$ *to the set A. For every* $x\in X$, $0,\leq \mu_A(x) + \nu_A(x)\leq 1$ and the amount $\pi_A(x) = 1-\mu_A(x)-\nu_A(x)$ is called the intuitionistic index or hesitation index, which may require to membership value, non-membership value or both.

2. Entropy Measures of Intuitionistic Fuzzy Sets.

2.1 Burillo and Bustince measure of entropy.

Burillo and Bustince[1996] were the first to state and propose an axiomatic theory of entropy for Intuitionistic fuzzy sets and interval-valued fuzzy sets.

A real valued function $E: X\to\Re^+$ is called an entropy on X, if E has the following properties:

(I) $E(A)=0$ *if only and if* $A\in X$

(ii) $E(A)$ *Cardinal (X) if and only if* $\mu_A(x) = 0$ *for all* $x\in X$

(iii) $E(A)=E(A^c)$ *for all* $A\in X$

(iv) $E(A)\geq E(B)$ *if* $\mu_A(x)\leq \mu_B(x)$ and $V_A(x)$ *for all* $x\in X$

Moreover, they proposed the following entropy measure for IFSs, satisfying the above mentioned system of axioms:

$$E(A)=\sum_{i=1}^{|X|}\pi_A(x_i) \qquad (2.1.1)$$

where, X denotes the cardinality of X.

(2.1.1) is the degree of intuitionism of in intuitionistic fuzzy set.

2.2 Szmidt and Kacprzyk measure of entropy.

Szmidt and Kacprzyk [2001] also proposed an axiomatic theory of entropy for Intuitionistic fuzzy sets. A real valued function $E: X\to\Re^+$ is called an entropy on X, if E has the following properties:

(i) $E(A)=0$ *if and only if* $A\in 2^X$

(ii) $E(A)=1$ *if and only if* $\mu_A(x)=\nu_A(x)$ *for all* $x\in X$

(iii) $E(A)=E(A^c)$ *for all* $A\in X$

(iv) $E(A)\leq E(B)$ *if*

$\mu_A(x)\leq\mu_B(x)$ *and* $\nu_A(x)\geq\nu_B(x)$ *for* $\mu_B(x)\leq\nu_B(x)$ *for all* $x\in X$ Or

$\mu_A(x)\geq\mu_B(x)$ *and* $\nu_A(x)\leq\nu_B(x)$ *for* $\mu_B(x)\geq\nu_B(x)$ *for all* $x\in X$

Along with their axiomatic theory, Szmidt and Kacprzyk [2000] proposed a new fuzzy measure of entropy by considering distances between IFSs as:

$$E(A)=\frac{1}{|X|}\sum_{i=1}^{|X|}\frac{\min\{\mu_A(x_i),\nu_A(x_i)\}+\pi_A(x_i)}{\max\{\mu_A(x_i),\nu_A(x_i)\}+\pi_A(x_i)} \qquad (2.2.1)$$

2.3 Hung and Yang measure of entropy.

Hung and Yang [2006] proposed an axiomatic theory of entropy for Intuitionistic fuzzy sets.

A real valued function $E: X\to\Re^+$ is called an entropy on X, if E has the following properties:

(i) $E(A)=0$ *if and only if* $A\in 2^X$

(ii) $E(A)$ *assumes a unique* max *imum if* $\mu_A(x)=\nu_A(x)=\pi_A(x)=\frac{1}{3}$ *for all* $x\in X$

(iii) $E(A)=E(A^c)$ *for all* $A\in X|$

(iv) $E(A)\leq E(B)$ *if*

$\mu_A(x)\leq\mu_B(x)$ *and* $\nu_A(x)\leq\nu_B(x)$ *for* $\max\{\mu_B(x),\nu_B(x)\}\leq\frac{1}{3}$ *for all* $x\in X$

Or

$\mu_A(x)\geq\mu_B(x)$ *and* $\nu_A(x)\geq\nu_B(x)$ *for* $\min\{\mu_B(x),\nu_B(x)\}\geq\frac{1}{3}$ *for all* $x\in X$

On the basis of Renyi's entropy [1960], Hung and Yang introduced the parametric entropy measure as:

$$E(A)=\sum_{i=1}^{|X|}\frac{1}{1-\beta}\log(\mu_A^{(k)}(x_i)+\nu_A^{(k)}(x_i)+\pi_A^{(k)}(x_i)) \quad \text{with } 0<k<1 \qquad (2.3.1)$$

Based on the above notations, the traditional entropy is based on the concepts of probability, and it could measure multiple attribute decision making (MADM). Nevertheless, the meaning of intuitionistic fuzzy entropy is different from the traditional entropy, because the

intuitionistic fuzzy entropy represents the credibility of the data while we apply it in multiple attribute decision making (MADM).

Szmidt and Kacprzyk [2001] proposed a new entropy method for IFS. They proposed that the IF entropy is a ratio of distances between the (A, A_{near}) and (A, A_{far})

We express it as: $E(A) = \dfrac{(A,\ A_{near})}{(A,\ A_{far})}$ (2.3.2)

where, (A, A_{near}) is the distance from A to the nearest point A_{near} among positive ideal point and negative ideal point and $(A,\ A_{far})$ is the distance from to the farther point A_{far} among positive ideal point and negative ideal point.

Szmidt and Kacprzyk [2000] also proved (2.2.1) which satisfies the entropy measure of Szmidt and Kacprzyk [2001]. The entropy concept of Szmidt and Kacprzyk constructs on distance, we need to concern the relative concepts of measuring distance for IFSs. Distances between IFSs should be calculated by taking into account three parameters describing an IFS.

Szmidt and Kacprzyk [2000] introduced two different distance measures for IFS as follows:

Let $X = \{x_1, x_2, \ldots\ldots, x_n\}$, the Hamming distance between two Intuitionistic fuzzy sets (IFSs) A and B in X is defined by

$$d_H(A, B) = \sum_{i=1}^{N} |\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)| + |\pi_A(x_i) - \pi_B(x_i)| \quad (2.3.3)$$

where, $0 \le d_H(A, B) \le N$

Also, the Euclidean distance between two intuitionistic fuzzy sets (IFSs) A and B in X is follows as:

$$d_E(A, B) = \left(\sum_{i=1}^{N} [\mu_A(x_i) - \mu_B(x_i)]^2 + [\nu_A(x_i) - \nu_B(x_i)]^2 + [\pi_A(x_i) - \pi_B(x_i)]^2 \right)^{\frac{1}{2}} \quad (2.3.4)$$

Clearly, $0 \le d_E(A, B) \le N$.

The other two distance measure between two intuitionistic fuzzy sets (IFSs) A and B in X as given by Wang and Xin [2005] are as follows:

$$d(A, B) = \frac{1}{N} \sum_{i=1}^{N} \left[\frac{|\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)|}{4} + \frac{Max\left[|\mu_A(x_i) - \mu_B(x_i)|, |\nu_A(x_i) - \nu_B(x_i)|\right]}{2} \right] \quad (2.3.5)$$

$$d(A, B) = \frac{1}{\sqrt[l]{N}} \sqrt[l]{\sum_{i=1}^{N} \left[\frac{|\mu_A(x_i) - \mu_B(x_i)|}{2} + \frac{+|\nu_A(x_i) - \nu_B(x_i)|}{2} \right]^l} \quad (2.3.6)$$

By combining (2.3.3-2.3.6) with (2.3.2), the following different IF entropy measures were introduced by Szmidt and Kacprzyk [2001] are as follows:

$$E(A) = \frac{|\mu_{near}(x_i) - \mu_A(x_i)| + |\nu_{near}(x_i) - \nu_A(x_i)| + |\pi_{near}(x_i) - \pi_A(x_i)|}{|\mu_{far}(x_i) - \mu_A(x_i)| + |\nu_{near}(x_i) - \nu_A(x_i)| + |\pi_{near}(x_i) - \pi_A(x_i)|} \quad (2.3.7)$$

$$E(A) = \frac{\left[[\mu_{near}(x_i) - \mu_A(x_i)]^2 + [\nu_{near}(x_i) - \nu_A(x_i)]^2 + [\pi_{near}(x_i) - \pi_A(x_i)]^2 \right]^{\frac{1}{2}}}{\left[[\mu_{far}(x_i) - \mu_A(x_i)]^2 + [\nu_{far}(x_i) - \nu_A(x_i)]^2 + [\pi_{far}(x_i) - \pi_A(x_i)]^2 \right]^{\frac{1}{2}}} \quad (2.3.8)$$

$$E(A) = \frac{\left[\frac{|\mu_{near}(x_i) - \mu_A(x_i)| + |\nu_{near}(x_i) - \nu_A(x_i)|}{4} + \frac{Max\left[|\mu_{near}(x_i) - \mu_A(x_i)|, |\nu_{near}(x_i) - \nu_A(x_i)|\right]}{2} \right]}{\left[\frac{|\mu_{far}(x_i) - \mu_A(x_i)| + |\nu_{far}(x_i) - \nu_A(x_i)|}{4} + \frac{Max\left[|\mu_{far}(x_i) - \mu_A(x_i)|, |\nu_{far}(x_i) - \nu_A(x_i)|\right]}{2} \right]} \quad (2.3.9)$$

$$E(A) = \frac{\frac{1}{\sqrt[l]{N}} \sqrt[l]{\sum_{i=1}^{N} \left[\frac{|\mu_{near}(x_i) - \mu_A(x_i)|}{2} + \frac{+|\nu_{near}(x_i) - \nu_A(x_i)|}{2} \right]^l}}{\frac{1}{\sqrt[l]{N}} \sqrt[l]{\sum_{i=1}^{N} \left[\frac{|\mu_{far}(x_i) - \mu_A(x_i)|}{2} + \frac{+|\nu_{far}(x_i) - \nu_A(x_i)|}{2} \right]^l}} \quad (2.3.10)$$

Conclusion.

Here, we studied three families of entropy measures on IFSs. These entropy measures satisfy the extended entropy measure axiom definition of IFSs. Moreover, these measures of entropy are used in the process of Decision Making.

REFERENCE

Atanassov K.(1986). Intuitionistic fuzzy sets. Fuzzy Set Systems,20, 87–96. | Burillo, P. and Bustince, H. (1996), "Entropy on Intuitionistic Fuzzy Sets and on Interval- valued Fuzzy Sets", Fuzzy Sets and Systems, 78, 3, 305 --316. | Hung, W.L. and Yang, M.S. (2006), "Fuzzy Entropy on Intuitionistic Fuzzy Sets", International Journal of Intelligent Systems, 21, 4, 443--451. | Seising, R.(2009). On two 60 years old theories and the theory of fuzzy sets and systems: Cybermetics and information theory.| Renyi, A. (1960), "Some fundamental questions of information theory", MTA III. Oszt. K¨ozl., 10, 251–282. | Szmidt E. and Kacprzyk J. (2000), "Distances between intuitionistic fuzzy sets", Fuzzy Sets and Systems, 114, 3, 505--518. | Szmidt E. and Kacprzyk J. (2001), "Entropy for intuitionistic fuzzy sets", Fuzzy Sets and Systems, 118, 467--477. | Wang, W. and Xin, X. (2005), "Distances Measure between Intuitionistic Fuzzy Sets", Pattern Recognition Letters, 26, 1, 2053--2069. | Zadeh, L.A.(1965). Fuzzy sets, Information and control, 8, 338-353.