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EFFECT OF HALL CURRENTS ON THE FLOW THROUGH A POROUS MEDIUM WITH VARIABLE PERMEABILITY IN SLIP FLOW REGIME WITH COUPLE STRESSES WITH MASS TRANSFER

Mathematics

KEYWORDS: Schmidt number=S_c, Magnetic Parameter = M, Permeability Parameter=K_0.

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ABSTRACT

This paper examines the problem of hall currents on MHD unsteady flow of a polar fluid through a porous medium of variable permeability bounded by an infinite horizontal porous plate in slip flow regime with mass transfer. The permeability of the porous medium decreases exponentially with time about a constant mean. Using perturbation technique the expressions for the velocity distribution, the mean angular velocity of rotation of particles, concentration distribution and skin friction are obtained. The effects of permeability parameter K_0 Magnetic parameter M, Schmidt number S_c and Hall currents parameter m entering into the problem on velocity and concentration distribution are shown graphically and discussed numerically. It can be observed that this velocity decreases with the increase in M,S_c and increases with the increase in K_0 M.

INTRODUCTION

The study of the flow through a porous medium is of great importance to geophysicists and fluid dynamicists. Brinkman [3]. Yamamoto [16]. Raptis et. al [10,11] have studied flow through a porous medium considering generalized Darcy's law. In all above research papers generalized Darcy's law is derived without taking into account the angular velocity of the fluid particles. These fluids are known as polar fluids in the literature. Aero et. al [2], D 'ep [5] derived flow equations of such fluids taking angular velocity into consideration. Rapits [9] considered magnetic effect on a polar fluid through a porous medium.In geothermal region situation may arise when the flow becomes unsteady and slip at the boundary may take place as well. In such situation of slip flow, ordinary continuum approach fails to yield satisfactory results. Many authors have solved problems taking slip conditions at the boundary. Recently Jain and Teneja [6] solved the problem of magnetopolar flow through a porous medium in slip flow regime. In above problems the permeability of porous medium was assumed to be constant. Taking permeability variations into consideration. Chandrasekhara et al. [4] and Vedhanayagam et al. [15] have studied the buoyancy induced flow behaviour adjacent to a horizontal surface in a porous medium. Singh et. al. [13]. Singh and Kumar [12], Acharya et al. [1] and Maharshi and Tak [8] have studied the effects of permeability variation on free convective flow through a porous medium. Khandelwal et al [7] have studied effect of couple stresses on the flow through a porous medium with variable permeability in slip flow regime. Recently, Varshney and sharma have presented effect of mass transfer on the flow through a porous medium with variable permeability in slip flow regime with couple stresses.

In the present paper an attempt has been made to study the effects of hall currents on MHD unsteady flow of a polar fluid through a porous medium with mass transfer of variable permeability in slip flow regime over an infinite porous flat plate.

FORMULATION AND SOLUTION OF THE PROBLEM

Let us consider the unsteady flow of a polar fluid through a porous medium of variable permeability in slip flow regime past an infinite horizontal porous plate in presence of a transverse magnetic field with mass transfer. The external velocity of the fluid is assumed as $U_0[1+\varepsilon e^{-nt}]$ The flow is chosen to be at small magnetic Reynolds number which enables us to neglect the induced magnetic field. The x-axis is taken along the plate and y-axis normal to it. Under these conditions the equations which govern the flow are:

$$\frac{\partial v}{\partial y} \quad .... \quad .... \quad .... \quad (1)$$

$$\frac{\partial u}{\partial y} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + (\nu+\nu_r)\frac{\partial^2 u}{\partial y^2} + 2\nu_r\frac{\partial \omega}{\partial y} + g\beta'(C-C_\infty) - \frac{\nu}{K(t)}u - \frac{\sigma B_0^2}{\rho(1+m^2)}u \quad ... (2)$$

$$\frac{\partial \omega}{\partial t} + v\frac{\partial \omega}{\partial y} + \frac{\gamma}{I}\frac{\partial^2 \omega}{\partial y^2} \quad .... \quad .... \quad .... \quad (3)$$

$$\left[\frac{\partial C}{\partial t} + v\frac{\partial C}{\partial y}\right] = D\frac{\partial^2 C}{\partial y^2} \quad .... \quad .... \quad .... \quad (4)$$

Where ν_r is the kinematic rotational viscosity, ω the mean angular velocity of rotation of the particles, K (t) the permeability of the porous medium, σ τ he electrical conductivity, B_0 the magnetic induction, m the hall currents parameter,I a scalar constant of dimension equal to that of the moment of inertia of unit mass and

$$\gamma = \frac{C_a + C_d}{I}$$

Where C_a and C_d are coefficients of couple stress viscosities. Remaining symbols have their usual meaning.

The boundary conditions relevant to the problem are taken as

$$u = L_1\frac{\partial u}{\partial y}, \quad \frac{\partial \omega}{\partial y} = -\frac{\partial^2 u}{\partial y^2}, \quad C = C_w \quad at \quad y = 0$$

$$u \to U(t) = U_0[1+\varepsilon e^{-nt}], \quad \omega \to 0, \quad C \to \infty \quad at \quad y = \infty \quad .... (5)$$

Where $L_1 = \left[\frac{2-m_1}{m_1}\right]L$,

L is the mean free path and m1 is the Maxwell's reflection coefficient.

These boundary conditions are derived from the assumptions that the couple stresses are dominant during the rotation at the particles (D'ep [5]).

$$K(t) = K_0[1 + A\varepsilon e^{-nt}] \quad .... \quad .... \quad .... \quad (6)$$

Where K_0 is the mean permeability of the medium, A and n are real positive constants, t is the time and ε is small such that εA < < 1. Integration of equation (1) for constant suction gives,

$$\nu = -v_0 \quad ..... \quad .... \quad (7)$$

We introduce the following non - dimensional quantities:

$$y^* = V_0\frac{y}{\nu}, \quad u^* = \frac{u}{U_0}, \quad \phi = \frac{C-C_\infty}{C_w-C_\infty},$$

$$t^* = V_0^2\frac{t}{\nu}, \quad n^* = \frac{n\nu}{v_0^2}, \quad \alpha = \frac{\nu_r}{\nu}, \quad \beta = \frac{Iv_0}{\nu}$$

$G_m = \frac{g\beta'\nu(C_w - C_\infty)}{U_0 V_0^2}$ (Grashof number of mass transfer)

$K_0 = \frac{K_0 V_0^2}{\nu^2}$ (Permeability parameter)

$S_c = \frac{\nu}{D}$ (Schimdt No.) , $\lambda = \frac{\lambda_1 V_0^2}{\nu}$ (Visco-elastic parameter),

$M^2 = \frac{\sigma B_0^2 \nu}{\rho V_0^2}$ (Magnetic parameter)

$\omega^* = \frac{L_1 \omega}{\nu_0 U_0}$ (Non dimensional amplitude ratio)

$h_1 = \frac{L_1 \nu_0}{\nu}$ (Rarefaction parameter)

In view of equations (6) and (7) and for the free stream velocity, the equations of motion in non-dimensional form after dropping the asterisks over them, reduce to:

$$(1+\alpha)\frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial y} + 2\alpha\frac{\partial \omega}{\partial y} - \left(\frac{M^2}{1+m^2} + \frac{1}{K_0(1+A\varepsilon e^{-nt})}\right)(u - U) - \frac{\partial u}{\partial t} = -\frac{\partial U}{\partial t} - G\phi \quad \ldots (8)$$

$$\frac{\partial^2 \omega}{\partial y^2} + \beta\frac{\partial \omega}{\partial y} = \beta\frac{\partial \omega}{\partial t} \quad \ldots \quad \ldots \quad \ldots \quad (9)$$

$$\frac{\partial^2 \phi}{\partial y^2} + S_c\frac{\partial \phi}{\partial y} = S_c\frac{\partial \phi}{\partial t} \quad \ldots \quad \ldots \quad \ldots \quad (10)$$

With corresponding boundary conditions

$$u = h_1\frac{\partial u}{\partial y}, \quad \frac{\partial \omega}{\partial y} = -\frac{\partial^2 u}{\partial y^2}, \quad \phi = 1 \quad \text{at } y = 0$$

$$u \to U(t) = [1 + \varepsilon e^{-nt}], \ \omega \to 0, \ \phi \to 0 \text{ at } y \to \infty \quad \ldots \quad (11)$$

Since e is small so let us assume u, ω and ϕ as

$$u(y, t) = u_0(y) + \varepsilon e^{-nt}u_1(y) \quad \ldots \quad \ldots \quad (12)$$

$$\omega(y, t) = \omega_0(y) + \varepsilon e^{-nt}\omega_1(y) \quad \ldots \quad \ldots \quad (13)$$

$$\phi(y, t) = \phi_0(y) + \varepsilon e^{-nt}\phi_1(y) \quad \ldots \quad \ldots \quad (14)$$

We get the solution after neglecting the coefficients of ε^2 and using corresponding boundary conditions:

$$u = (1 + C_3e^{-R_2y} + Z_2C_1e^{-\beta y} - Z_2e^{-S_cy}) + \varepsilon e^{-nt}(C_4 + e^{-R_4y} - Z_6C_3e^{-R_2y} - Z_7e^{-\beta y} + Z_8e^{-S_cy} + Z_9C_2e^{-m_1y} + 1 + \frac{M^2A}{(1+m^2)(N^2-n)}) \quad \ldots (15)$$

$$\omega = C_1e^{-\beta y} + \varepsilon e^{-nt}C_2e^{-m_1y} \quad \ldots \quad \ldots \quad \ldots \quad (16)$$

$$\phi = e^{-S_cy} \quad \ldots \quad \ldots \quad \ldots \quad (17)$$

Where

$$R_1 = \frac{-1 + \sqrt{1+4(1+\alpha)N^2}}{2(1+\alpha)}, \qquad R_2 = \frac{1 + \sqrt{1+4(1+\alpha)N^2}}{2(1+\alpha)}$$

$$R_3 = \frac{-1 + \sqrt{1+4(1+\alpha)(N^2-n)}}{2(1+\alpha)}, \qquad R_4 = \frac{1 + \sqrt{1+4(1+\alpha)(N^2-n)}}{2(1+\alpha)}$$

$$m_1 = \frac{\beta\sqrt{\beta^2 - 4n\beta}}{2}, \qquad N^2 = \frac{M^2}{1+m^2} + \frac{1}{K_0}$$

$$C_1 = \frac{Z_3Z_2 - h_1Z_2R_2^2}{Z_3Z_5 - Z_1Z_4R_2^2}, \qquad C_2 = \frac{R_4^2Z_{12} - Z_{11}Z_{14}}{Z_9Z_{10} + Z_{11}Z_{13}},$$

$$C_3 = \frac{h_1Z_1Z_5 - Z_1Z_2Z_4}{Z_3Z_5 - Z_1Z_4R_2^2}, \qquad C_4 = \frac{Z_{12}Z_{13} + Z_9Z_{10}Z_{14}}{Z_9Z_{10}R_4^2 + Z_{11}Z_{13}}$$

$$Z_1 = \frac{2\alpha\beta}{(1+\alpha)\beta^2 - \beta - N^2}, \qquad Z_2 = \frac{G_m}{(1+\alpha)S_c - S_c - N^2}$$

$$Z_3 = 1 + h_1R_2, \quad Z_4 = 1 + h_1\beta, \quad Z_5 = 1 + h_1\beta^2$$

$$Z_6 = \frac{\{(1+\alpha)R_2 - 1\}AR_2}{(1+\alpha)R_2^2 - R_2 - (N^2-n)}, \qquad Z_7 = \frac{\{(1+\alpha)\beta - 1\}Z_1C_1 - 2\alpha}{(1+\alpha)\beta_2^2 - \beta - (N^2-n)}$$

$$Z_8 = \frac{\{(1+\alpha)S_c - 1\}AZ_2S_c}{(1+\alpha)S_c^2 - S_c - (N^2-n)}, \qquad Z_9 = \frac{2\alpha m_1}{(1+\alpha)m_1^2 - m_1 - (N^2-n)}$$

$$Z_{10} = 1 + h_1m_1, \qquad Z_{11} = 1 + h_1R_4$$

$$Z_{12} = (1 + h_1\beta)Z_7 - (1 + h_1S_c)Z_8 + (1 + h_1R_2)Z_6C_3 + 1 + \frac{M^2A}{(1+m^2)(N^2-n)}$$

$$Z_{13} = 1 - Z_9m_1^2, \quad Z_{14} = Z_6R_2^2C_3 + Z_7\beta^2 - Z_8S_c^2$$

The non-dimensional skin friction at the plate is given by

$$\tau = \frac{\tau_w}{\rho U_0 \nu_0} = (1+\alpha)\left(\frac{\partial u}{\partial y}\right)_{y=0} = (1+\alpha)(-R_2C_3 - \beta Z_2C_1 + S_cZ_2) + \varepsilon e^{-nt}(-R_4 + R_2Z_6C_3 + \beta Z_7 - S_cZ_8 - m_1Z_9C_2) \quad \ldots (18)$$

Results and discussion:

The zeroth-order velocity profile u_0 is plotted in Fig.-1 having Graph-1 to 5 at α = 0.2, β = 2, h_1 = 0.4, ε = 0.01, A =0.5, G_m = 5 and different values of K_0, M, S_c and m.

	K_0	M	S_c	m
For Graph-1	1	0.2	0.3	1
For Graph-2	2	0.2	0.3	1
For Graph-3	1	2.0	0.3	1
For Graph-4	1	0.2	0.7	1
For Graph-5	1	0.2	0.3	2

From the Graphs of Fig.-1 it is clear that the zeroth-order velocity u_0 decreases on increasing y. It can also be observed that this velocity decreases with the increase in M, S_c and increases with the increase in K_0, m.

The concentration profile is plotted in Fig.-2 having Graph-1 & 2 at different values of S_c as taken for velocity. From the Graphs of Fig.-2 it is clear that the concentration field ϕ decreases with the increase in y and S_c.

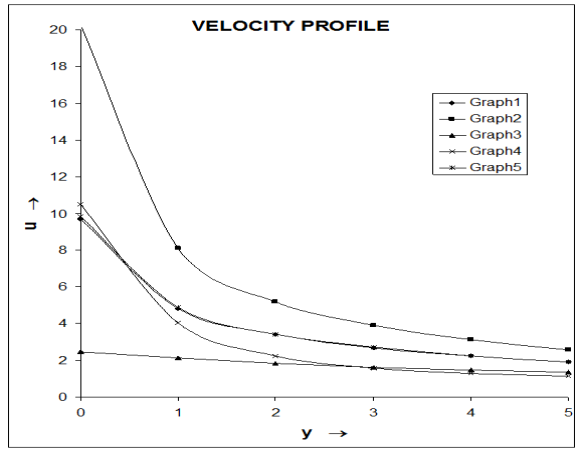

Fig.-1

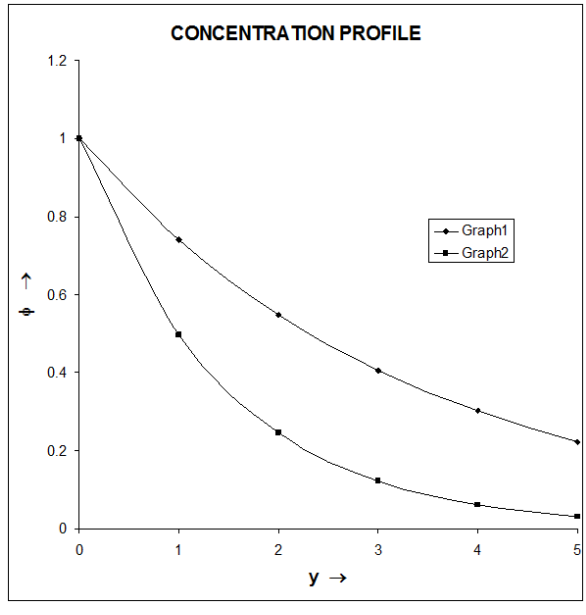

Fig.-2

Table-1: Values of velocity at $\beta = 2$, $h_1 = 0.4$, $\varepsilon = 0.01$, A =0.5, $G_m = 5$ $\alpha = 0.2$ **and different values of** K_0, **M,** S_c **and** m_1.

y	Graph-1	Graph-2	Graph-3	Graph-4	Graph-5
0	9.6852	20.2406	2.4654	10.4916	9.8168
1	4.84052	8.099893	2.14324	4.057535	4.88047
2	3.408602	5.183641	1.854509	2.247954	3.431453
3	2.704794	3.921181	1.634033	1.57205	2.721059
4	2.24728	3.130166	1.469835	1.274971	2.2593
5	1.920716	2.57053	1.348079	1.13468	1.929631

Table-2 : Values of concentration at different values of S_c

y	Graph-1	Graph-2
0	1	1
1	0.740818	0.496585
2	0.548812	0.246597
3	0.40657	0.122456
4	0.301194	0.06081
5	0.22313	0.030197

REFERENCE

[1] Acharya, M.; Dash, G.C. and Singh, I.P.: Indian J. Pure Appl. Math., 31, 1 (2000). | [2] Aero, E.L.; Bulygin, A.N. and Kuvschinski, E.V.: Prikl. Math. Mech., 29, 297 (1965); J. Appl. Math. Mech., 29, 333 (1965). | [3] Brinkman, H.C.: Appl. Sci. Res. Al, 27 (1947). | [4] Chandrasekhara, B.C., Namboodiri, P.M.S. and Hanumanthappa, A.R. : Warmeund -Stoffubertragung, 18, 17 (1984). | [5] D'ep, N.V.: Prikl. Mech., 32 (1968), 748; J. Appl. Math. Mech., 32, 777 (1968). | [6] Jain, N.C. and Taneja, Rajeev: Ganita Sandesh, 14, 87 (2000). | [7] Khandelwal, K. Anil; Gupta, Poonam and Jain, N.C. : Ganita, Vol. 54, no. 2, 203-212 (2003). | [8] Maharshi, Arvind and Tak, S.S.: Ganita Sandesh, 14, 77 (2000). | [9] Raptis, A.: Bull. de la classe des Sc. 69, 530 (1983). | [10] Rapits, A.; Peridikis, C. and Tzivanidis, G.: J. Phys. D. Appl. Phys. 14, L 99 (1981). | [11] Raptis, A.; Tzivanidis, G. and Kafousias, N. : Letters in Heat and Mass Transfer, 8, 417 (1981). | [12] Singh, K.D. and Kumar, S.: Jour. Math. Phys. Sci., 27, 141 (1993). | [13] Singh, P.; Mishra, J.K. and Narayan, K.A.: Int. J. Numerical and Analytical Methods Geomechanics, 13, 443 (1991). | [14] Varshney, N.K. and Sharma, Vineet Kumar: International conference on Modelling of Engg. Tech. and 9th Biannual National conference, 14 to 16 January, 2009, BMAS Engg. College, Agra | [15] Vedhanayagam, M.; Jain, P. and Fairweather, G.: Int. Comm. Heat Mass Transfer, 14, 485 (1987). | [16] Yamamoto, K.: J. Phys. Soc. Japan, 34, 814 (1973).