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RELIABILITY ANALYSIS OF A CIRCULAR CONSECUTIVE REPAIRABLE SYSTEM

Mathematics

KEYWORDS: Circular consecutive-3-out-of-8:F system, Critical component, Generalized transition probability.

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ABSTRACT

System reliability is an issue of the utmost importance to the system engineer. This investigation is concerned with the reliability analysis of a circular consecutive-3-out-of-8: F repairable system with a single repairman. In case of more than one failed component, priority for repair is assigned to the critical failed component. The state transition probability matrix is obtained using concept of critical components and generalized transition probability. Expressions for some performance measures such as system reliability, availability and mean time to first failure are established. Numerical results are presented to analyse the effect of traffic intensity upon the performance indices.

1. INTRODUCTION

Reliability is a measure of the "operational success" of a machining system. Consequently, reliability analysis forms an integral part in the planning, designing and operation of all systems. A system designer has to keep high reliability and low cost as the guiding parameters. Both these demands are met in the consecutive-k-out-of-n: F systems wherein the system fails if and only if at least k consecutive components (out of a total of n) fail.

Kontoleon (1980) is ascribed the credit of introducing the consecutive-k-out-of-n: F system reliability models. Since then, considerable work is reported in literature on these models such as those of Chiang and Niu (1981), Derman et al. (1982), Malon (1984), Du and Hwang (1986), Ge and Wang ((1990) , Zuo and Wu (1996) and others.

Zhang and Wang (1996) proposed a linear consecutive-2-out-of-n:F system for which repair is facilitated using the queue discipline "first in first out". Later, Flynn and Chung (2004) presented an algorithm to determine optimal replacement policies for the consecutive-k-out-of-n systems. Krishnan & Somasundaram, (2011) provided reliability analysis of repairable consecutive-k-out-of-n: G systems with sensor and repairmen.

A circular consecutive-k-out-of-n: F system can be visualized as having n components arranged along a circle wherein components n and 1 are considered consecutive. This system will fail if and only if at least k consecutive components in the system fail. Zhang et al. (2000) studied a circular consecutive-2-out-of-n: F repairable system by providing priority to the failed components for repair. A repairable circular consecutive-k-out-of-n:F system with single repairman was investigated by Yam et al. (2003).

This paper deals with a circular consecutive-3-out-of-8: F repairable system with single repairman. Priority is assigned in repair to failed components by introducing the concept of critical component. Rest of the paper is organized as follows: Section 2 is devoted to model description stating underlying assumptions and useful notations. In the next section 3, governing equations are constructed to generate the transition rate matrix. The performance metrics of the system are established in section 4. Numerical results are presented in section 5. We draw curtains in section 6 with some concluding remarks on the work done.

2. MODEL DESCRIPTION

A circular consecutive-3-out-of-8: F system is considered having provision of a single repairman. The system is explicitly described using the following underlying assumptions:

- The system consists of 8 components arranged along a circular path wherein components 1 and 8 are considered to be consecutive.
- The system is considered failed when at least 3 consecutive components in the system fail.
- The life times and repair times of the components are assumed to be exponentially distributed.
- All the components are working at time t=0.
- The repair facility consists of a single repairman.
- Upon repair, a component works as good as a new one.
- The component which when repaired restores the system to working state is called a 'critical component'. Such critical components, upon failure, are given priority in repair vis-à-vis other components.

Some notations employed for modeling purpose are given below:

λi : Failure rate of i^{th} component

μi : Repair rate of ith component

$[x]$: The greatest integer less than or equal to x.

$p_{ij}(\Delta t)$: The generalized transition probability from state i to state j in time Δt

M_{-i} : he number of possible cases for state –i for a circular cosecutive-3-out-of-8: F system

It should be noted that a state –i implies that i components have failed but the system still works. On the other hand, j state means a failed state of the system wherein j components stop working.

3. MATHEMATICAL ANALYSIS

Assume $\{N(t), t \geq 0\}$ to be a continuous-time homogeneous Markov process with finite state space Ω. For a given state $i \in \Omega$, we may assume M_i cases.

Define $p_{im} = \Pr\{\text{System is in case m of the state i} \mid N(t) = I\}$, $m=1,2,\ldots,M_i$

It is clear that $\sum_{m=1}^{M_i} p_{im} = 1$

Now, $p_{m(i)j}(t)$ $\Pr\{N(t+t) = j \mid \text{system is in case m of state i at time t}\}$

$$p_{ij}(\Delta t) \ \Pr\{N(t+\Delta t) = j \mid N(t) = I\} = \sum_{m=1}^{M_i} p_{im} p_{m(i)j}(\Delta t) \qquad (1)$$

$p_{ij}(\Delta t)$ is called the generalized transition probability from state i to state j in time Δt.

If $i \neq j$, assume $pm(i)j(\Delta t) = q_{m(i)j}\Delta t + O(\Delta t)$

Then (1) can be rewritten as,

$$p_{ij}(\Delta t) = q_{ij}\Delta t + O(\Delta t) \text{ for } i \neq j \qquad (2)$$

where, $q_{ij} = \sum_{m=1}^{M_i} p_{im} q_{m(i)j}$ and

$$p_{ii}(\Delta t) = 1 - \sum_{i \neq j} q_{ij}\Delta t + O(\Delta t) = 1 - q_i \Delta t + O(\Delta t) \quad (3)$$

$$\text{where, } q_i = \sum_{i \neq j} q_{ij}$$

In case of equal probability of occurrence of every case of state I,

$p_{im} = \frac{1}{M_i}$, so that the (2) reduces to

$$p_{ij}(\Delta t) = \frac{1}{M_i} \sum_{m=1}^{M_i} \left(q_{m(i)j}\Delta t\right) + O(\Delta t) \quad (4)$$

while (3) is rewritten as

$$p_{ii}(\Delta t) = 1 - \frac{1}{M_i} \sum_{i \neq j} \sum_{m=1}^{M_i} \left(q_{m(i)j}\Delta t\right) + O(\Delta t) \quad (5)$$

Let the state of the system at time t be denoted by N(t). Then,

N(t) =
- 0, if all the components work and the system works at time t
- -1, if one component fails and the system works at time t
- -2, if two components fail and the system works at time t
- .....
- -[2n/3], if [2n/3] components fail and the system still works at time t
- 3, if three components fail and the system fails
- 4, if four components fail and the system fails
- .....
- [2n/3]+1, if [2n/3]+1 components fail and the system fails

Thus, for the model under consideration for n = 8, the set of working states is denoted by W = {0,-1,-2,-3,-4,-5} while the set of failed states is described as F = {3,4,5,6}.

The set of equations is derived using generalized transition probability and the concept of critical component, as follows:

$p0\text{-}1(\Delta t) = 8\lambda(\Delta t) + O(\Delta t)$; (6)

$p00(\Delta t) = 1 - 8\lambda(\Delta t) + O(\Delta t)$; (7)

$p0j(\Delta t) = O(\Delta t)$; $j \neq 0,-1$ (8)

$p\text{-}10(\Delta t) = \mu(\Delta t) + O(\Delta t)$; (9)

$p\text{-}1\text{-}2(\Delta t) = 7\lambda(\Delta t) + O(\Delta t)$; (10)

$p\text{-}1\text{-}1(\Delta t) = 1 - (7\lambda+\mu)(\Delta t) + O(\Delta t)$; (11)

$p\text{-}1j(\Delta t) = O(\Delta t)$; $j \neq 0,-1,-2$ (12)

$p\text{-}i\text{-}(i\text{-}1)(\Delta t) = \mu(\Delta t) + O(\Delta t)$; $i = 2,3,4,5$ (13)

$p\text{-}2\text{-}3(\Delta t) = (\Delta t) + O(\Delta t)$; (14)

$p\text{-}3\text{-}4(\Delta t) = (\Delta t) + O(\Delta t)$; (15)

$p\text{-}4\text{-}5(\Delta t) = (\Delta t) + O(\Delta t)$; (16)

$p\text{-}23(\Delta t) = (\Delta t) + O(\Delta t)$; (17)

$p\text{-}34(\Delta t) = (\Delta t) + O(\Delta t)$; (18)

$p\text{-}45(\Delta t) = (\Delta t) + O(\Delta t)$; (19)

$p\text{-}56(\Delta t) = 3\lambda(\Delta t) + O(\Delta t)$; (20)

$p\text{-}i\text{-}i(\Delta t) = 1 - \{(8\text{-}i)\lambda+\mu\}(\Delta t) + O(\Delta t)$; $i = 2,3,4,5$ (21)

$p\text{-}ij(\Delta t) = O(\Delta t)$; $j \neq i, -(i\text{-}1), -(i+1), (i+1)$ (22)

$pi\text{-}(i\text{-}1)(\Delta t) = \mu(\Delta t) + O(\Delta t)$; $i = 3,4,5,6$ (23)

$pii(\Delta t) = 1 - \mu(\Delta t) + O(\Delta t)$; $i = 3,4,5,6$ (24)

$pij(\Delta t) = O(\Delta t)$; $j \neq -(i\text{-}1), i$ (25)

Equations (6)-(25) can be utilized to obtain the transition rate matrix Q as follows:

$Q = (qij)$, $i,j \in \Omega$

where,

$$q_{ij} = \lim_{\Delta t \to 0} \frac{p_{ij}(\Delta t)}{\Delta t} \quad , \quad i,j \in \Omega \text{ and } i \neq j$$

$$q_{jj} = \lim_{\Delta t \to 0} \frac{1 - p_{jj}(\Delta t)}{\Delta t} \quad , \quad j \in \Omega$$

Thus, the transition rate matrix for the circular consecutive-3-out-of-8: F system is expressed as:

$Q = (q_{ij})$

$$= \begin{bmatrix}
-8\lambda & 8\lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\mu & -(7\lambda+\mu) & 7\lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \mu & -(6\lambda+\mu) & \frac{36\lambda}{7} & 0 & 0 & \frac{6\lambda}{7} & 0 & 0 & 0 \\
0 & 0 & \mu & -(5\lambda+\mu) & \frac{19\lambda}{6} & 0 & 0 & \frac{11\lambda}{6} & 0 & 0 \\
0 & 0 & 0 & \mu & -(4\lambda+\mu) & 20\lambda/19 & 0 & 0 & \frac{56\lambda}{19} & 0 \\
0 & 0 & 0 & 0 & \mu & -(3\lambda+\mu) & 0 & 0 & 0 & 3\lambda \\
0 & 0 & \mu & 0 & 0 & 0 & -\mu & 0 & 0 & 0 \\
0 & 0 & 0 & \mu & 0 & 0 & 0 & -\mu & 0 & 0 \\
0 & 0 & 0 & 0 & \mu & 0 & 0 & 0 & -\mu & 0 \\
0 & 0 & 0 & 0 & 0 & \mu & 0 & 0 & 0 & -\mu
\end{bmatrix} \quad (26)$$

4. PERFORMANCE METRICS

4.1. System Reliability

For determining the system reliability R(t), consider a continuous-time homogeneous Markov process {N(t), t 0} with five absorbing states. It is obvious that the set of failed states in {N(t), t 0} becomes the set of absorbing states in {(t), t 0}.

$$\text{Define, } \bar{p}_j(t) = P\{\bar{N}(t) = j\} \quad , \quad j \in \Omega$$

The system reliability then assumes the form

$$R(t) = \bar{p}_0(t) + \bar{p}_{-1}(t) + \bar{p}_{-2}(t) + \bar{p}_{-3}(t) + \bar{p}_{-4}(t) + \bar{p}_{-5}(t) \quad (27)$$

The following systems of the differential equations can be easily derived:

$$\bar{p}'_w(t) = \bar{p}_w(t).B \quad (28)$$

$$\text{where } \bar{p}_w(t) = (\bar{p}_0(t), \bar{p}_{-1}(t), \bar{p}_{-2}(t), \bar{p}_{-3}(t), \bar{p}_{-4}(t), p_{-5}(t)) \text{ with}$$

$$\bar{p}_w(0) = (1,0,0,0,0,0)$$

and

$$B = \begin{bmatrix}
-8\lambda & 8\lambda & 0 & 0 & 0 & 0 \\
\mu & -(7\lambda+\mu) & 7\lambda & 0 & 0 & 0 \\
0 & \mu & -(6\lambda+\mu) & \frac{36\lambda}{7} & 0 & 0 \\
0 & 0 & \mu & -(5\lambda+\mu) & \frac{19\lambda}{6} & 0 \\
0 & 0 & 0 & \mu & -(4\lambda+\mu) & \frac{20\lambda}{19} \\
0 & 0 & 0 & 0 & \mu & -(3\lambda+\mu)
\end{bmatrix} \quad (29)$$

Laplace transform of equation (28) is obtained using initial conditions as,

$$s\bar{p}^*_0(s) = -8\lambda \bar{p}^*_0(s) + \mu \bar{p}^*_{-1}(s) + 1 \quad , \quad (30)$$

$$s\bar{p}^*_{-1}(s) = 8\lambda \bar{p}^*_0(s) - (7\lambda+\mu)\bar{p}^*_{-1}(s) + \mu \bar{p}^*_{-2}(s) \quad , \quad (31)$$

$$s\bar{p}^*_{-2}(s) = 7\lambda\bar{p}^*_{-1}(s) - (6\lambda+\mu)\bar{p}^*_{-2}(s) + \mu\bar{p}^*_{-3}(s) \quad , \quad (32)$$

$$s\bar{p}^*_{-3}(s) = \frac{36\lambda}{7}\bar{p}^*_{-2}(s) - (5\lambda+\mu)\bar{p}^*_{-3}(s) + \mu\bar{p}^*_{-4}(s) \quad , \quad (33)$$

$$s\bar{p}^*_{-4}(s) = \frac{19\lambda}{6}\bar{p}^*_{-3}(s) - (4\lambda+\mu)\bar{p}^*_{-4}(s) + \mu\bar{p}^*_{-5}(s) \quad , \quad \ldots.(34)$$

$$s\bar{p}^*_{-5}(s) = \frac{20\lambda}{19}\bar{p}^*_{-4}(s) - (3\lambda+\mu)\bar{p}^*_{-5}(s) \quad , \quad \ldots.(35)$$

Equations (30)-(35) can be algebraically simplified to yield,

$$\bar{p}^*_{-5}(s) = \frac{20\lambda/19}{s+3\lambda+\mu}\bar{p}^*_{-4}(s) \quad , \quad \ldots.(36)$$

$$\bar{p}^*_{-3}(s) = \frac{6\left[(s+4\lambda+\mu)(s+3\lambda+\mu) - 20\lambda\mu/19\right]}{19\lambda(s+3\lambda+\mu)}\bar{p}^*_{-4}(s) \quad , \quad \ldots.(37)$$

$$\bar{p}^*_{-2}(s) = \frac{7g_1(s,\lambda,\mu)}{114\lambda^2(s+3\lambda+\mu)}\bar{p}^*_{-4}(s) \quad , \quad \ldots.(38)$$

$$\bar{p}^*_{-1}(s) = \frac{g_2(s,\lambda,\mu)}{798\lambda^3(s+3\lambda+\mu)}\bar{p}^*_{-4}(s) \quad , \quad \ldots.(39)$$

$$\bar{p}^*_{0}(s) = \frac{g_3(s,\lambda,\mu)}{6384\lambda^4(s+3\lambda+\mu)}\bar{p}^*_{-4}(s) \quad , \quad \ldots.(40)$$

$$\bar{p}^*_{-4}(s) = \frac{6384\lambda^4(s+3\lambda+\mu)}{f_1(s,\lambda,\mu)} \quad , \quad \ldots.(41)$$

where,

$$\left.\begin{aligned} g_1(s,\lambda,\mu) &= (s+5\lambda+\mu)(s+4\lambda+\mu)(s+3\lambda+\mu) \\ &\quad - \frac{\lambda\mu}{114}(481s + 1683\lambda + 481\mu) \\ g_2(s,\lambda,\mu) &= 7(s+6\lambda+\mu)g_1(s,\lambda,\mu) - 36\lambda\mu(s+4\lambda+\mu)(s+3\lambda+\mu) \\ &\quad + \frac{720\lambda^2\mu^2}{19} \\ g_3(s,\lambda,\mu) &= (s+7\lambda+\mu)g_2(s,\lambda,\mu) - 49\lambda\mu g_1(s,\lambda,\mu) \\ f_1(s,\lambda,\mu) &= (s+8\lambda)g_3(s,\lambda,\mu) - 8\lambda\mu g_2(s,\lambda,\mu) \end{aligned}\right\} \quad \ldots.(42)$$

The Laplace transform of the system's reliability R(t) is given by

$$\begin{aligned} R^*(s) &= \bar{p}^*_{0}(s) + \bar{p}^*_{-1}(s) + \bar{p}^*_{-2}(s) + \bar{p}^*_{-3}(s) + \bar{p}^*_{-4}(s) + \bar{p}^*_{-5}(s) \\ &= [g_3(s,\lambda,\mu) + 8\lambda g_2(s,\lambda,\mu) + 392\lambda^2 g_1(s,\lambda,\mu) \\ &\quad + 336\lambda^3(s+3\lambda+\mu)(6s+43\lambda+6\mu) - (40320/19)\lambda^4\mu \\ &\quad + 6720\lambda^5] / f_1(s,\lambda,\mu) \end{aligned} \quad \ldots.(43)$$

4. 2 System Mean Time to First Failure (MTTF)

The system mean time to first failure is evaluated using

$$MTTF = 1\lim_{s\to 0} R^*(s) \quad \text{to yield}$$

$$= \frac{12697776\lambda^5 + 5782074\lambda^4\mu + 1129000\lambda^3\mu^2 + 143191\lambda^2\mu^3 + 13277\lambda\mu^4 + 798\mu^5}{16087680\lambda^5 + 5885712\lambda^5\mu + 719376\lambda^4\mu^2 + 38304\lambda^3\mu^3} \quad \ldots.(44)$$

4.3. System Availability

We define

$$p_j(t) = P\{N(t) = j\} \quad , \quad j \in \Omega$$

The Fokker-Planck equation can be obtained as follows:

$$p'_j(t) = \sum_{k\in\Omega} p_k(t)q_{kj} \quad , \quad j \in \Omega \quad \ldots.(45)$$

with the initial conditions given by

$$p_j(0) = \begin{cases} 1, & j = 0 \\ 0, & j \in \Omega \text{ and } j \neq 0 \end{cases}$$

Denote $P(t) = \{p_0(t), p_{-1}(t), p_{-2}(t), p_{-3}(t), p_{-4}(t), p_{-5}(t), p_3(t), p_4(t), p_5(t), p_6(t)\}$
Use the transition rate matrix to replace equation (45) by

$$p'(t) = P(t).Q, \quad \ldots.(46)$$

We derive the Laplace transform of equation (46) as

$$sp^*_0(s) = -8\lambda p^*_0(s) + \mu p^*_{-1}(s) + 1 \quad , \quad \ldots.(47)$$
$$sp^*_{-1}(s) = 8\lambda p^*_0(s) - (7\lambda+\mu)p^*_{-1}(s) + \mu p^*_{-2}(s) \quad , \quad \ldots.(48)$$
$$sp^*_{-2}(s) = 7\lambda p^*_{-1}(s) - (6\lambda+\mu)p^*_{-2}(s) + \mu p^*_{-3}(s) + \mu p^*_3(s), \quad \ldots.(49)$$
$$sp^*_{-3}(s) = p^*_{-2}(s) - (5\lambda+\mu)p^*_{-3}(s) + \mu p^*_{-4}(s) + \mu p^*4(s) \quad \ldots.(50)$$
$$sp^*_{-4}(s) = p^*_{-3}(s) - (4\lambda+\mu)p^*_{-4}(s) + \mu p^*_{-5}(s) + \mu p^*_5(s) \quad \ldots.(51)$$
$$sp^*_{-5}(s) = {}^*_{-4}(s) - (3\lambda+\mu)^*_{-5}(s) + \mu p^*_6(s), \quad \ldots.(52)$$
$$sp^*_3(s) = p^*_{-2}(s) - \mu p^*_3(s) \quad , \quad (53)$$
$$sp^*_4(s) = p^*\text{-}3(s) - \mu p^*4(s) \quad , \quad (54)$$
$$sp^*5(s) = p^*_{-4}(s) - \mu p^*_5(s) \quad , \quad (55)$$
$$sp^*_6(s) = 3\lambda p^*_{-5}(s) - \mu p^*_6(s) \quad , \quad (56)$$

The above Laplace transform equations (47) – (56) are solved to give

$$p^*6(s) = \frac{3\lambda}{(s+\mu)}p^*_{-5}(s), \quad (57)$$

$$p^*_{-4}(s) = \frac{19(s+3\lambda+\mu)(s+\mu) - 57\lambda\mu}{20\lambda(s+\mu)}p^*_{-5}(s) \quad , \quad (58)$$

$$p^*_5(s) = \frac{14(s+3\lambda+\mu)(s+\mu) - 42\lambda\mu}{5(s+\mu)^2}p^*_{-5}(s), \quad (59)$$

$$p^*_{-3}(s) = \frac{g_4(s,\lambda,\mu)}{380\lambda^2(s+\mu)^2}p^*_{-5}(s) \quad , \quad (60)$$

$$p^*4(s) = \frac{11g_4(s,\lambda,\mu)}{2280\lambda(s+\mu)^3}p^*_{-5}(s) \quad , \quad (61)$$

$$p^*_{-2}(s) = \frac{7f_2(s,\lambda,\mu)}{82080\lambda^3(s+\mu)^3}p^*_{-5}(s) \quad , \quad (62)$$

$$p^*0(s) = \frac{(s+7\lambda+\mu)f_3(s,\lambda,\mu) - 49\lambda\mu(s+\mu)f_2(s,\lambda,\mu)}{4596480\lambda^5(s+\mu)^4}p^*_{-5}(s) \quad , \quad (65)$$

$$p^*_{-5}(s) = \frac{4596480\lambda^5(s+\mu)^4}{[(s+8\lambda)(s+7\lambda+\mu) - 8\lambda\mu]f_3(s,\lambda,\mu) - 49\lambda\mu(s+\mu)(s+8\lambda)f_2(s,\lambda,\mu)} \quad , \quad (66)$$

The system availability at time t is defined as

$$A(t) = p_0(t) + p_{-1}(t) + p_{-2}(t) + p_{-3}(t) + p_{-4}(t) + p_{-5}(t) \quad (67)$$

Laplace transform of A(t) yields

$$A^*(s) = p^*0(s) + p^*_{-1}(s) + p^*_{-2}(s) + p^*_{-3}(s) + p^*_{-4}(s) + p^*_{-5}(s)$$

$$= \frac{\left[\begin{aligned}&(s+15\lambda+\mu)f_3(s,\lambda,\mu) + 49\lambda(s+\mu)(8\lambda-\mu)f_2(s,\lambda,\mu)\\ &+ 12096\lambda^3(s+\mu)^2 g_4(s,\lambda,\mu) + g_5(s,\lambda,\mu)\end{aligned}\right]}{[(s+8\lambda)(s+7\lambda+\mu) - 8\lambda\mu]f_3(s,\lambda,\mu) - 49\lambda\mu(s+\mu)(s+8\lambda)f_2(s,\lambda,\mu)} \quad (68)$$

where

$$g_4(s,\lambda,\mu) = 114(s+4\lambda+\mu)(s+3\lambda+\mu)(s+\mu)^2 - 342\lambda\mu(s+4\lambda+\mu)(s+\mu) - 336\lambda\mu(s+3\lambda+\mu)(s+\mu) + 1008\lambda^2\mu^2 - 120\lambda\mu(s+\mu)^2$$

$$f_2(s,\lambda,\mu) = [6(s+\mu)(s+5\lambda+\mu) - 11\lambda\mu]g_4(s,\lambda,\mu) - 114\lambda(s+\mu)^2[19\mu(s+3\lambda+\mu)(s+\mu) - 57\lambda\mu^2]$$

$$f_3(s,\lambda,\mu) = [7(s+\mu)(s+6\lambda+\mu) - 6\lambda\mu]f_2(s,\lambda,\mu) - 216\lambda\mu(s+\mu)^2 g_4(s,\lambda,\mu)$$

$$g_5(s,\lambda,\mu) = 229824\lambda^4(s+\mu)^3[19(s+3\lambda+\mu)(s+\mu) - 37\lambda\mu + 20\lambda s]$$

(69)

The steady-state availability of the system is derived using Tauberian theorem and L'Hospital's rule as

$$A = \lim_{t\to\infty} A(t) = \lim_{s\to 0} sA^*(s)$$

$$= \frac{4788\mu^9 + 38304\mu^8\lambda + 268128\mu^7\lambda^2 + 1378944\mu^6\lambda^3 + 4366656\mu^5\lambda^4 + 4596480\mu^4\lambda^5}{[\,4788\mu^9 + 38304\mu^8\lambda + 269248\mu^7\lambda^2 + 1608768\mu^6\lambda^3 + 6894720\mu^5\lambda^4 + 17466624\mu^4\lambda^5 + 13789440\mu^3\lambda^6\,]}$$

(70)

5. NUMERICAL RESULTS

In this section, we present numerical results to illustrate how the system availability and mean time to failure (MTTF) are affected by varying traffic intensity .

We observe from figure 1 that the system mean time to failure (MTTF) decreases with the increase in . The decreasing trend is sharper for lower values of and then attains the asymptotic limit. Figure .2 depicts variation of steady state availability against the traffic intensity. It is clearly illustrated that the machine availability decreases with corresponding increase in .

Overall, it can be concluded that both the system availability and mean time to failure decrease as traffic intensity increases. This is also in accordance with the physical situations.

6.CONCLUDING REMARKS

In this paper, a circular consecutive-3-out-of-8: F repairable system with single repairman is studied. Maintenance is initiated when the number of failed components exceeds the critical level. By introducing the concept of generalized transition probability and critical component, explicit expressions for Mi are obtained. The equations for performability measures such as reliability and availability are obtained. Numerical results are employed to provide insight on the impact of traffic intensity upon the MTTF and system availability. It is hoped that the work done would provide greater theoretical meaning to the consecutive-k-out-of-n: F systems.

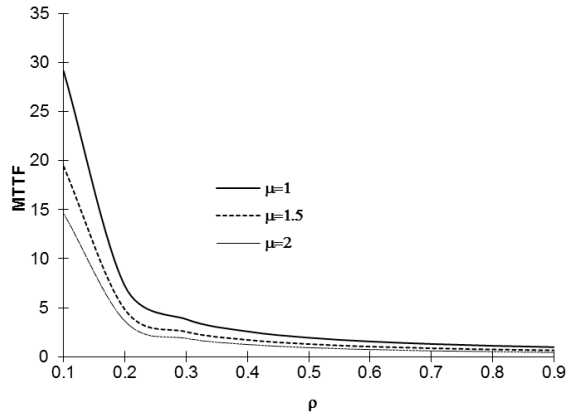


Fig. 1: Mean time to failure(MTTF) vs. traffic intensity

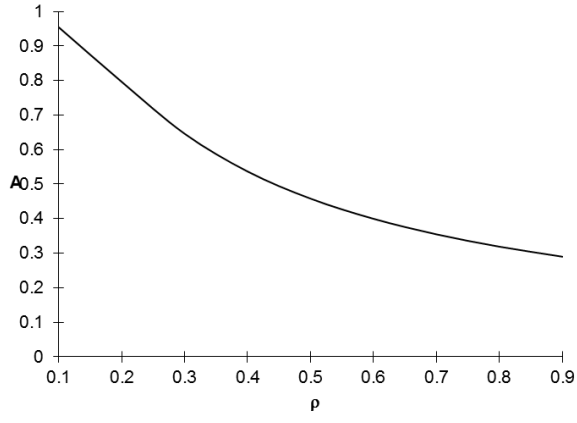


Fig. 2: Steady state availability(A) vs. traffic intensity

REFERENCE

1) Chiang, D.T. and Niu, S.C. (1981) : Reliability of a consecutive-k-out-of-n: F system, IEEE Trans. Reliab., Vol. 30, pp. 87-89. | 2) Derman C., Lieberman, G.J., and Ross, S.M. (1982) : On the consecutive-k-out-of-n: F system, IEEE Trans. Reliab., Vol. 31, pp. 57-63. | 3) Du, D.Z. and Hwang, F.K. (1986) : Optimal consecutive-2-out-of-n: F system, Math. Oper. Res., Vol. 11, pp. 187-191. | 4) Flynn, J. and Chung, C.S. (2004) : A heuristic algorithm for determining replacement policies in consecutive k-out-of-n system, Comput. Oper. Res., Vol. 31(8), pp. 1335-1348. | 5) Ge, G.P. and Wang, L.S. (1990) : Exact reliability formula for consecutive-k-out-of-n: F system with homogeneous Markov dependence, IEEE Trans. Reliab., Vol. 39, pp. 600-602. | 6) Krishnan, R. & Somasundaram, S. (2011): Reliability analysis of repairable consecutive-k-out-of-n: G systems with sensor and repairmen", International Journal of Quality & Reliability Management, Vol. 28 Iss: 8, pp.894 – 908. | 7) Malon, D.M. (1984) : Optimal consecutive-2-out-of-n: F component sequencing, IEEE Trans. Reliab., Vol. 33, pp. 414-418. | 8) Yam, R.C.M., Zuo, M.J. and Zhang, Y.L. (2003) : A method for evaluation of reliability indices for repairable circular consecutive-k-out-of-n: F systems, Reliab. Engg. Syst. Safety, Vol. 79(1), pp. 1-9. | 9) Zhang, Y.L., Zuo, M.J. and Yam, R.C.M. (2000) : Reliability analysis for a circular consecutive-2-out-of-n: F repairable system with priority in repair, Reliab. Engg. Syst. Safety, Vol. 68, pp. 113-120. | 10) Zuo, M. and Wu, Y. (1996) : Reliability evaluation of a furnace system using the k-out-of-n and the reliability models, Proceedings of the IEEE International Conference on Systems, Man, and Cybernetics, Beijing, pp. 3119-3123.