# TSALLI'S ENTROPY MEASURE UNDER INTUITIONISTIC FUZZY ENVIRONMENT: A HYPOTHETICAL CASE STUDY FOR THE SELECTION OF PROPERTY

Mathematics

**KEYWORDS:** Intuitionistic fuzzy sets, Fuzzy relations, Tsalli's Entropy, Real estate companies.

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## ABSTRACT

The real estate companies are offering various solutions to customers for the purchase of house. For a genuine buyer, it is mandatory to take best possible decision from the available set of options. The present work provides the buyer a decision making method for the selection of house as per their need and desire. For this purpose, we develop a hypothetical case study based on the information about customers, available real estate projects and their factors, which affect the purchase of the house with assigned degree of membership, non- membership and hesitation index. Finally, we use Tsallis entropy measure under intuitionistic fuzzy environment for the better decision making.

## 1. INTRODUCTION

Everyone wants a house of their dreams and is indeed a fundamental need of a human being. Thus, the selection of a house by an individual has become a complex and highly tedious task and therefore requires decision making. The requirements of the customers are changing variably along with the pace of the technology. Now a days, the real estate companies provide a wide range of options to attract customers, both in budget as well as luxury segment houses. While selecting a house, many questions arises, but a common man's selection for a particular house is based on his need. These questions are usually vague but practically when a customer opts for a house, the key factors for that are: Budget, Carpet Area, Basic Amenities, Quality of Construction, Builder's Reputation etc. The projected cost of the house may depend on what kind of house (budget or luxury) actually a customer is going to purchase. People need a house having maximum carpet area. This feature adds great impact on the selection of the house but the actual need of the area is based on the one's minimum required area. Basic Amenities are the need of the day. It comprises parking facility, safety, club facility, power backup etc. and is the one of the selling feature of the project. Nobody wants a house which requires immediate repair, thus quality of construction is prime while selecting a house. Learned costumers also looks upon the reputation of the builder such as timely delivery of their previous projects, quality delivered etc. before making a purchase. Zadeh[1965] introduced the concept of Fuzzy sets to deal with uncertainty and vagueness and is an efficient tool in the process of decision making. Attanasov[1986] introduced the concept of IFS, which is the generalization of fuzzy sets. Gupta et. al. [2014] has applied Sanchez's method and Shannon's entropy for the fuzzy diagnosis procedure of the types of TB. Adlassnig[1986], described applications of fuzzy theory to medical decision making. Szmidt[2003] described the concept of distances and other measures of generalized fuzzy theory and proposed various decision making methods. Shan(2007) explained the method of fuzzy distance by trapezoidal fuzzy numbers for the selection of house. In this paper, we propose a mathematical model for the selection of house by using Generalized fuzzy information theory measure of Tsallis. The present study considers only the mid-segment houses(less than 40 lacks.) which is generally purchased by a common man.

### 1.1 Intuitionistic Fuzzy Sets(IFS)

Atanassov [1986], For a fixed set $X$, an IFS of $A$ is defined as : $A=\{<x,\mu A(x),\nu A(x)>|x\in X\}$,where $\mu A(x)X :\rightarrow[0,1]$ and $\nu A(x)$: $X\rightarrow$ [0,1] define the degree of membership and degree of non-membership of the element $x\in X$ to the set $A$. For every $x\in X$, $0\leq \mu A(x)+\nu A(x)\leq 1$ and the amount $\pi A(x)=1-\mu A(x)-\nu A(x)$ is called the intuitionistic index or hesitation index, which may require to membership value, non-membership value or both.

### 1.2 Intuitionistic Fuzzy Relations (IFRs).

Biswas[1997], If $A$ is an IFS of $X$, the max-min composition of the IFR $R(X\rightarrow Y)$ with is an IFS $B$ of $Y$ denoted by $B=R\circ A$, and is denoted by the membership function $\mu_B(y)=\max_{x\in X}\{\min[\mu_A(x),\mu_R(x,y)]\}\ \forall\ y\in Y$ and the non-membership function $\nu_B(y)=\min_{x\in X}\{\max[\nu_A(x),\nu_R(x,y)]\},\ \forall\ y\in Y$

Let $F=\{f_1,f_2,.....f_m\}$; $B=\{b_1,b_2,.....b_n\}$ and $C=\{c_1,c_2,.....c_p\}$be the finite set of factors, Real Estate companies and customers respectively.

According to Adlassnig [1986] , two fuzzy relations(FR), $Q$ and $R$ are defined as:

$$Q=\left\{<(c,f),\mu_Q(c,f),\nu_Q(c,f)>|(c,f)\in C\times F\right\}$$

$$R=\left\{<(f,b),\mu_R(f,b),\nu_R(f,b)>|(f,b)\in F\times B\right\}$$

where, $\mu_Q(c,f)$ indicate the degree to which the factor $f$ affects the customer $c$ and $\nu_Q(c,f)$ indicate the degree to which the factor does not appears in customer whereas, $\mu_R(f,b)$ indicate the degree to which the factor confirm the brand $b$ and $\nu_R(f,b)$ indicate the degree to which the factor does not confirms the brand .

The composition $T$ of IFRs $R$ and $Q(T=R\circ Q)$ is given as:

$$\mu_T(c_i,b)=\max_{s\in S}\left\{\min[\mu_Q(c_i,b),\mu_R(f,b)]\right\}$$

and

$$\nu_T(c_i,b)=\min_{s\in S}\left\{\max[\nu_Q(c_i,f),\nu_R(f,b)]\right\},$$

$$\forall\ c_i\in C\ and\ b\in B\ .$$

### 1.3 Tsallis Measure of Entropy.

Tsallis [1988] introduced the entropy based on non-extensive generalization of Boltzmann-Shannon-Gibbs theory. For a random variable $X$with probability mass function $p=(p_1,p_2,.....p_n)$, the Tsalllis measure of entropy is defined as:

$$H_\alpha^{(T)}(p)=\frac{1}{1-\alpha}\left[\sum_{i=1}^{N}(p_i)^\alpha-1\right]$$

where, $0>\alpha$ and is called as non-extensive index.

In the limiting case, when $\alpha \to 1$, the measure reduces to Shannon's measure of entropy.

Thus, Intuitionistic fuzzy Tsallis measure of entropy is defined as:

$$H_{\alpha}^{(T)}(p) = \frac{1}{1-\alpha}\left[\sum_{i=1}^{N}[(\mu_i)^{\alpha} + (\nu_i)^{\alpha} + (\pi_i)^{\alpha}] - 1\right], \alpha > 0$$

**2. METHODOLOGY.**

In order to accomplish the objectives of the study for the selection of suitable house as per the given facts. Firstly, we establish IF relation between customers and factors that affect the selection for the purchase of house with assigned degree of membership and non membership elements as explained in table(1.1). Secondly we establish IF relation between factors and the available projects with assigned degree of membership and non membership as explained in table (1.2). By taking information about different available projects, we apply Max-Min composition and Min-Max composition of IF relations as explained in table(1.3). Finally, we used Tsalli's entropy under IF environment for different values of the parameter(α) as explained in tables(1.4.1-1.4.8). The minimum weight for each customer from the possible analysis has been taken. If there is a tie in weight elements, then the customer may opt for both types of houses. This information plays a significant role in the process of decision making.

**Case Study.**

To see the application of the method, let us frame a hypothetical case study of factors characterize for each customer and the type of house for each customer as:

Let F={Budget, Carpet Area, Basic Amenities, Quality of construction, Builder's reputation}; C={ $c_1$, $c_2$ ,$c_3$ $c_4$} and B= { Ansal Buildtech, Omaxe ltd., ATS Builders, Unitech, Tata Housing } be the finite set of factors, customers and brands respectively.

Suppose the intuitionistic fuzzy relation (IFR) $Q(C \to F)$ is given by (hypothetically):

**Table(1.1)**

| $Q$ | $f_1$ | | $f_2$ | | $f_3$ | | $f_4$ | | $f_5$ | |
|---|---|---|---|---|---|---|---|---|---|---|
| | $\mu$ | $\nu$ | $\mu$ | $\nu$ | $\mu$ | $\nu$ | $\mu$ | $\nu$ | $\mu$ | $\nu$ |
| $c_1$ | 0.7 | 0.2 | 0.6 | 0 | 0.3 | 0.7 | 0.5 | 0.2 | 0 | 0.6 |
| $c_2$ | 0 | 0.6 | 0.5 | 0.3 | 0.6 | 0.1 | 0 | 0.6 | 0 | 0.9 |
| $c_3$ | 0.5 | 0.2 | 0.7 | 0.2 | 0 | 0.5 | 0.2 | 0.3 | 0.3 | 0.4 |
| $c_4$ | 0.4 | 0 | 0.3 | 0.5 | 0.2 | 0.3 | 0.8 | 0.1 | 0.5 | 0.3 |

**Table (1.2)**

| $R$ | $b_1$ | | $b_2$ | | $b_3$ | | $b_4$ | | $b_5$ | |
|---|---|---|---|---|---|---|---|---|---|---|
| | $\mu$ | $\nu$ | $\mu$ | $\nu$ | $\mu$ | $\nu$ | $\mu$ | $\nu$ | $\mu$ | $\nu$ |
| $f_1$ | 0.7 | 0.1 | 0.3 | 0.1 | 0.4 | 0.4 | 0.2 | 0.6 | 0.2 | 0.7 |
| $f_2$ | 0.4 | 0.5 | 0.8 | 0 | 0.5 | 0.3 | 0.3 | 0.3 | 0 | 0.6 |
| $f_3$ | 0.1 | 0.5 | 0.2 | 0.7 | 0.6 | 0.2 | 0.5 | 0.3 | 0.2 | 0.4 |
| $f_4$ | 0.3 | 0.2 | 0.5 | 0.3 | 0.6 | 0.1 | 0.7 | 0.2 | 0.4 | 0.3 |
| $f_5$ | 0.2 | 0.5 | 0.1 | 0.6 | 0.2 | 0.7 | 0.3 | 0.6 | 0.9 | 0 |

**Table (1.3)**

| $T$ | $b_1$ | | $b_2$ | | $b_3$ | | $b_4$ | | $b_5$ | |
|---|---|---|---|---|---|---|---|---|---|---|
| | $\mu$ | $\nu$ | $\mu$ | $\nu$ | $\mu$ | $\nu$ | $\mu$ | $\nu$ | $\mu$ | $\nu$ |
| $c_1$ | 0.7 | 0.2 | 0.6 | 0 | 0.5 | 0.2 | 0.5 | 0.2 | 0.4 | 0.3 |
| $c_2$ | 0.4 | 0.5 | 0.5 | 0.3 | 0.6 | 0.2 | 0.5 | 0.3 | 0.2 | 0.4 |
| $c_3$ | 0.5 | 0.2 | 0.7 | 0.2 | 0.5 | 0.3 | 0.3 | 0.3 | 0.3 | 0.3 |
| $c_4$ | 0.4 | 0.1 | 0.5 | 0.1 | 0.6 | 0.1 | 0.7 | 0.2 | 0.5 | 0.3 |

**Table (1.4.1) when α=0.1**

| $T$ | $b_1$ | $b_2$ | $b_3$ | $b_4$ | $b_5$ |
|---|---|---|---|---|---|
| $c_1$ | 1.7896 | 0.9585 | 1.8566 | 1.8566 | 1.8729 |
| $c_2$ | 1.8220 | 1.8566 | 1.8365 | 1.8566 | 1.8625 |
| $c_3$ | 1.8566 | 1.7896 | 1.8566 | 1.8729 | 1.8729 |
| $c_4$ | 1.8220 | 1.8220 | 1.8123 | 1.7896 | 1.8566 |

**Table (1.4.2) when α=0.3**

| $T$ | $b_1$ | $b_2$ | $b_3$ | $b_4$ | $b_5$ |
|---|---|---|---|---|---|
| $c_1$ | 1.4525 | 0.8822 | 1.6088 | 1.6088 | 1.6476 |
| $c_2$ | 1.5330 | 1.6088 | 1.5600 | 1.6088 | 1.6234 |
| $c_3$ | 1.6088 | 1.4525 | 1.6088 | 1.6476 | 1.6476 |
| $c_4$ | 1.5330 | 1.5330 | 1.5085 | 1.4525 | 1.6088 |

**Table (1.4.3) when α=0.5**

| $T$ | $b_1$ | $b_2$ | $b_3$ | $b_4$ | $b_5$ |
|---|---|---|---|---|---|
| $c_1$ | 1.2002 | 0.8141 | 1.4041 | 1.4041 | 1.4558 |
| $c_2$ | 1.3116 | 1.4041 | 1.3380 | 1.4041 | 1.4242 |
| $c_3$ | 1.4041 | 1.2002 | 1.4041 | 1.4558 | 1.4558 |
| $c_4$ | 1.3116 | 1.3116 | 1.2771 | 1.2002 | 1.4041 |

**Table (1.4.4) when α=0.7**

| $T$ | $b_1$ | $b_2$ | $b_3$ | $b_4$ | $b_5$ |
|---|---|---|---|---|---|
| $c_1$ | 1.0090 | 0.7531 | 1.2341 | 1.2341 | 1.2919 |
| $c_2$ | 1.1388 | 1.2341 | 1.1588 | 1.2341 | 1.2575 |
| $c_3$ | 1.2341 | 1.0090 | 1.2341 | 1.2919 | 1.2919 |
| $c_4$ | 1.1388 | 1.1388 | 1.0980 | 1.0090 | 1.2341 |

**Table (1.4.5) when α=0.9**

| $T$ | $b_1$ | $b_2$ | $b_3$ | $b_4$ | $b_5$ |
|---|---|---|---|---|---|
| $c_1$ | 0.8623 | 0.6983 | 1.0919 | 1.0919 | 1.1515 |
| $c_2$ | 1.0016 | 1.0919 | 1.0129 | 1.0919 | 1.1169 |
| $c_3$ | 1.0919 | 0.8623 | 1.0919 | 1.1515 | 1.1515 |
| $c_4$ | 1.0016 | 1.0016 | 0.9572 | 0.8623 | 1.0919 |

**Table (1.4.6) when α=1.1**

| $T$ | $b_1$ | $b_2$ | $b_3$ | $b_4$ | $b_5$ |
|---|---|---|---|---|---|
| $c_1$ | 0.7483 | 0.6490 | 0.9725 | 0.9725 | 1.0308 |
| $c_2$ | 0.8907 | 0.9725 | 0.8934 | 0.9725 | 0.9978 |
| $c_3$ | 0.9725 | 0.7483 | 0.9725 | 1.0308 | 1.0308 |
| $c_4$ | 0.8907 | 0.8907 | 0.8448 | 0.7483 | 0.9725 |

**Table (1.4.7) when α=1.3**

| $T$ | $b_1$ | $b_2$ | $b_3$ | $b_4$ | $b_5$ |
|---|---|---|---|---|---|
| $c_1$ | 0.6584 | 0.6046 | 0.8714 | 0.8714 | 0.9268 |
| $c_2$ | 0.7996 | 0.8714 | 0.7948 | 0.8714 | 0.8962 |
| $c_3$ | 0.8714 | 0.6584 | 0.8714 | 0.9268 | 0.9268 |
| $c_4$ | 0.7996 | 0.7996 | 0.7536 | 0.6584 | 0.8714 |

**Table (1.4.8) when α=1.5**

| $T$ | $b_1$ | $b_2$ | $b_3$ | $b_4$ | $b_5$ |
|---|---|---|---|---|---|
| $c_1$ | 0.5865 | 0.5645 | 0.7854 | 0.7854 | 0.8368 |
| $c_2$ | 0.7237 | 0.7854 | 0.7127 | 0.7854 | 0.8092 |
| $c_3$ | 0.7854 | 0.5865 | 0.7854 | 0.8368 | 0.8368 |
| $c_4$ | 0.7237 | 0.7237 | 0.6786 | 0.5865 | 0.7854 |

**Conclusion.**

From the tables, it is clear that the customers $c_1$ and $c_3$can purchase a house build by the builder $b_2$. Customer $c_2$can purchase a house build by the builder $b_3$ and customer $c_4$can purchase a house build by the builder $b_4$.

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